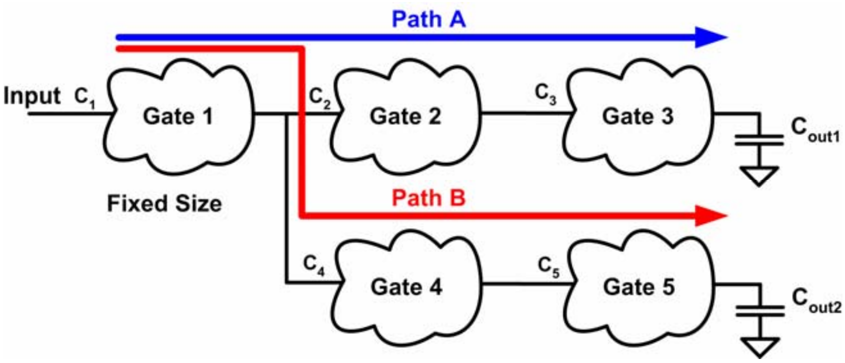
Branching



Find hi for each gate:

$$h_1 = \frac{C_2 + C_4}{C_1} \qquad h_2 = \frac{C_3}{C_2} \qquad h_3 = \frac{C_{out1}}{C_3}$$

$$h_4 = \frac{C_5}{C_4} \qquad h_5 = \frac{C_{out2}}{C_5}$$

For a path we want to find $\prod h_i$ along a path, so for **Path A** we want to find:

$$h_1 \cdot h_2 \cdot h_3 = \frac{C_2 + C_4}{C_1} \frac{C_3}{C_2} \frac{C_{out1}}{C_3}$$

Another way of doing this:

Remember that for **Path A**:

$H = \dfrac{C_{out1}}{C_1}$ which we can also express as $H = \dfrac{C_2}{C_1} \dfrac{C_3}{C_2} \dfrac{C_{out1}}{C_3}$

To factor H from $\prod h_i$, we need to introduce the concept of branching. Branching at the output of a gate is defined as

$$b = \frac{C_{on} + C_{off}}{C_{on}}$$

(where C_{on} refers to the input capacitance of the gate on the path being analyzed and C_{off} refers to the total input capacitance of the gates that are not on the path being analyzed).

For example, the branching at the output of gate 1 when analyzing path A is:

$$b = \frac{C_2 + C_4}{C_2}$$

While the branching at the output of gate 1 when analyzing path B is:

$$b = \frac{C_4 + C_2}{C_4}$$

If we multiply the branching at the output of a gate by C_{on}/C_{in} of the gate we obtain h_i for the gate. For example when analyzing gate 1 for Path A.

$$h_1 = b_1 \frac{C_2}{C_1} = \left(\frac{C_2 + C_4}{C_2}\right) \cdot \frac{C_2}{C_1} = \frac{C_2 + C_4}{C_1}$$

For paths that don't branch $b_i = 1$, so $b_i * C_{on}/C_{in} = h_i$.

So we can express $\prod h_i$ as:

$\prod h_i = H \cdot \prod b_i$ (remember that we can express H as $H = \frac{C_2}{C_1}\frac{C_3}{C_2}\frac{C_{out1}}{C_3}$)

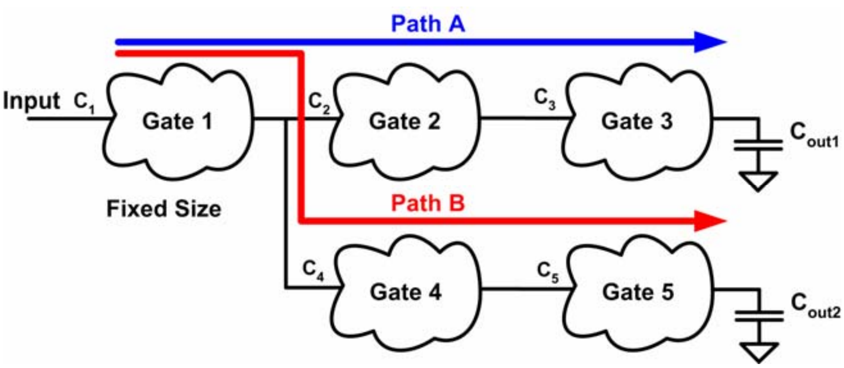
to simplify we refer to $\prod b_i$ as B.

so $\prod h_i = H \cdot B$

This allows us to express F as:

$$F = GBH$$

How to solve for b_i?



Here we will solve for b_1 when analyzing **Path A**.

$$b_1 = \frac{C_2 + C_4}{C_2}$$

Need to compute C_2 and C_4

Solve for C_2 and C_4

$$C_2 = \frac{g_2 g_3 C_{out1}}{f_2 f_3} \qquad C_4 = \frac{g_4 g_5 C_{out2}}{f_4 f_5}$$

The optimal delay occurs when the delay of each is equal. From LE, we know that the optimal delay occurs when $f_2 = f_3$, and $f_4 = f_5$. As mentioned in class we will ignore the parasitic delay difference of each path. By doing this the delay of each path is equal when $f_2 = f_3 = f_4 = f_5$.

Now lets go back to solving for b_1.

$$b_1 = \frac{\dfrac{g_2 g_3 C_{out1}}{f_2 f_3} + \dfrac{g_4 g_5 C_{out2}}{f_4 f_5}}{\dfrac{g_2 g_3 C_{out1}}{f_2 f_3}}$$

Since $f_2 = f_3 = f_4 = f_5$ we can simplify to

$$b_1 = \frac{g_2 g_3 C_{out1} + g_4 g_5 C_{out2}}{g_2 g_3 C_{out1}}$$

If $C_{out1} = C_{out2}$

$$b_1 = \frac{g_2 g_3 + g_4 g_5}{g_2 g_3}$$

From this, it can be seen that if the gates on each path are identical, then $b_1 = 2$.

These results hold for branches which have the same number of stages on each path (and we ignore the parasitic delay difference). We will address the issue of different number of stages in branches in the next posting.