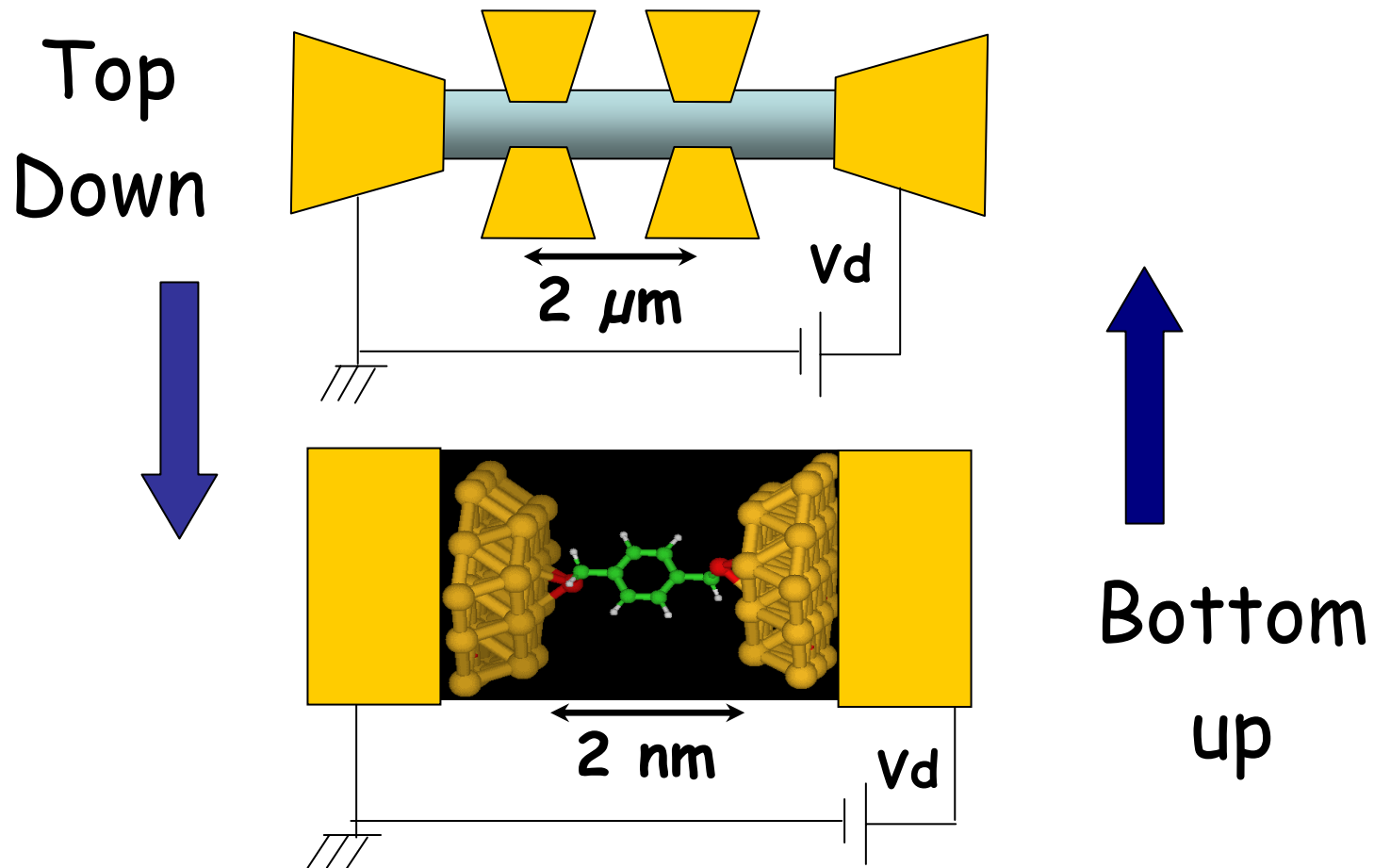




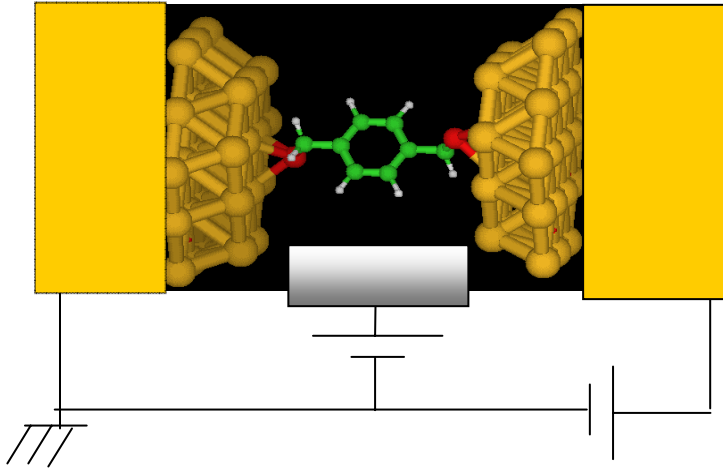
Understanding Nanoscale Conduction

ISLPED'04





Outline

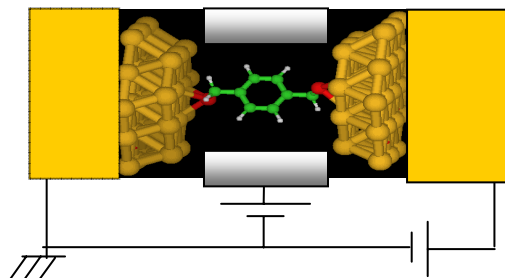


- Qualitative picture
- Quantitative models
 - Examples
- Coulomb blockade
 - Summary/Open questions

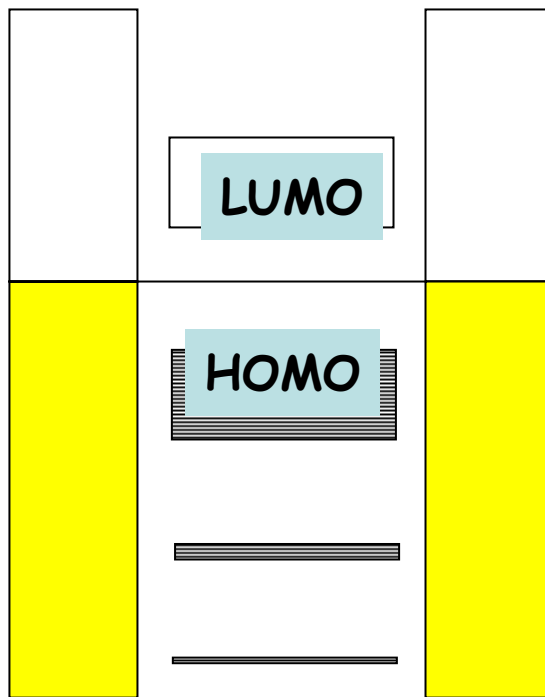
INAC



"Band-Diagram"



Vacuum
Level



E

μ

f

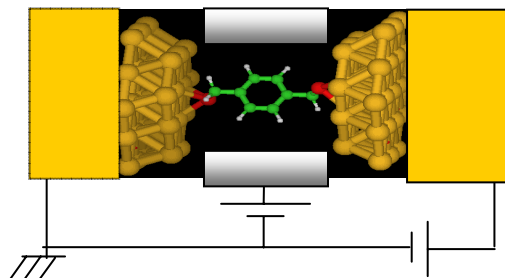
Fermi
function

Electrochemical
Potential

INAC



"Band-Diagram"



n-type

Vacuum

Level

$V_G > 0$

E

μ

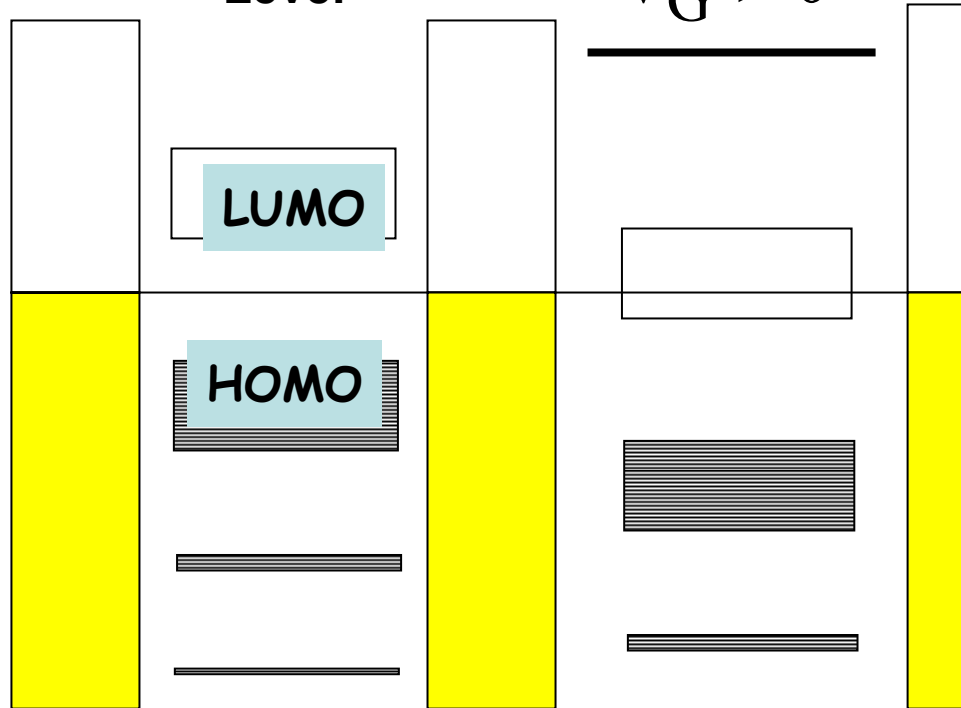
f

LUMO

HOMO

Fermi
function

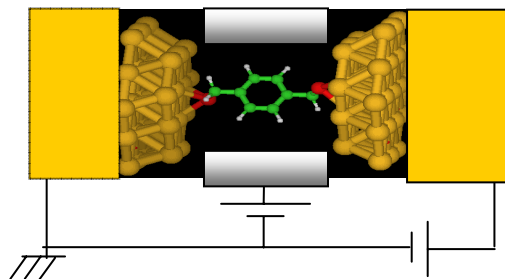
Electrochemical
Potential



INAC



"Band-Diagram"



n-type

$$V_G < 0$$

Vacuum
Level

$$V_G > 0$$

E

LUMO

HOMO

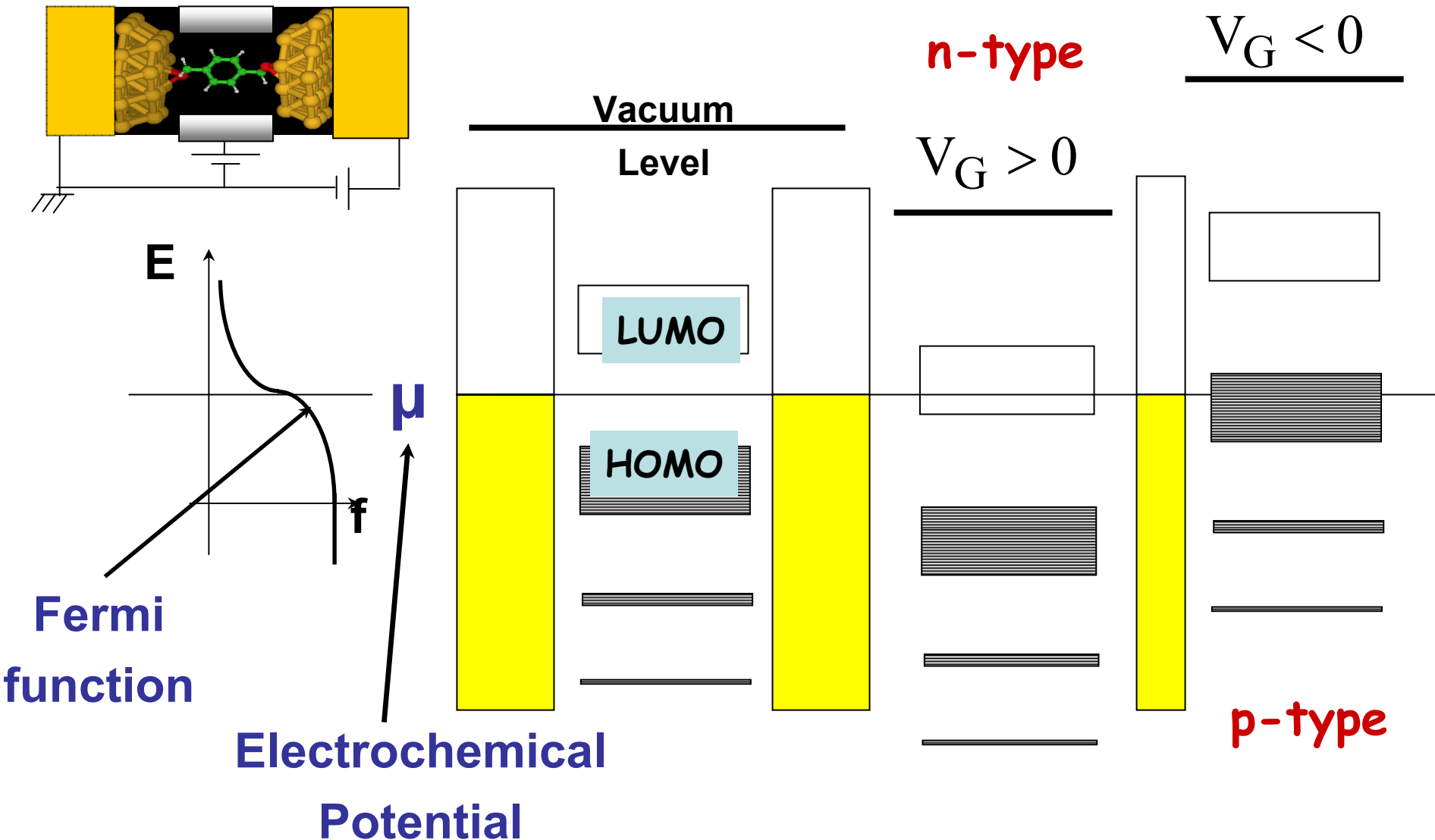
Fermi
function

μ

f

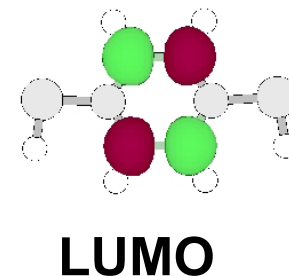
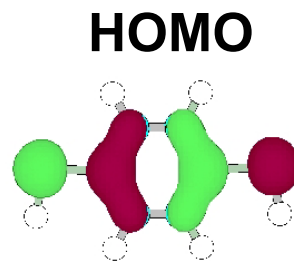
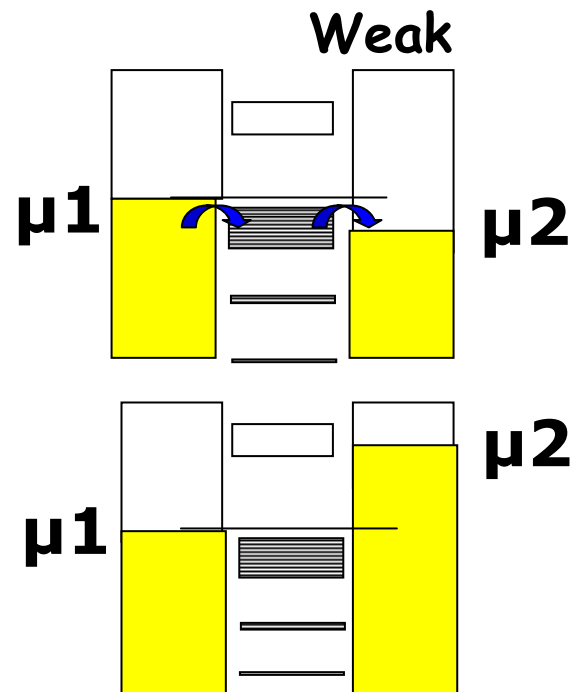
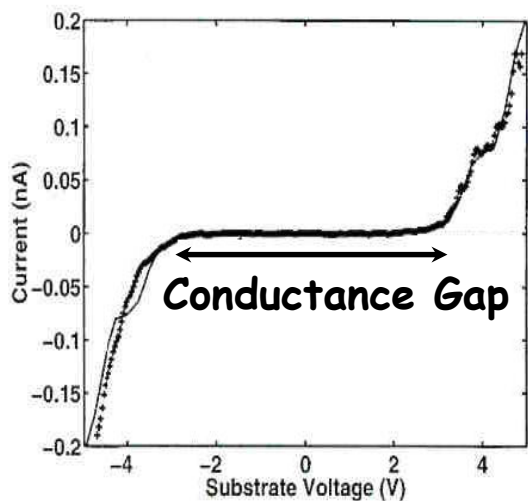
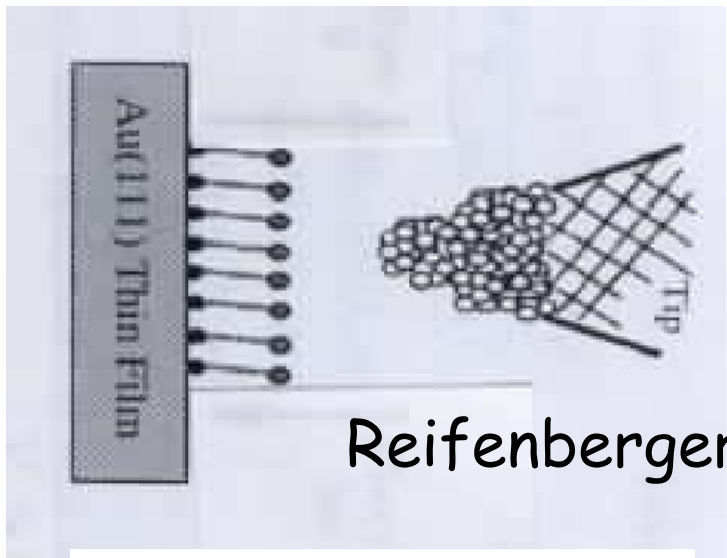
Electrochemical
Potential

p-type



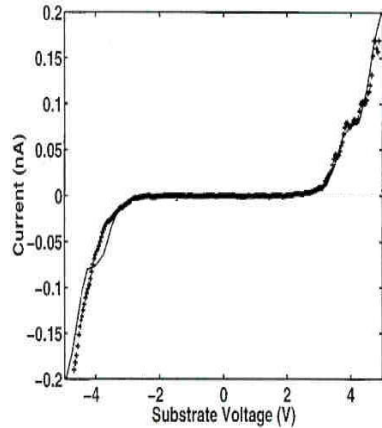


Conductance Gap = ???





Conductance Gap = ???



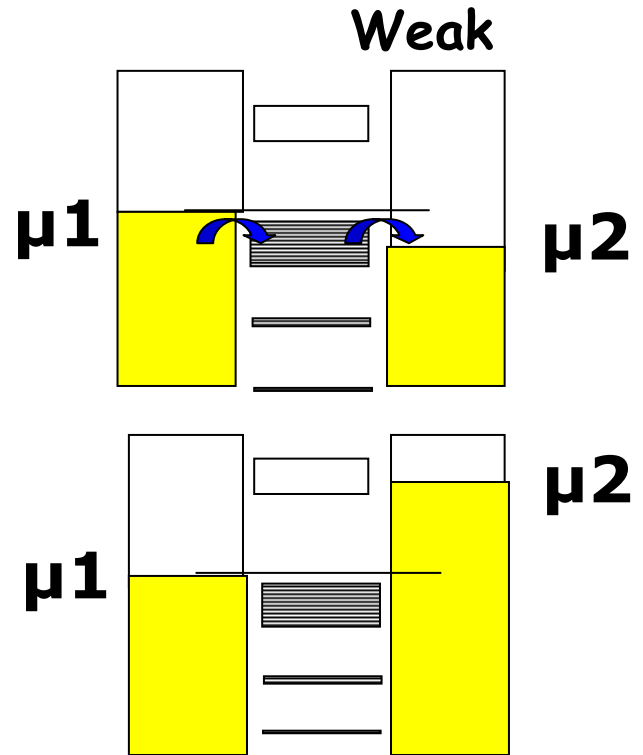
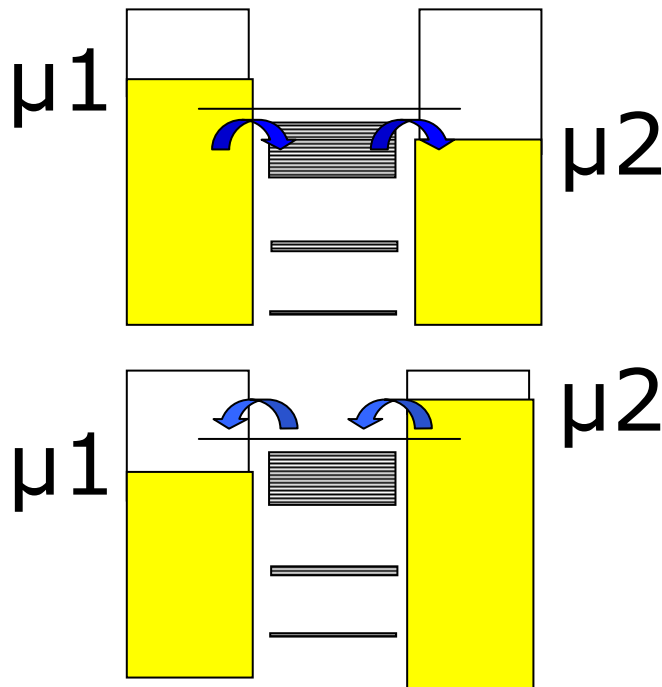
PRL 1997

Levels move

Levels fixed

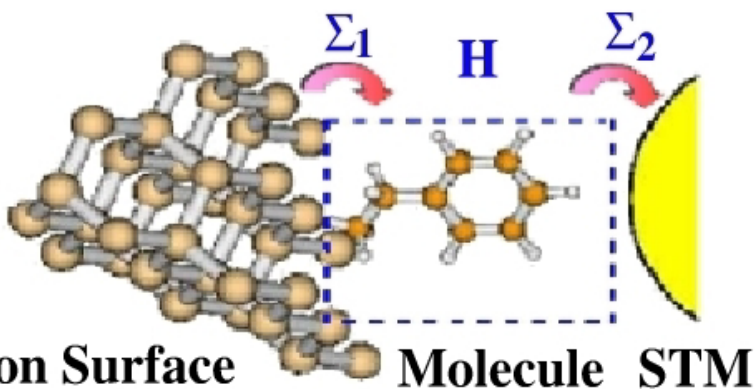
$4 * (E_f - \text{HOMO})$

LUMO - HOMO

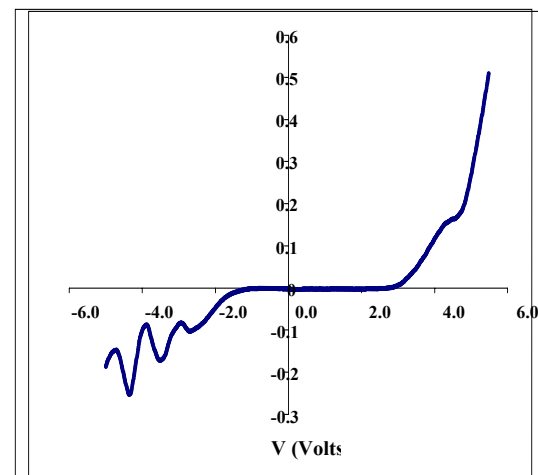
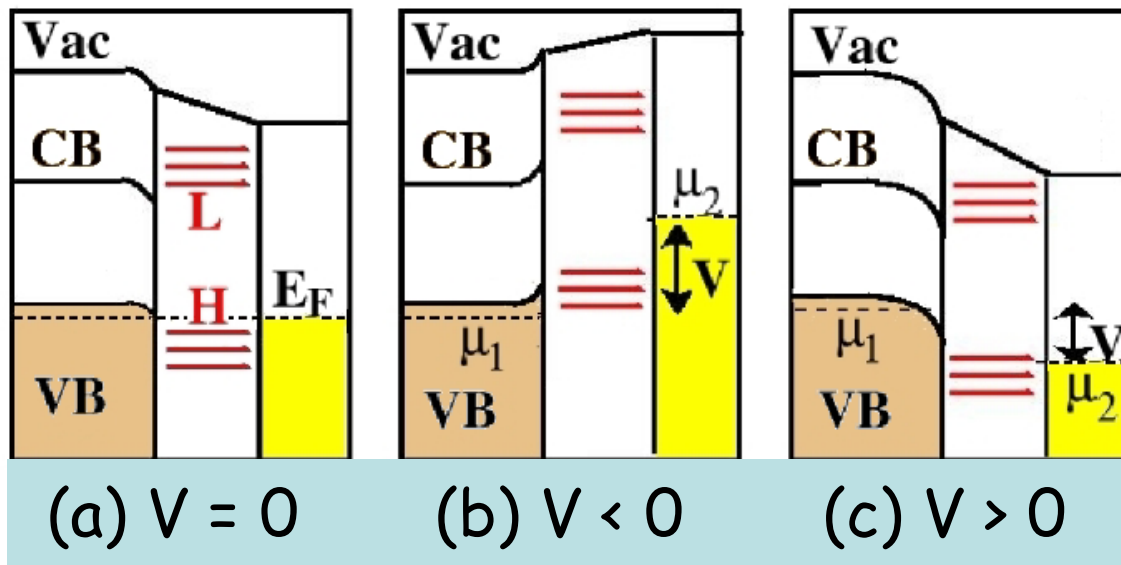




Molecular devices on silicon



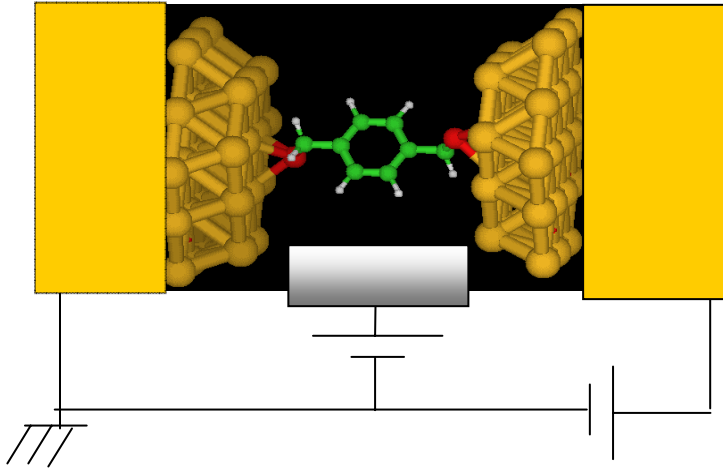
Expt: Mark Hersam
 Nanoletters, 01/04
 Cover story



Room temperature



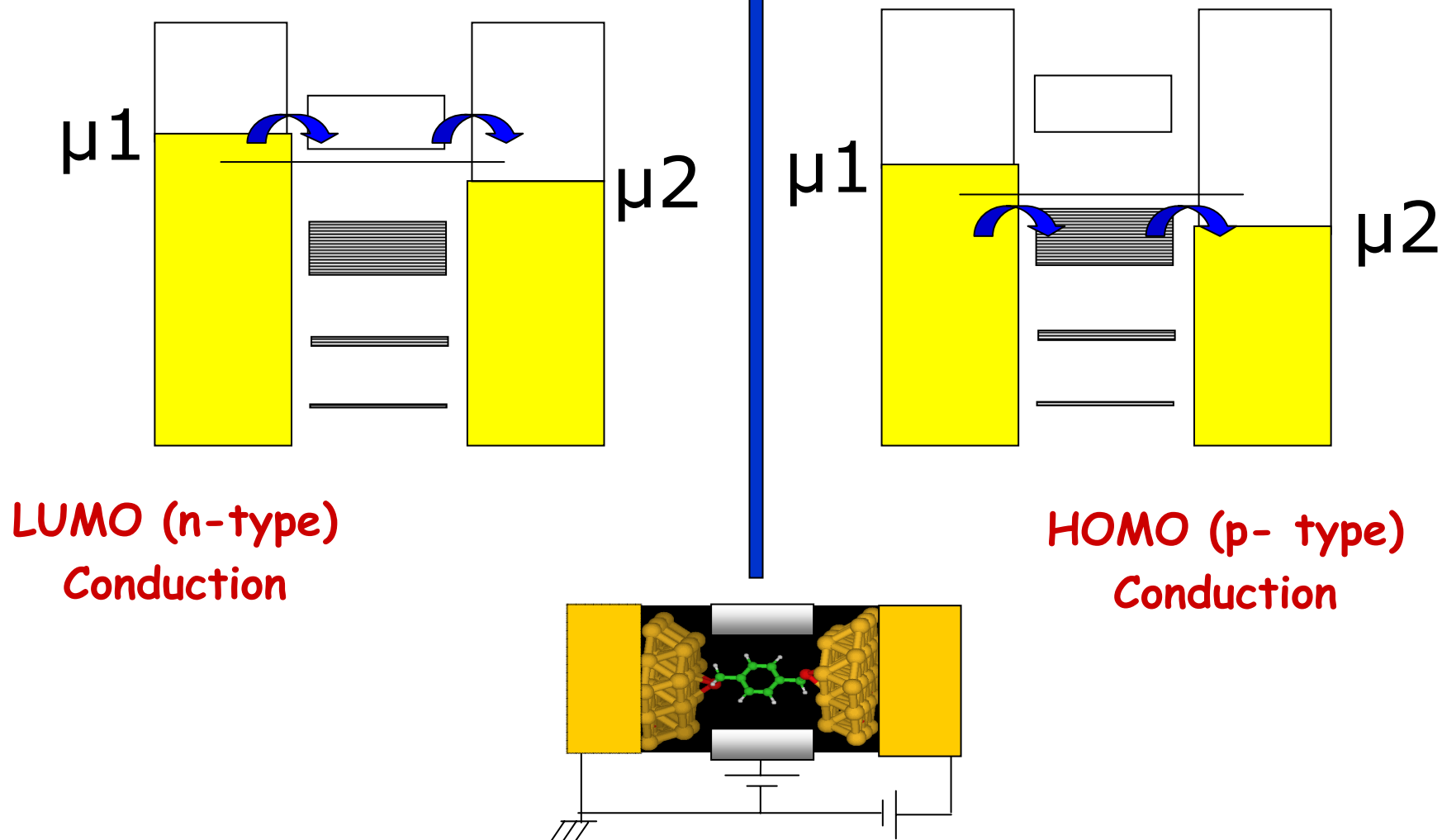
Outline



- Qualitative picture
- Quantitative models
 - Examples
- Coulomb blockade
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What makes electrons flow?

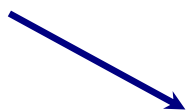




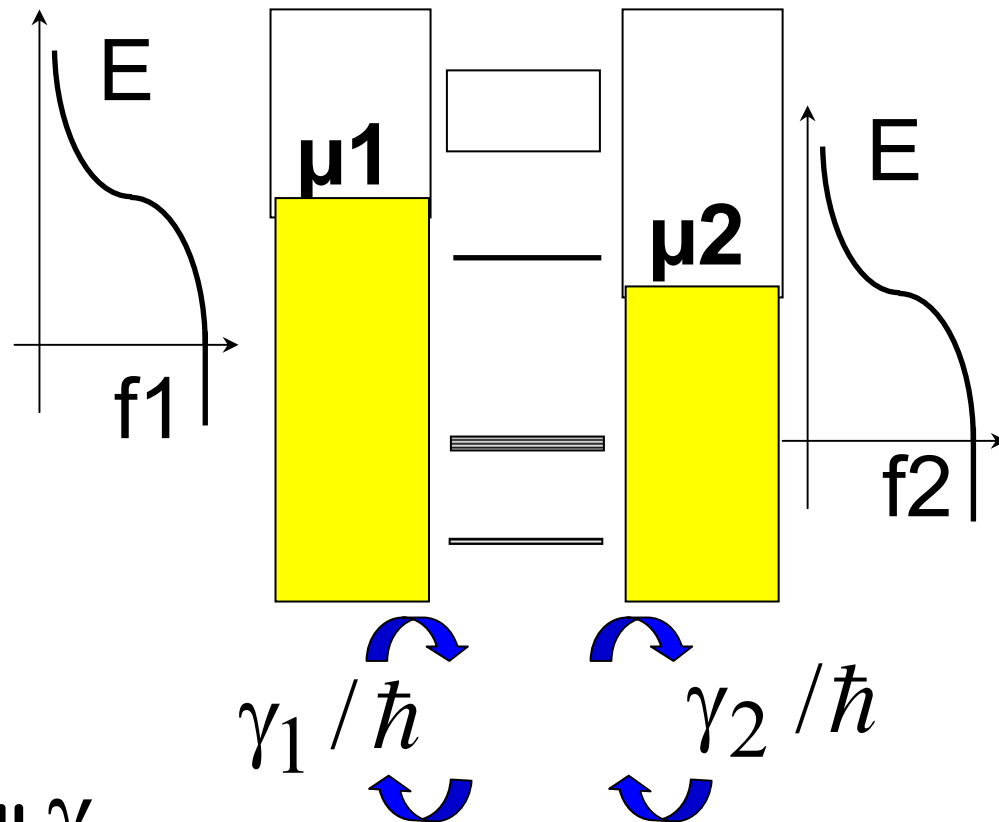
Toy model: Escape time

$$I_1 = q \frac{\gamma_1}{\hbar} [f_1 - N]$$

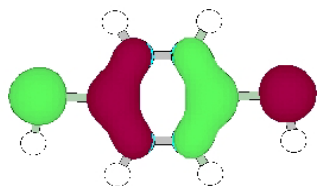
$$I_2 = q \frac{\gamma_2}{\hbar} [N - f_2]$$



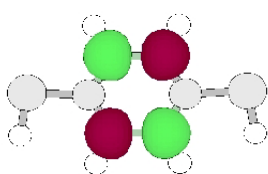
$$I = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$



Large γ

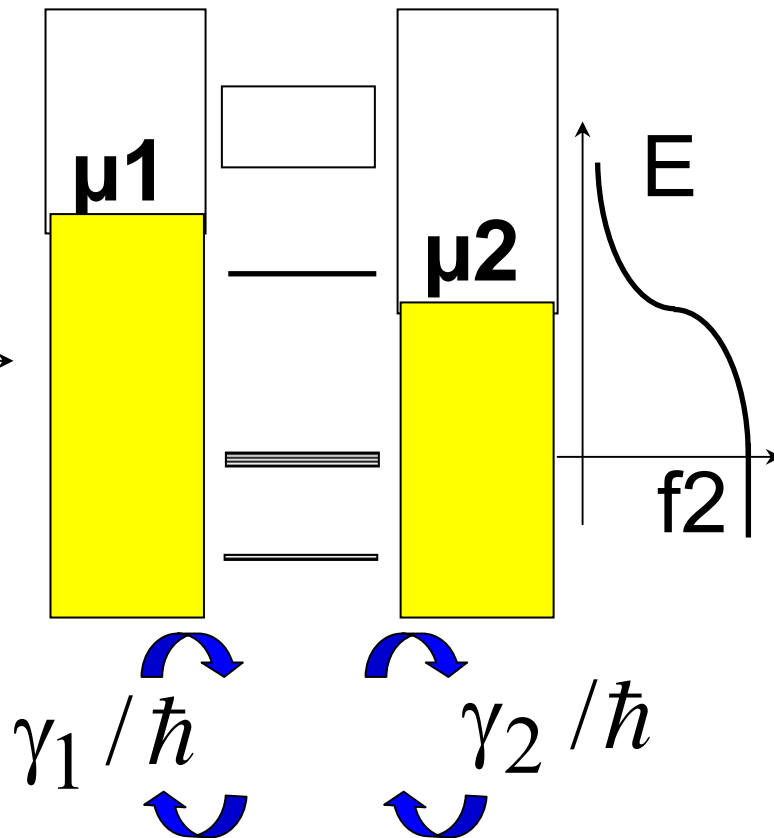
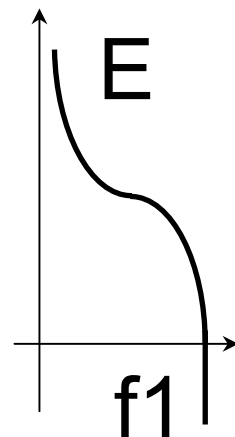
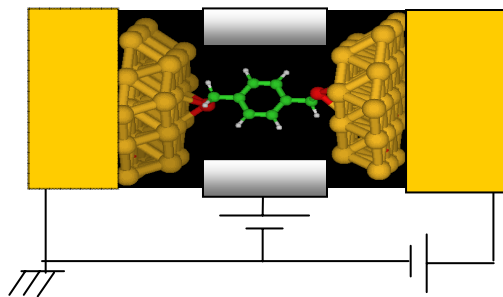


Small γ





Toy model: Max. G ?

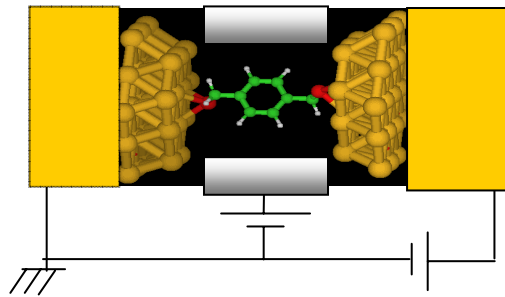


$$I = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$$\sim \frac{q}{2\hbar} \gamma$$



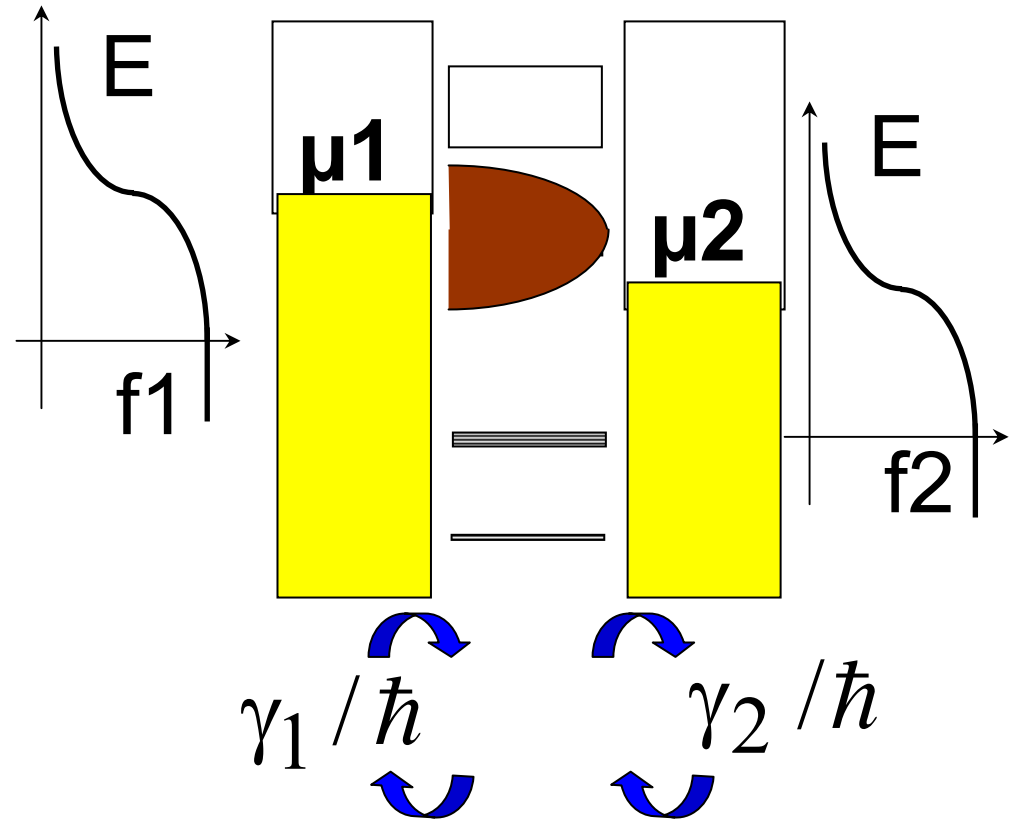
Toy model: Broadening



$$I \sim \frac{q\gamma}{2\hbar} \frac{qV}{\gamma}$$

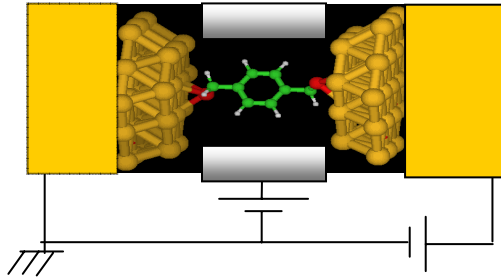
Maximum conductance

$$\frac{I}{V} \sim q^2/h \sim \frac{1}{25.8 \text{ K}\Omega}$$





Broadening

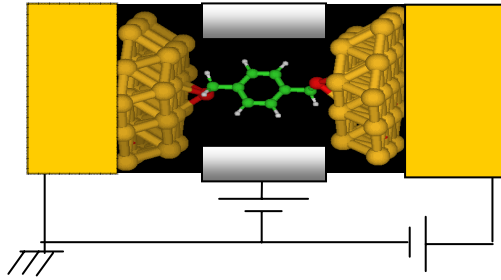


$$N = \int dE D(E) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{h} \int dE D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$



Broadening



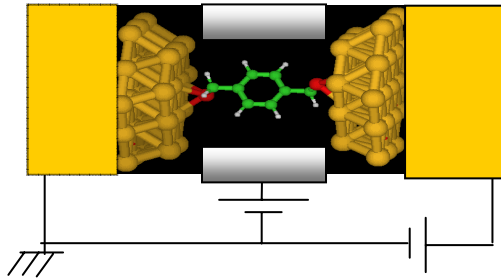
$$N = \int dE D(E - U) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{h} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$$U = U_L + U_0(N - N_0)$$



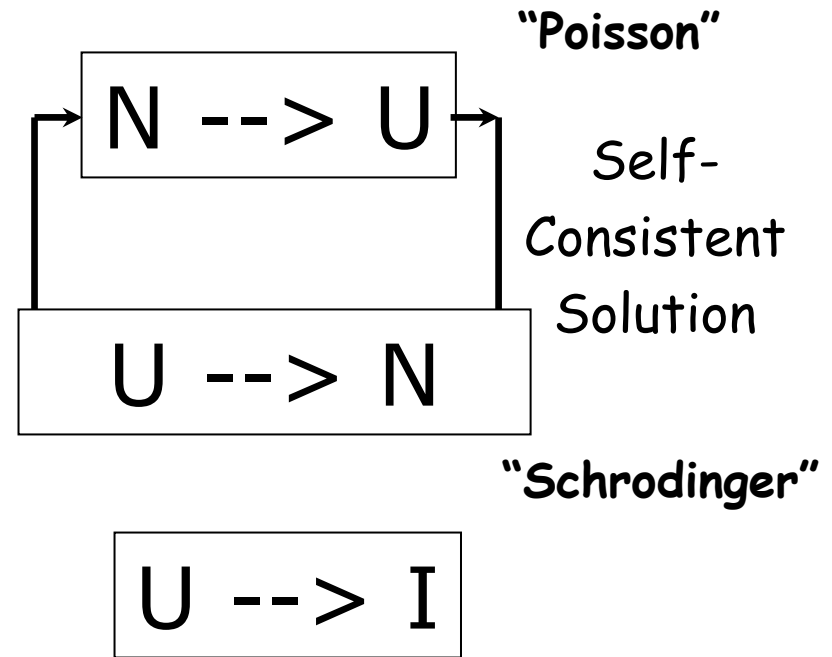
Broadening + Charging



$$N = \int dE D(E - U) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{\hbar} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

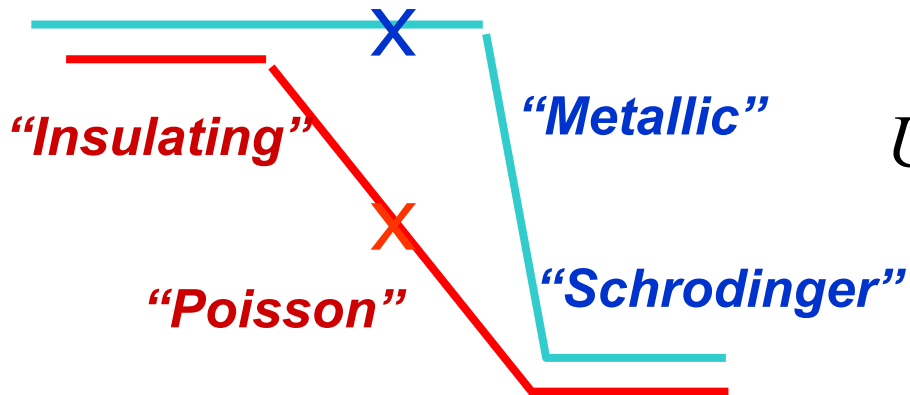
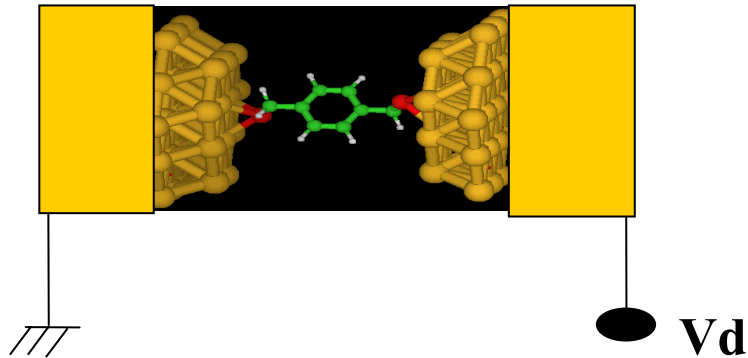
$$U = U_L + U_0(N - N_0)$$





Where is the voltage drop ?

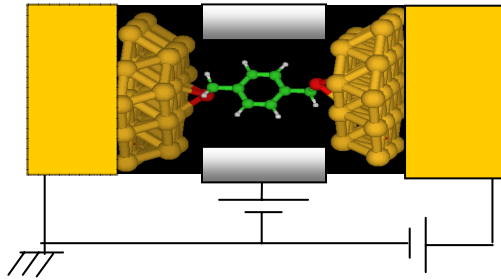
Weak



$$U = U_L + U_0(N - N_0)$$



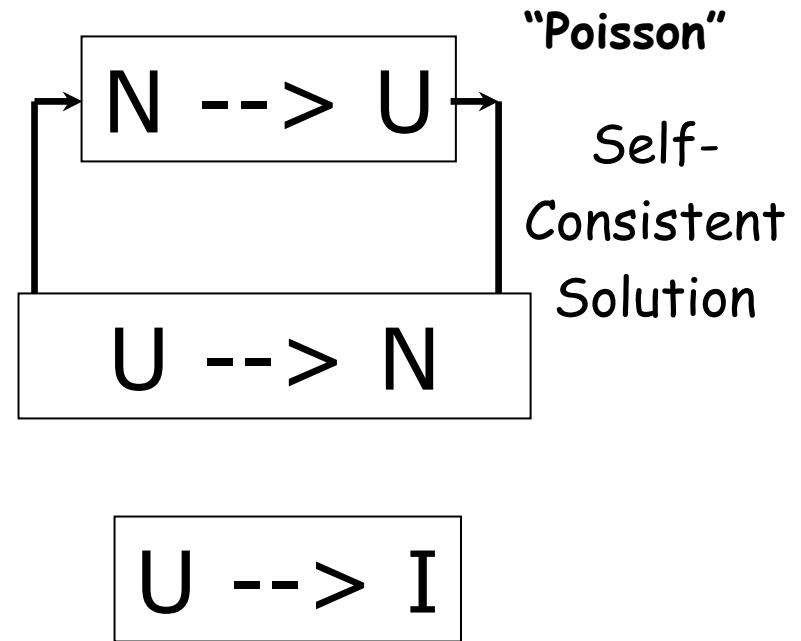
Minimal Model



$$N = \int dE D(E - U) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{\hbar} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

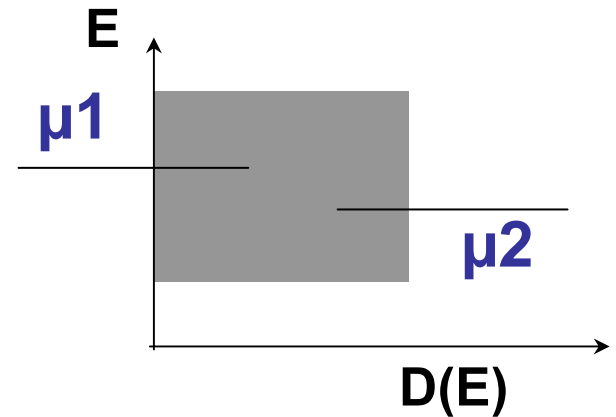
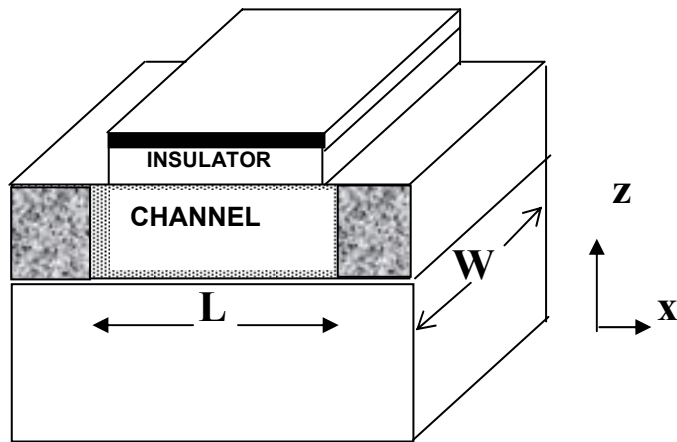
$$U = U_L + U_0(N - N_0)$$



Nanowires / Nanotubes / Molecules



Ohm's Law ?



$$I = \frac{q}{\hbar} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$D(E) \sim \text{Area} \times \text{Length}$

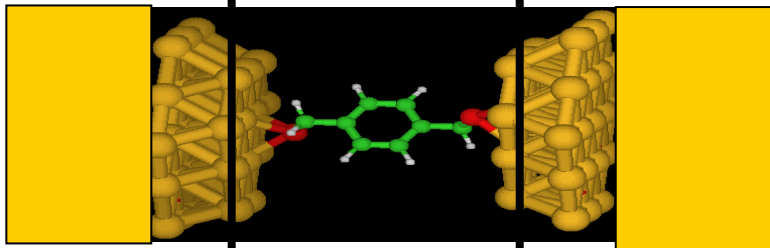
$\gamma \sim \text{Velocity} / \text{Length}$

$I \sim \text{Area}$



Numbers --> Matrices: NEGF

Surface Physics Quantum Chemistry Surface Physics



$[\Sigma_1]$ $[H]$ $[\Sigma_2]$

$[\Sigma_s]$

Incoherent Scattering

www.nanohub.org

$$\varepsilon \rightarrow [H]$$

$$\gamma \rightarrow [\Gamma], [\Sigma]$$

$$D(E) \rightarrow [A(E)]$$

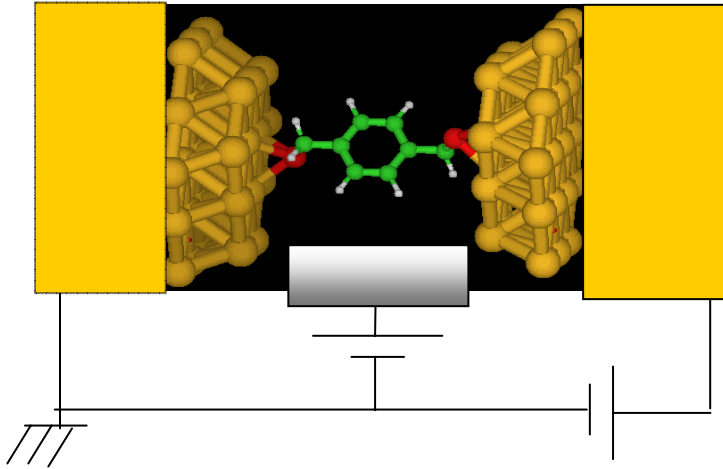
$$U \rightarrow [U]$$

$$N \rightarrow [\rho]$$

Ghosh, Liang, Rakshit,
Zahid, Damle, Paulsson



Outline



- Qualitative picture
- Quantitative models
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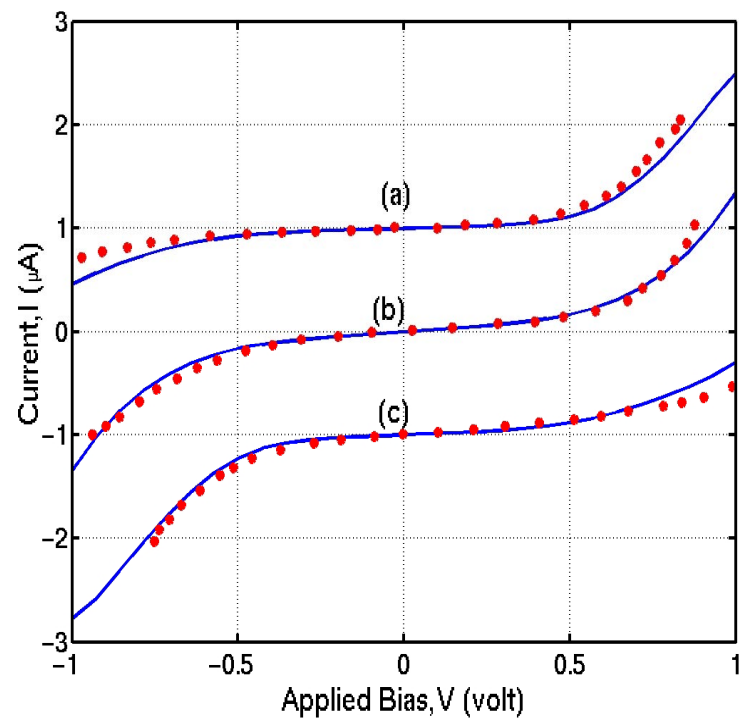
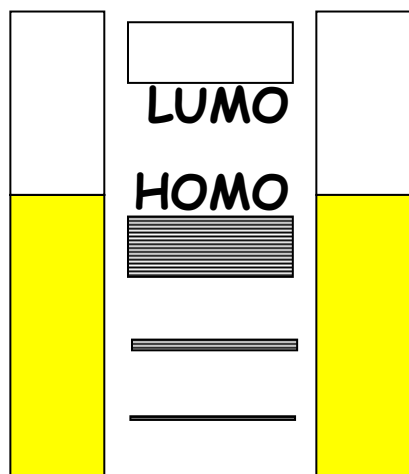
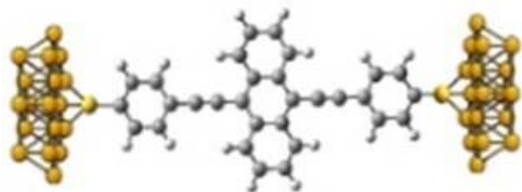
Experiment vs. Theory

Experiment:

Reichert et.al. PRL (2002)

Theory:

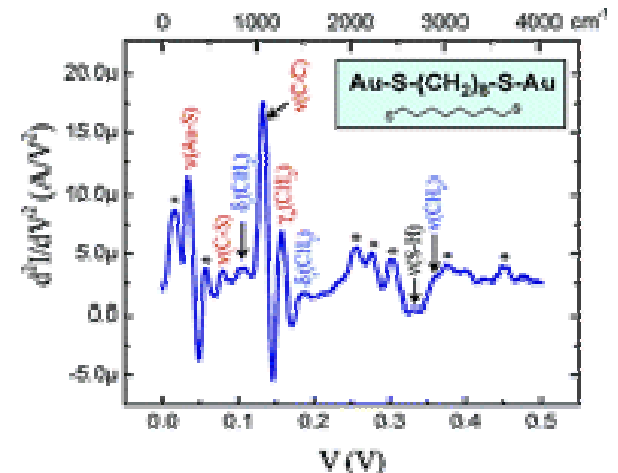
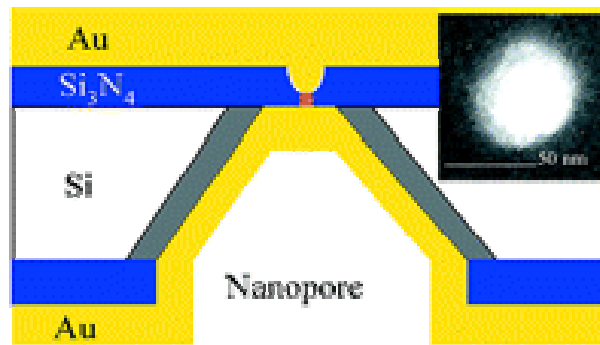
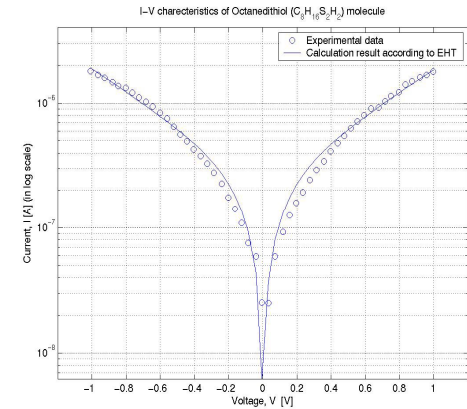
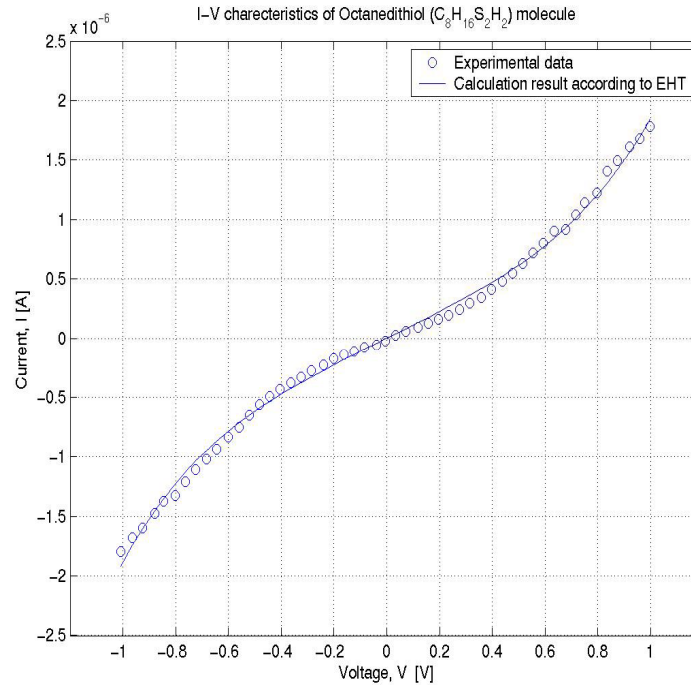
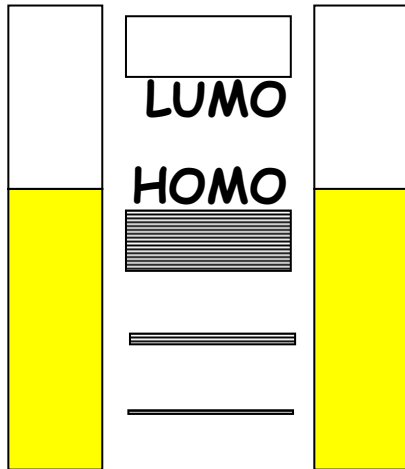
Zahid, Ghosh et.al.





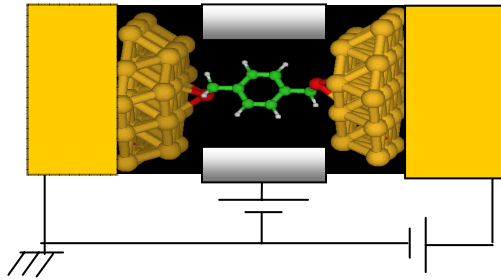
Experiment vs Theory

Expt: Wang, Lee and Reed
 Alkane- thiols
 Theory: Zahid and Siddiqui





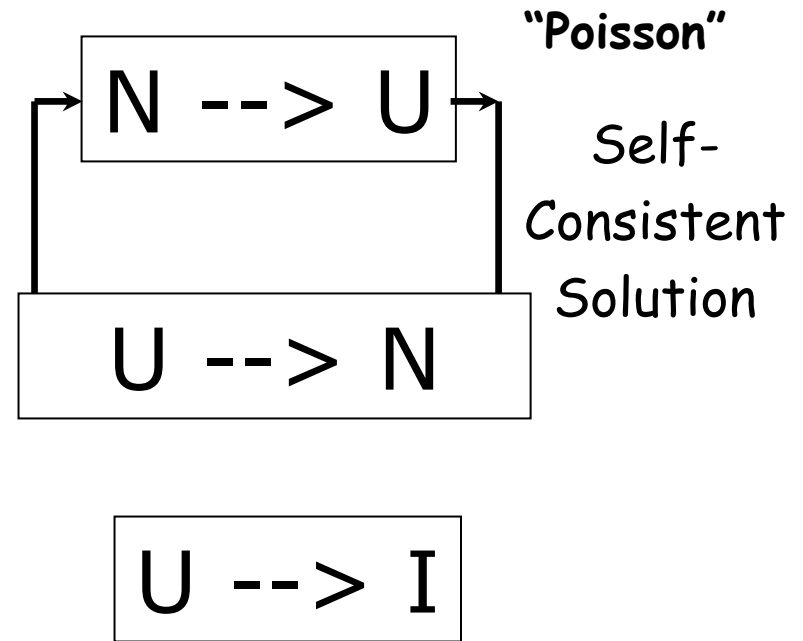
Minimal Model



$$N = \int dE D(E - U) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

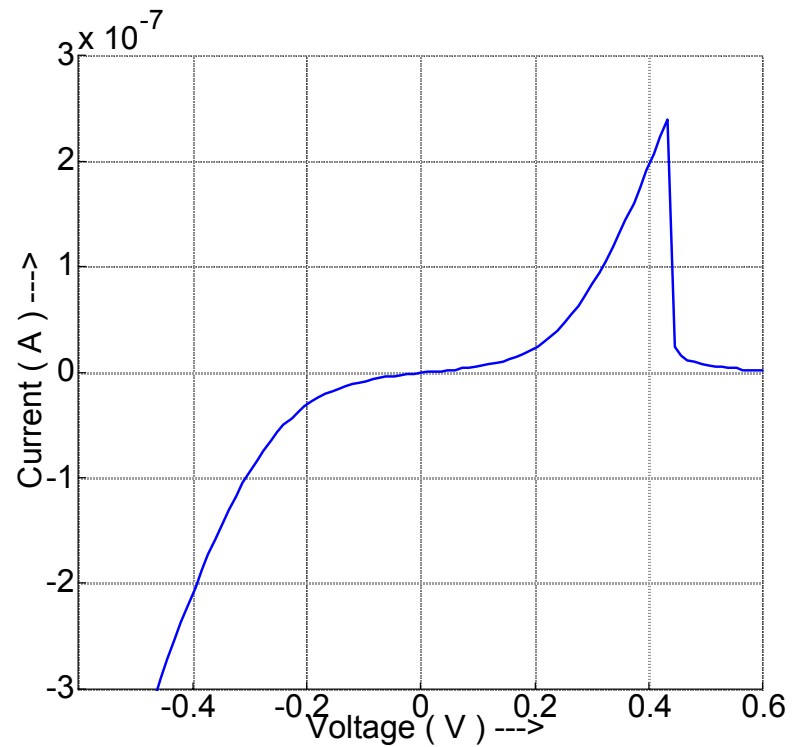
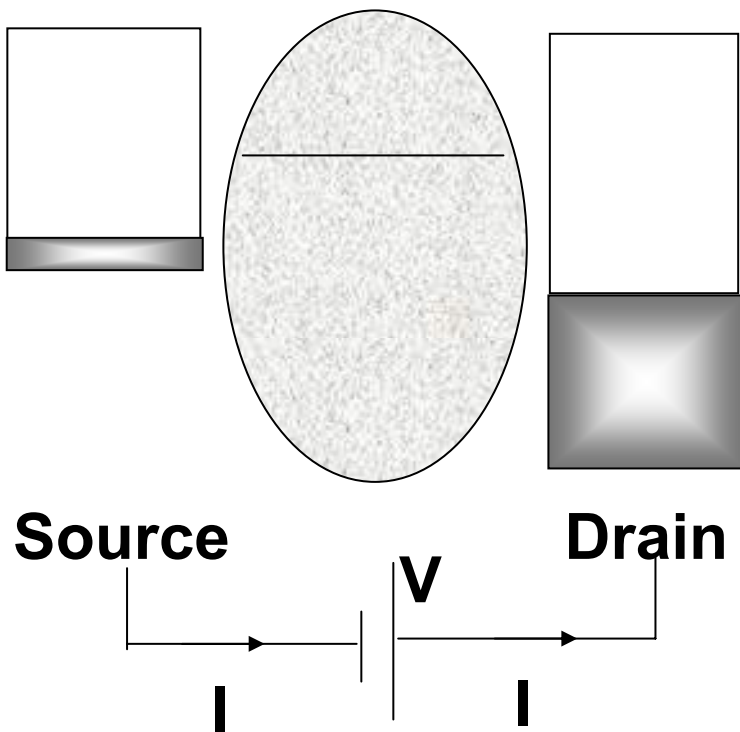
$$I = \frac{q}{\hbar} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$$U = U_L + U_0(N - N_0)$$





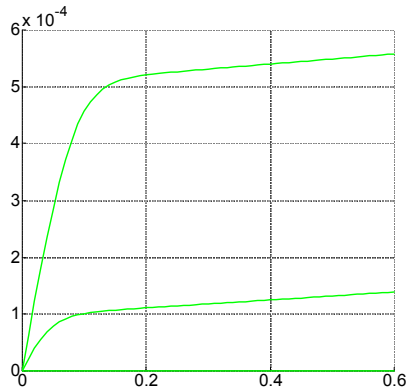
Negative Differential Resistance (NDR)





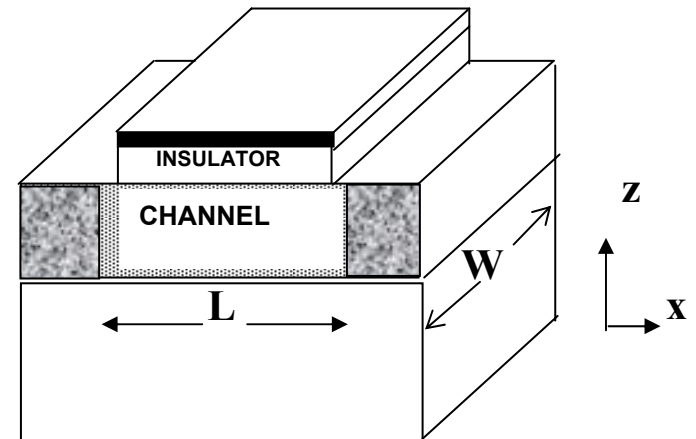
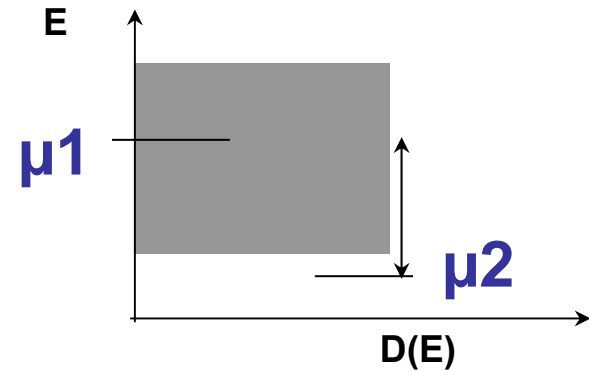
FET: Why current "saturates" ?

Drain current



Drain voltage

$$U = U_L + U_0(N - N_0)$$

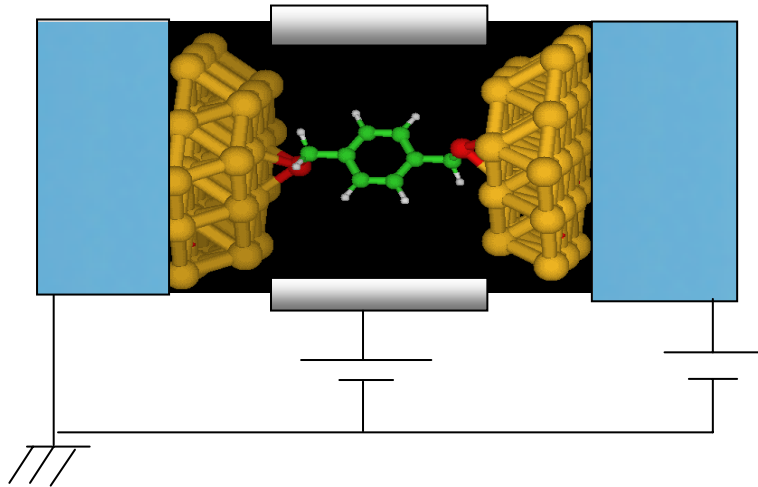




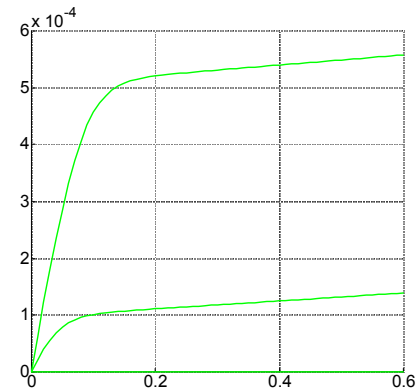
Molecular FET's ?

$$U = U_L + U_0(N - N_0)$$

Gate should exercise greater control than *drain*



Drain current

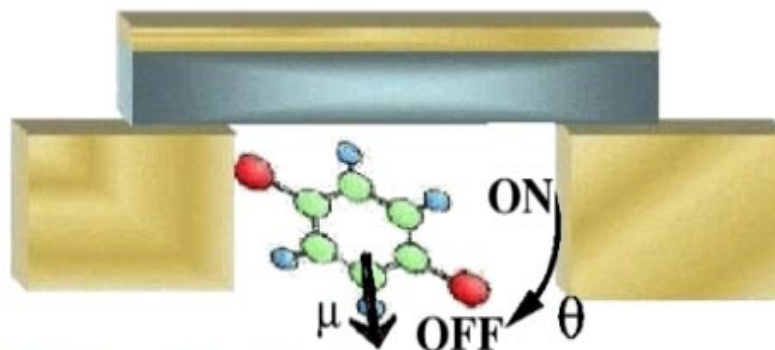


Drain voltage

Damle, Rakshit, Paulsson,
IEEE Transactions on
Nanotechnology (2002)



Conformational Transistors ?



Molecular Relay

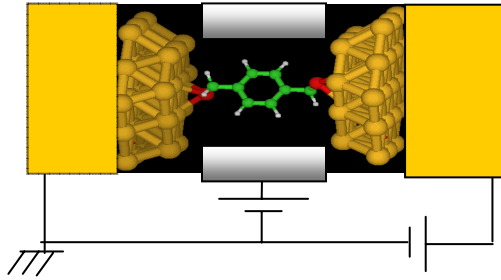
$$(S_{es})_{\min} = 2.3(k_B T/e) \approx 60 \text{ mV/dec.}$$

$$(S_{\text{conf}})_{\min} = 2.3(k_B T/e) \cdot (et_{ox}/\mu)$$

Ghosh, Rakshit
(Nanoletters, 2004)



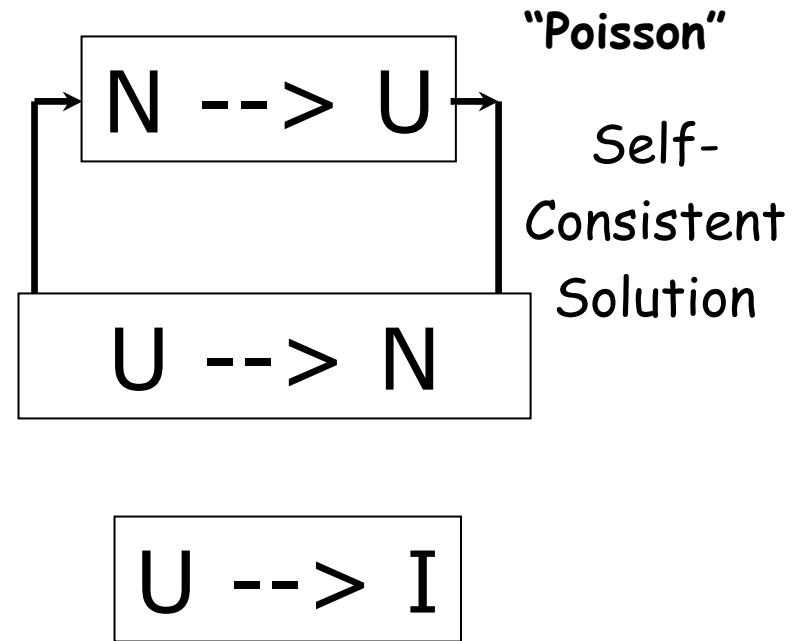
Minimal Model



$$N = \int dE D(E - U) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = \frac{q}{\hbar} \int dE D(E - U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

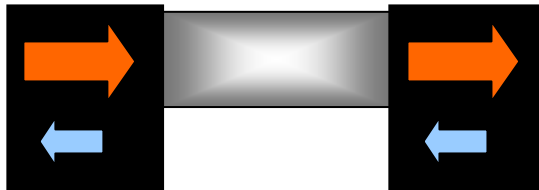
$$U = U_L + U_0(N - N_0)$$





Spin Valves

Source Channel Drain



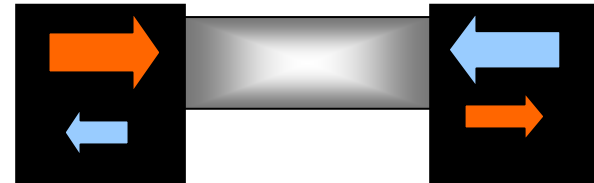
Insulating substrate

$$\alpha \text{ --- } \alpha$$

$$\beta \text{ --- } \beta$$

$$I_P \sim \frac{\alpha \cdot \alpha}{\alpha + \alpha} + \frac{\beta \cdot \beta}{\beta + \beta} = \frac{\alpha + \beta}{2}$$

Source Channel Drain



Insulating substrate

$$\alpha \text{ --- } \beta$$

$$\beta \text{ --- } \alpha$$

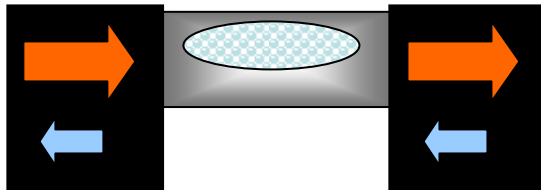
$$I_{AP} \sim \frac{2\alpha\beta}{\alpha + \beta}$$

$$= \left(\frac{\alpha + \beta}{2} \right) - \frac{(\alpha - \beta)^2}{2(\alpha + \beta)}$$

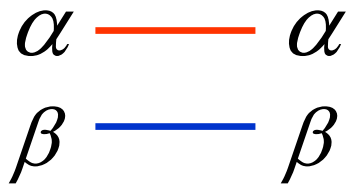


Spintronics

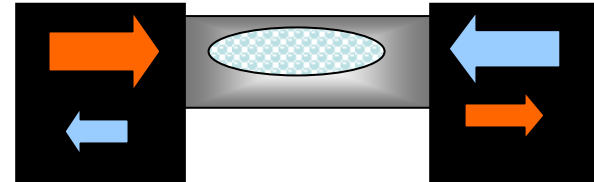
Source Channel Drain



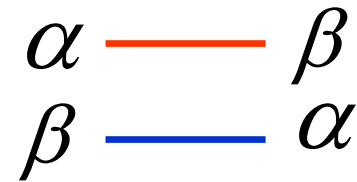
Insulating substrate



Source Channel Drain

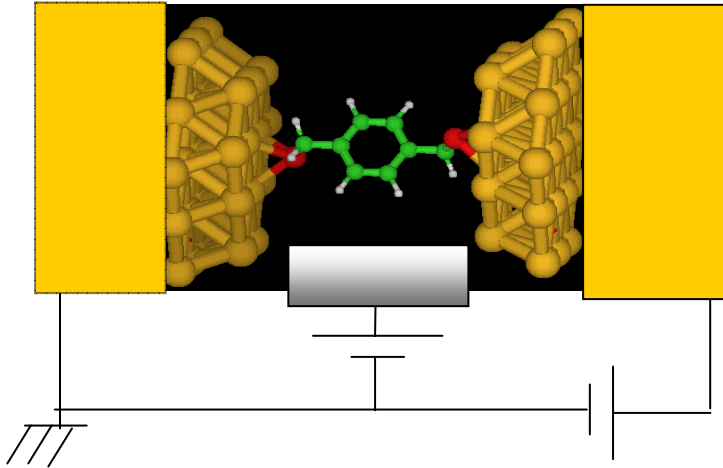


Insulating substrate





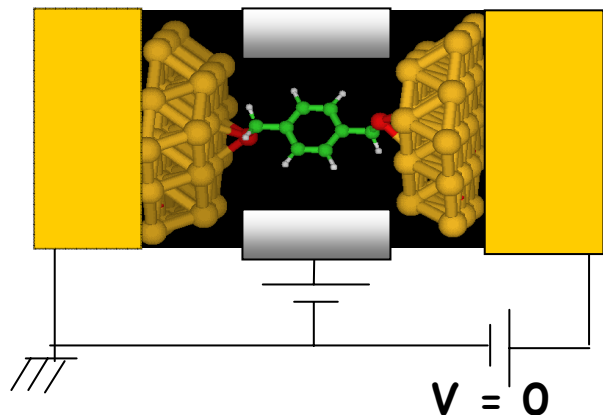
Outline



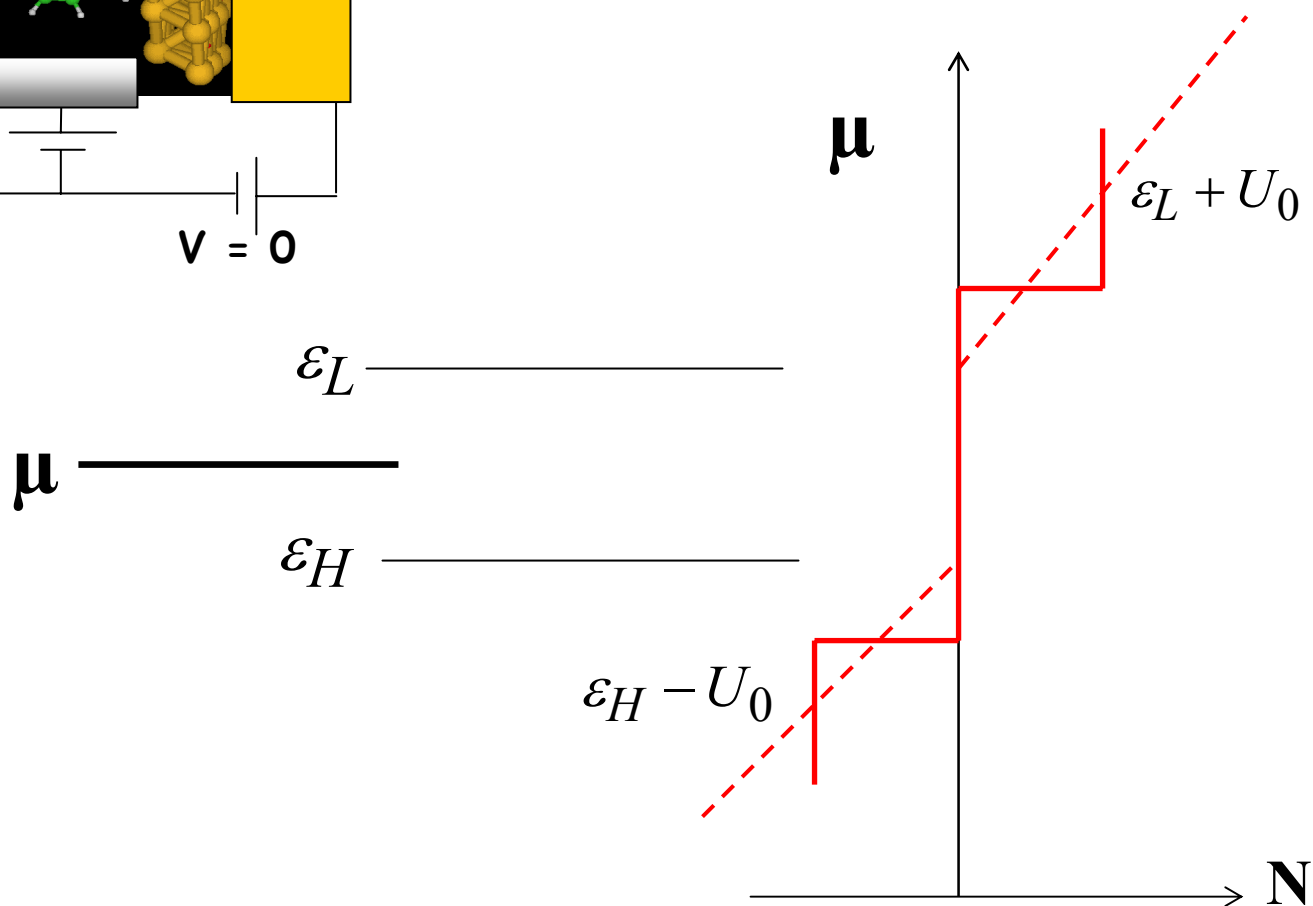
- Qualitative picture
- Quantitative models
 - Examples
- Coulomb blockade
- Summary/Open questions



Electron addition and removal

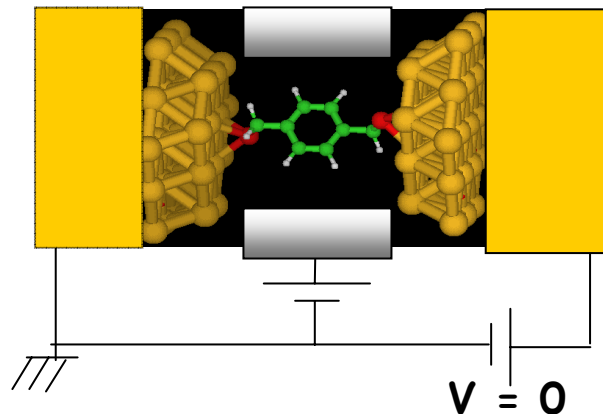


$$U = U_L + U_0 \Delta N$$



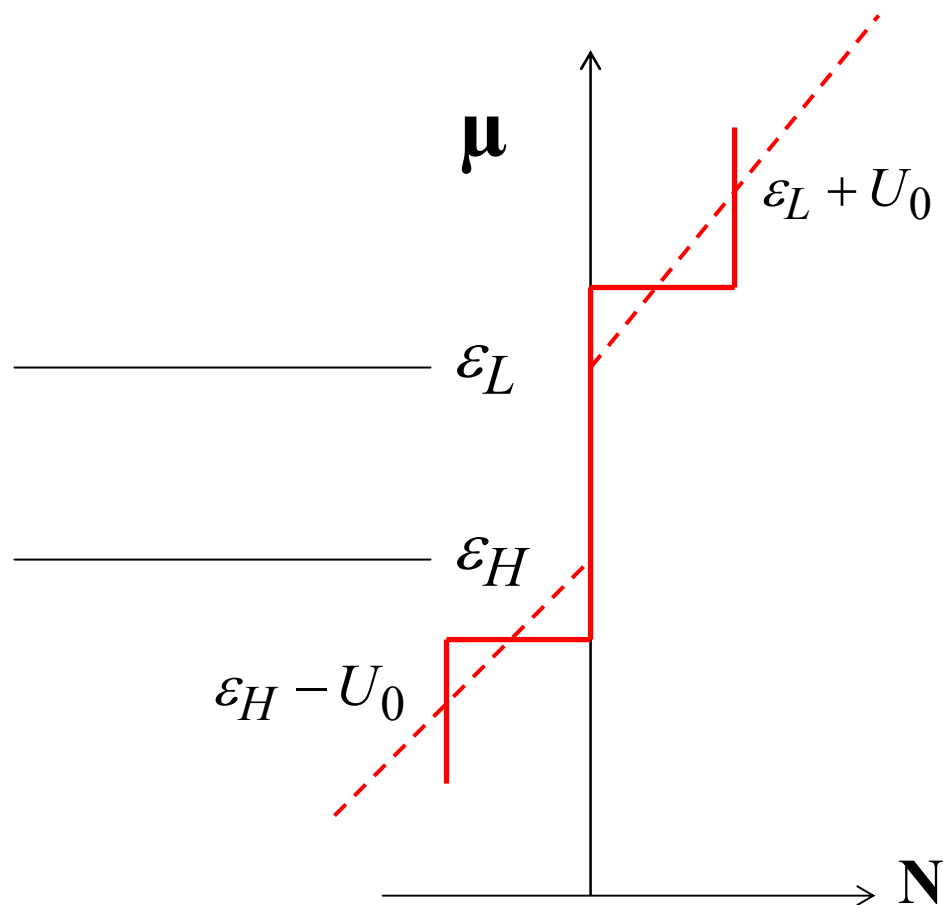


Self-interaction correction



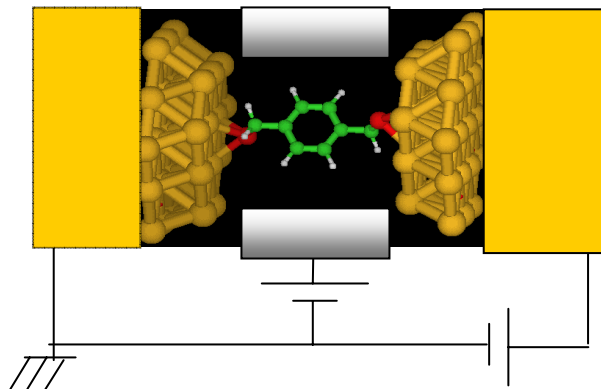
$$U_j = U_L + U_0(\Delta N - \Delta n_j)$$

CB: $U_0 \gg \gamma$
 SCF: $\gamma \gg U_0$





Non-equilibrium



$$U_j = U_L + U_0(\Delta N - \Delta n_j)$$

μ (Drain)

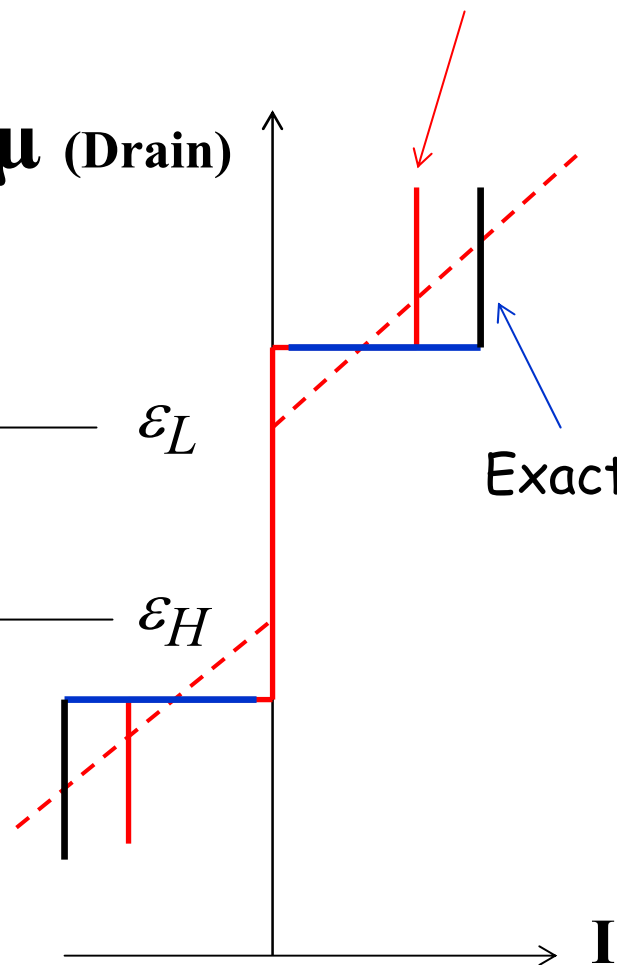
μ (Source)

ε_L

ε_H

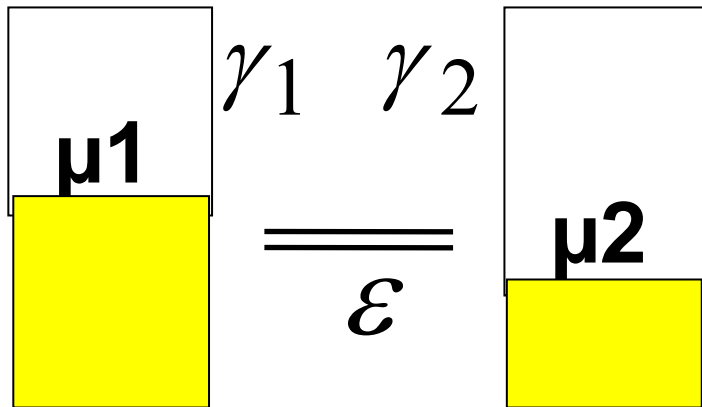
Exact

CB: $U_0 \gg \gamma$
 SCF: $\gamma \gg U_0$

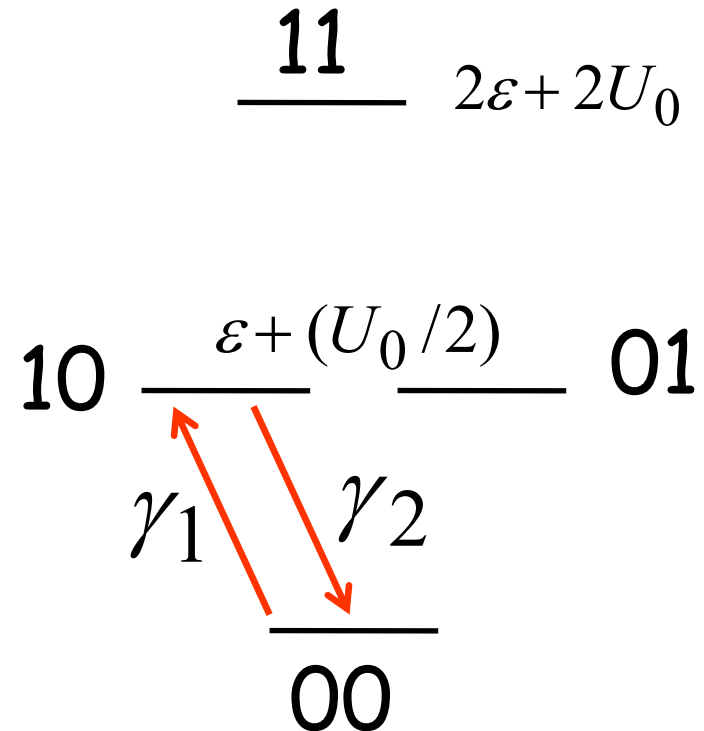




One-electron vs. Multielectron



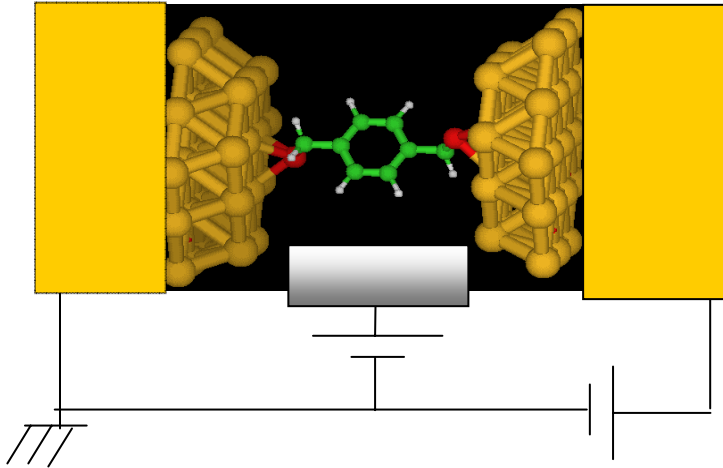
$$\begin{aligned}
 U_{scf} &= \partial U_{ee} / \partial N \\
 &= U_0(N - N_0)
 \end{aligned}$$



$$U_{ee} = (U_0/2) (N - N_0)^2$$



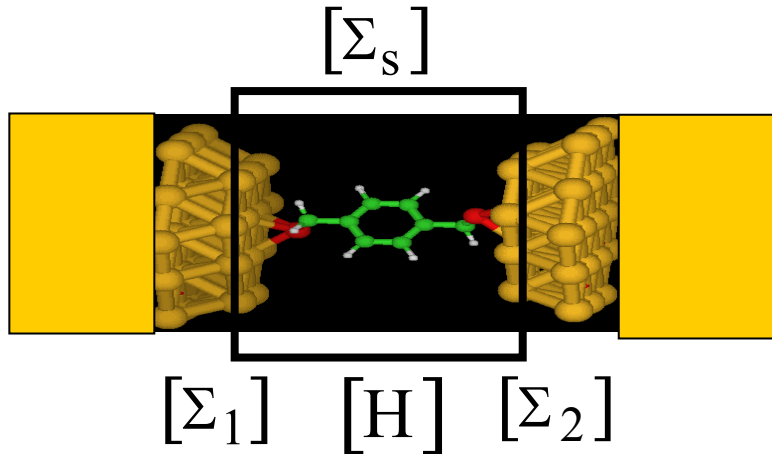
Outline



- Qualitative picture
- Quantitative models
 - Examples
- Coulomb blockade
 - Summary/Open questions



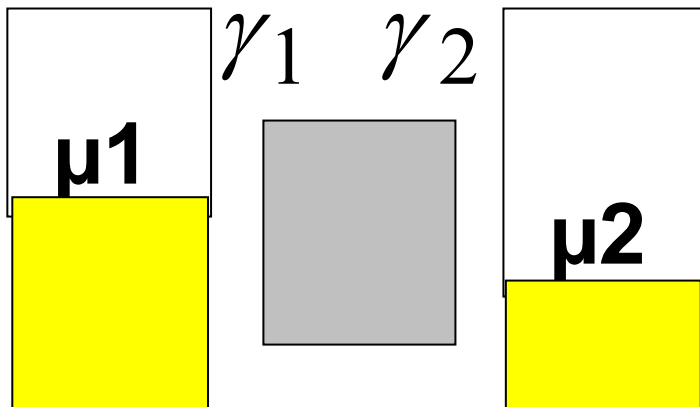
Summary



$$U = U_L + U_0 \Delta N$$

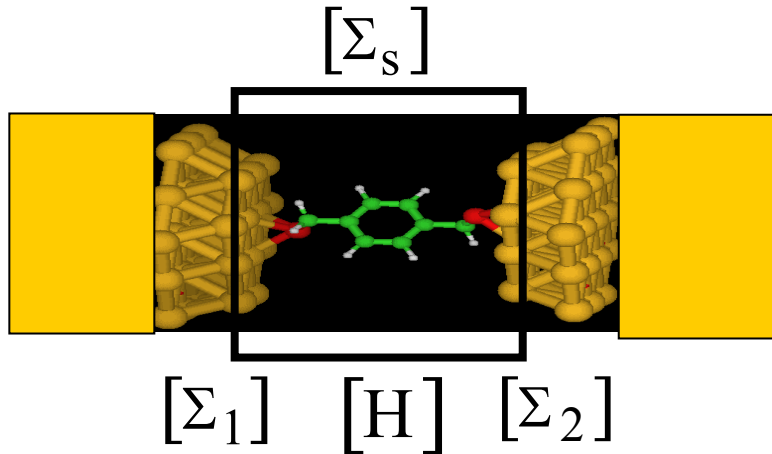
$$\text{SCF: } \gamma \gg U_0$$

$$\text{CB: } U_0 \gg \gamma$$





Summary / Open questions

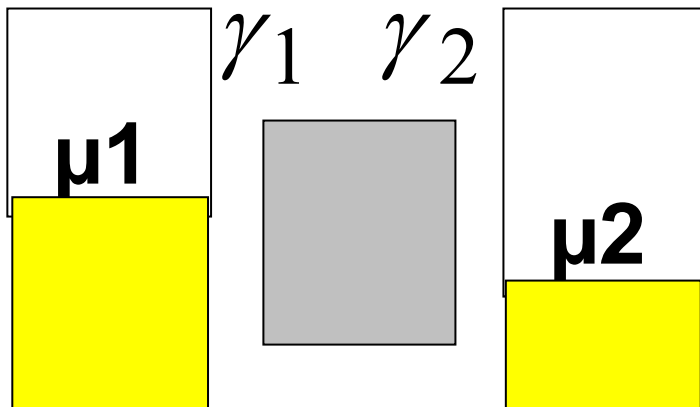


$$U = U_L + U_0 \Delta N$$

$$\text{SCF: } \gamma \gg U_0$$

$$\text{CB: } U_0 \gg \gamma$$

$$\gamma \sim U_0 : \text{??????}$$



Cooperative Phenomena ?



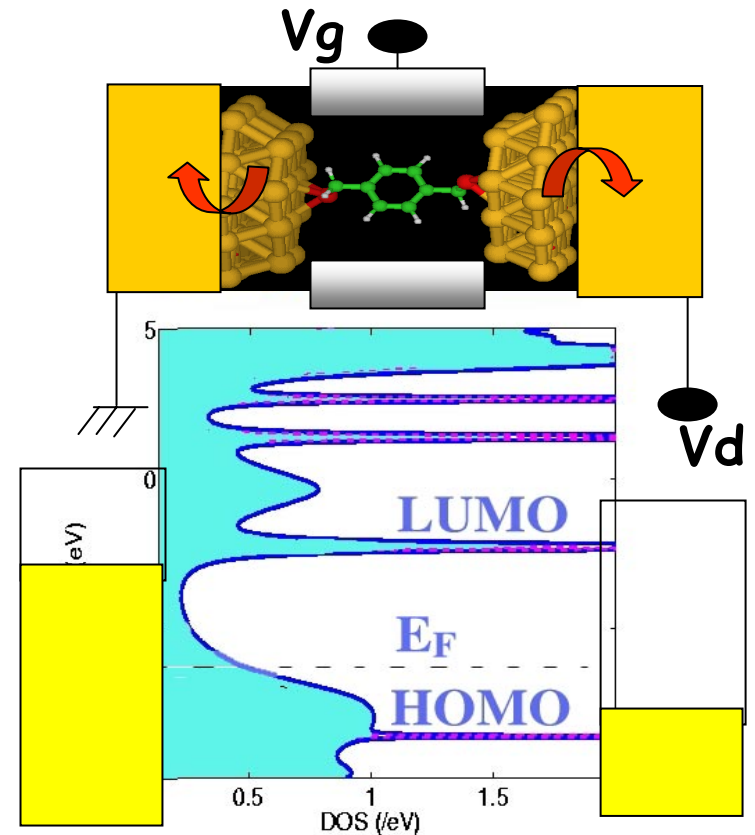
Inflow / Outflow

$$\hbar \frac{d}{dt} N + \gamma_1 N = \gamma_1 f_1$$

Out In

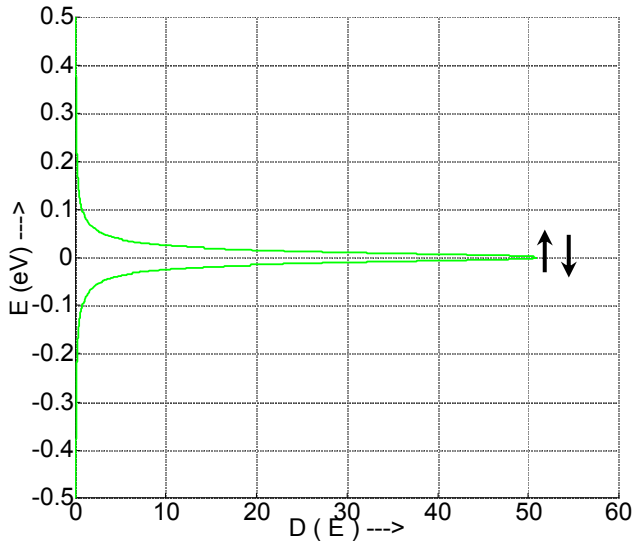
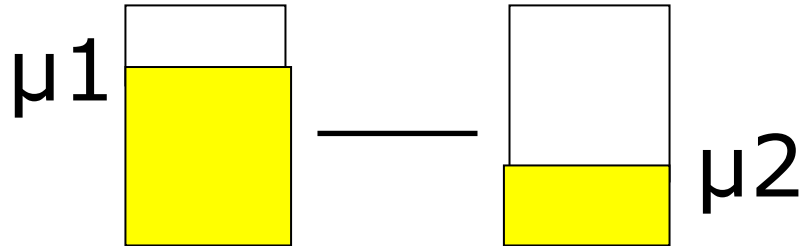
$$i\hbar \frac{d}{dt} \psi - H \psi - \Sigma_1 \psi = S_1$$

$$i\hbar \frac{d}{dt} \begin{Bmatrix} \psi_L \\ \psi \\ \psi_R \end{Bmatrix} = \begin{bmatrix} H_L & \tau^+ & 0 \\ \tau & H & \tau \\ 0 & \tau^+ & H_R \end{bmatrix} \begin{Bmatrix} \psi_L \\ \psi \\ \psi_R \end{Bmatrix}$$

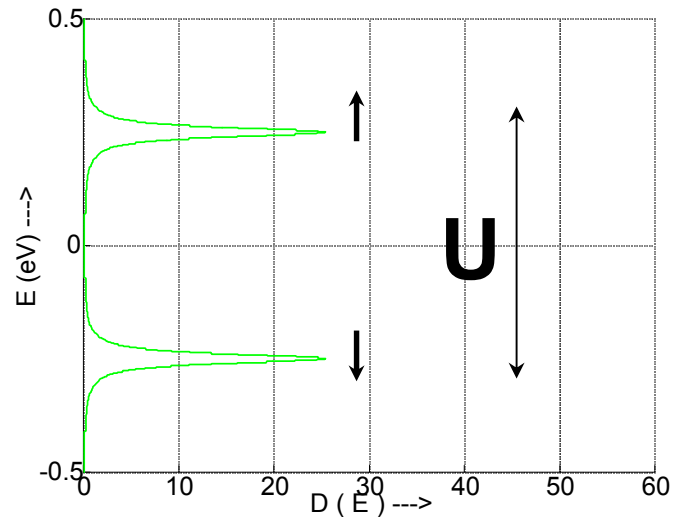




Coulomb blockade ($\Gamma \ll U$)



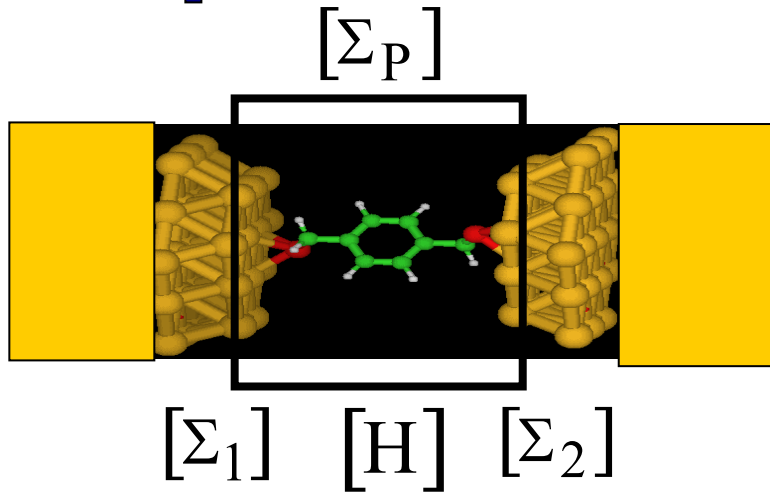
μ



$$\Gamma \ll U \sim \frac{q^2}{4\pi\epsilon R}$$



Kondo Resonance ($\Gamma \sim U$)

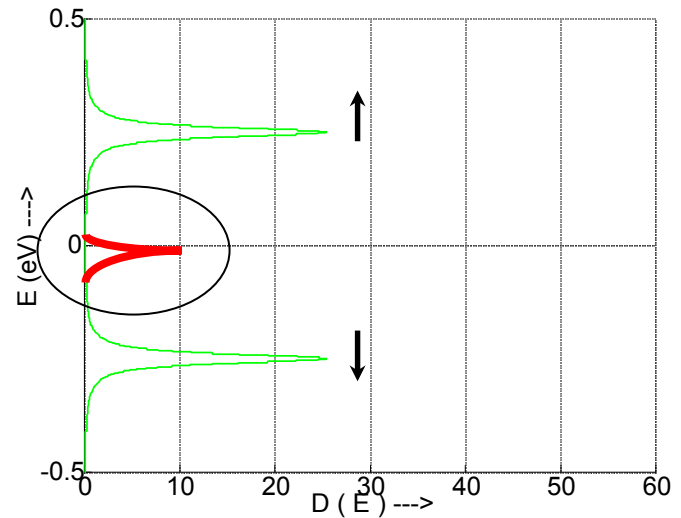
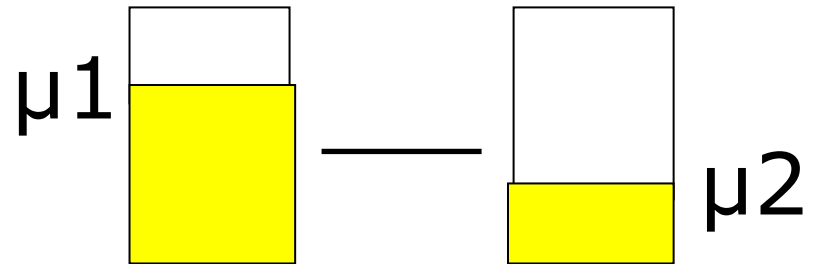


$$i\hbar \frac{d}{dt} \psi - H \psi - \Sigma \psi = S$$

In

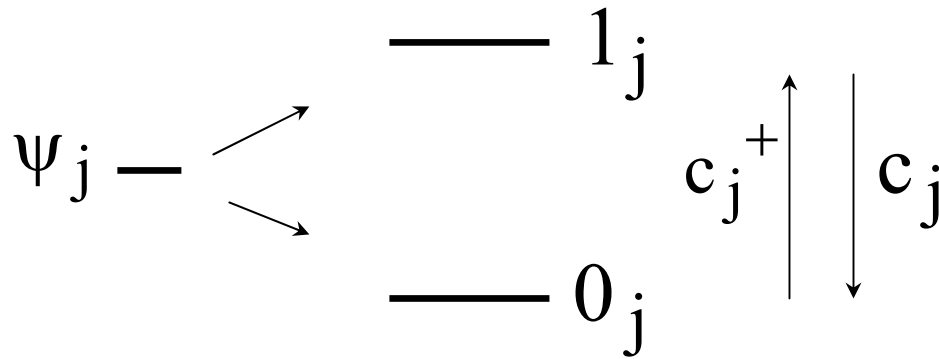
Out

Σ, S depend on 'f'





Second quantization

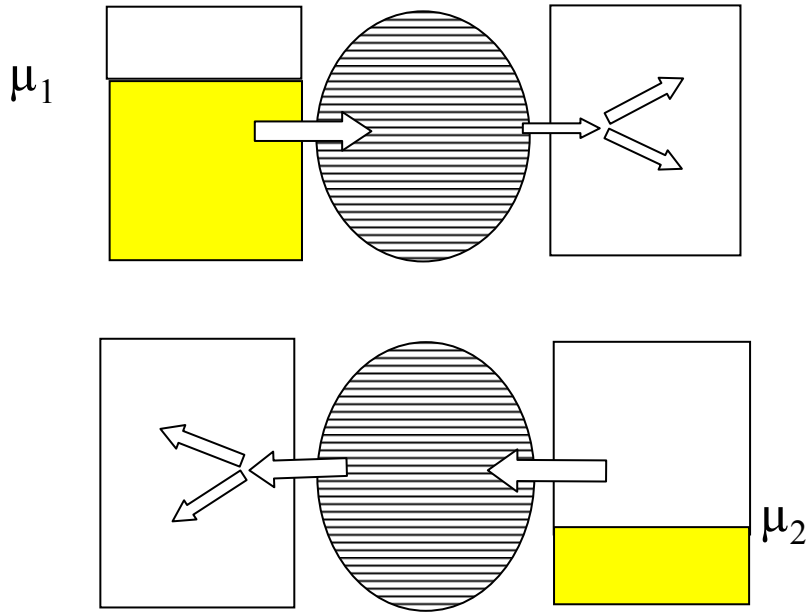


$$i\hbar \frac{d}{dt} \begin{Bmatrix} c_L \\ c \\ c_R \end{Bmatrix} = \begin{bmatrix} H_L & \tau^+ & 0 \\ \tau & H & \tau \\ 0 & \tau^+ & H_R \end{bmatrix} \begin{Bmatrix} c_L \\ c \\ c_R \end{Bmatrix}$$

$$i\hbar \frac{d}{dt} c - Hc - \Sigma c = S + U d^+ dc$$



Pauli blocking?



$$\hbar \frac{d}{dt} N + (\gamma_1 + \gamma_2) N = \gamma_1 f_1 + \gamma_2 f_2$$

↙ Out ↘ In

$$i\hbar \frac{d}{dt} \psi - H\psi - (\Sigma_1 + \Sigma_2)\psi = S_1 + S_2$$

$$|1'\rangle = \exp[-iHt/\hbar]|1\rangle$$

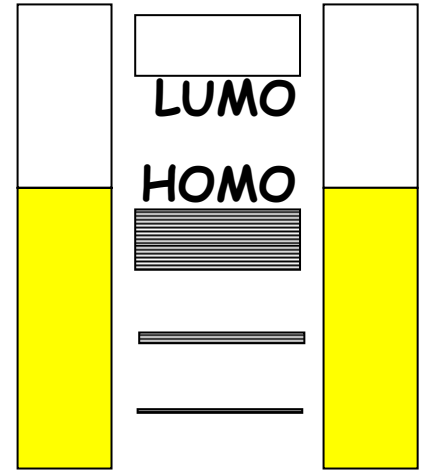
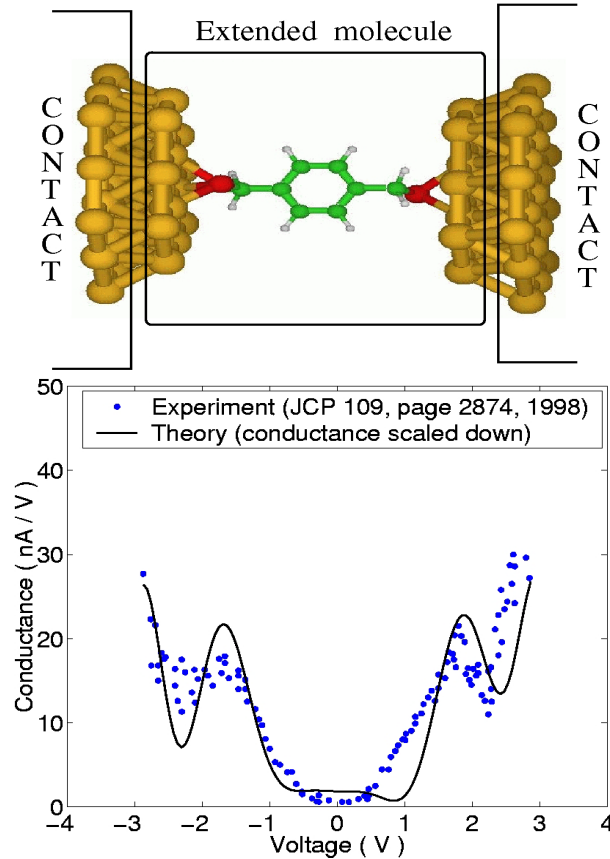
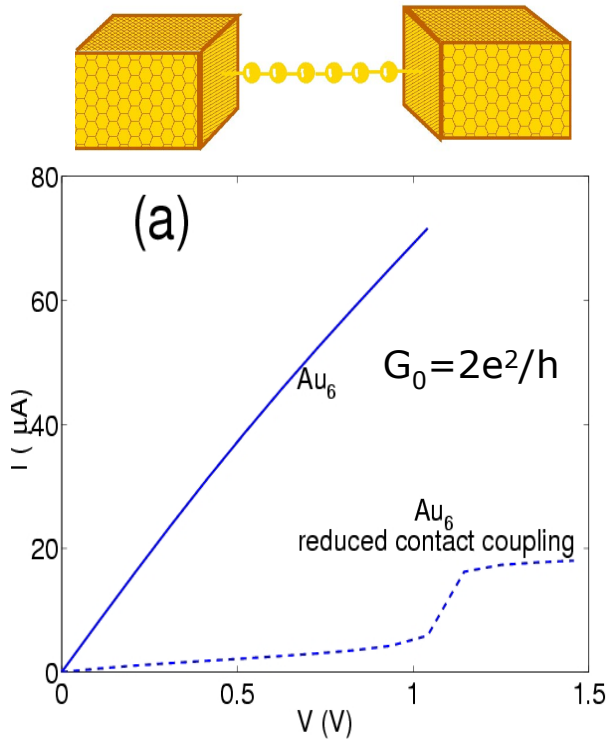
$$|2'\rangle = \exp[-iHt/\hbar]|2\rangle$$

$$\langle 1' | 2' \rangle = \langle 1 | 2 \rangle$$

**NOT applicable to
phase-breaking processes**



Experiment vs Theory

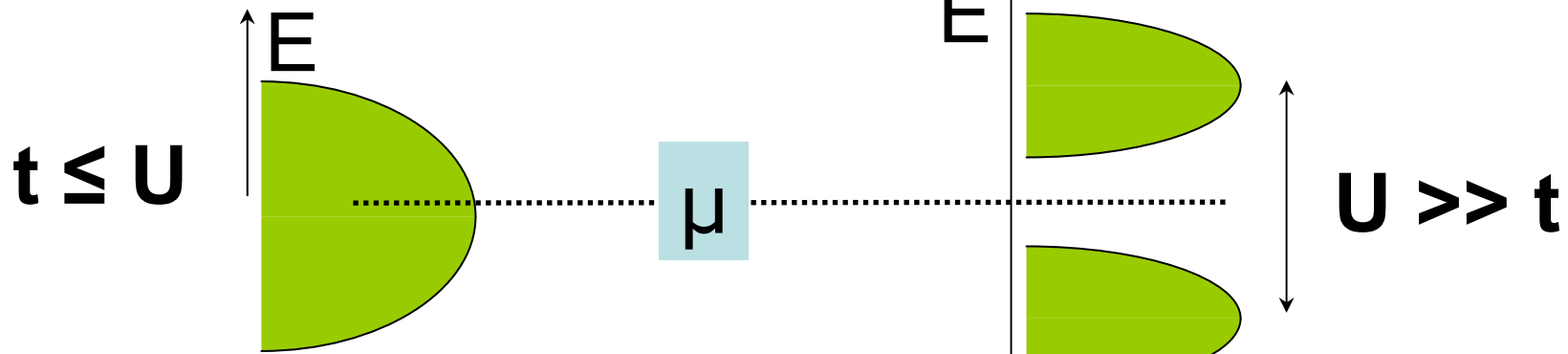


Expt:
Reifenberger,
Kubiak et.al.

Theory: Damle, Ghosh et.al.
PRB 64, 201403 R (2001)

Solid state \leftrightarrow Molecular

$$t \leftrightarrow \Gamma$$



Band limit

Solids

Atomic limit

Scf method

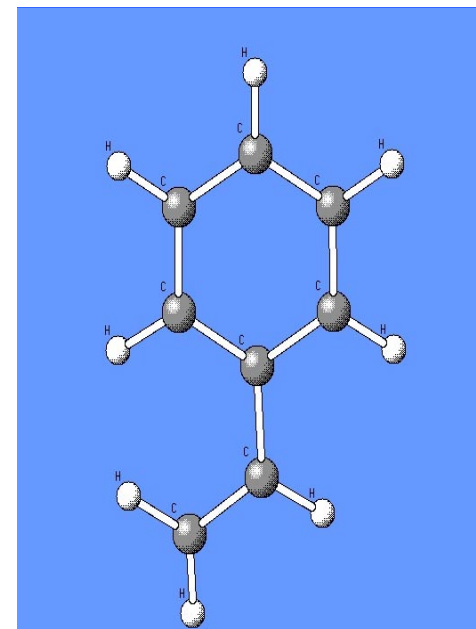
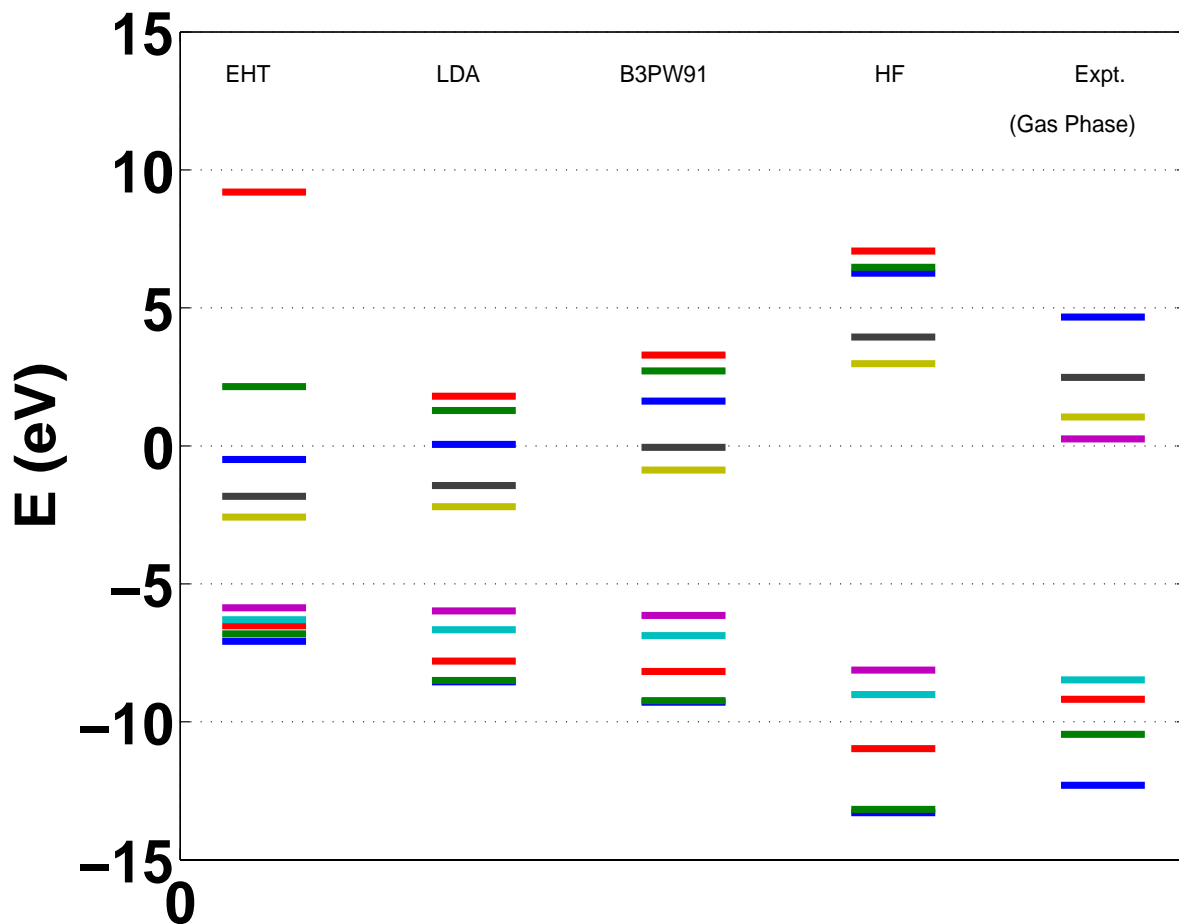
"Molecules"

Coulomb blockade



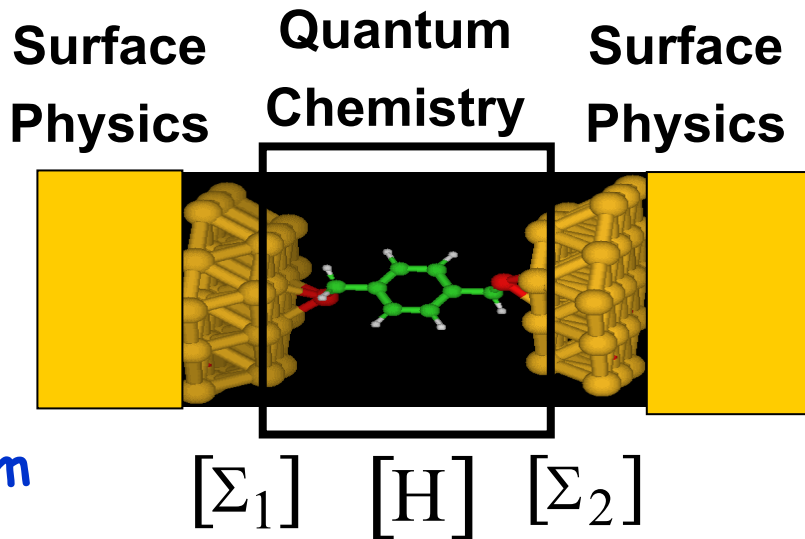
Energy Levels: Styrene

Styrene Energy levels





Real (?) Models



Incoherent Scattering, $[\Sigma_s]$

Open System, Out-of-equilibrium

Closed System

