

Solutions for Homework # 3

5.3 (a)

	a	0	1
b c	00	1	
	01		1
	11		
	10	1	1

$$f1 = a'e' + a'b'c + b'e'$$

5.3 (b)

	d	0	1
e f	00	1	1
	01	1	
	11		
	10	1	

$$f2 = d'e' + d'f' + e'f'$$

5.3 (c)

	r	0	1
s t	00	1	1
	01	1	
	11	1	
	10	1	1

$$f3 = r' + t'$$

5.3 (d)

	x	0	1
y z	00	0	1
	01	1	0
	11	1	1
	10	1	1

$$f4 = x'z + y + xz'$$

5.4 (a)

	A B	00	01	11	10
C D	00	1	1	1	1
	01			1	
	11	1		1	1
	10	1	1	1	1

$$F = B'D' + B'CD + ABC + ABC'D + B'D'$$

5.4 (b)

	A B	00	01	11	10
C D	00	1	1	1	1
	01			1	
	11	1		1	1
	10	1	1	1	1

$$F = D' + B'C + AB$$

5.4 (c)

	A B	00	01	11	10
C D	00	1	1	1	1
	01	0	0	1	0
	11	1	0	1	1
	10	1	1	1	1

$$F = (A + B + D')(B + C + D')$$

5.6 (a)

	A B	00	01	11	10
C D	00	1*		1*	
	01	1*	1*		
	11	1*	1*		1*
	10		1	1	

$$F = \underline{A'B'C'} + \underline{A'D} + \underline{B'CD} + \underline{ABD'} + B'CD'$$

$$\text{Alt: } F = \underline{A'B'C'} + \underline{A'D} + \underline{B'CD} + \underline{ABD'} + A'BC$$

A (*) indicates a minterm that makes the corresponding prime implicant essential.

$$A'D \rightarrow m_5; A'B'C' \rightarrow m_0; B'CD \rightarrow m_{11}; ABD' \rightarrow m_{12}$$

5.6 (b)

	A B	00	01	11	10
C D	00	1	1	0	1*
	01	0	1	1*	0
	11	1*	1	1	0
	10	1	1	0	1

$$F = \underline{A'C} + \underline{B'D'} + \underline{BD} + AD'$$

$$\text{Alt: } F = \underline{A'C} + \underline{B'D'} + \underline{BD} + AB$$

(*) Indicates a minterm that makes the corresponding prime implicant essential.

$$BD \rightarrow m_{13} \text{ or } m_{15}; A'C \rightarrow m_3; B'D' \rightarrow m_8 \text{ or } m_{10}$$

5.6 (c)

C D \ A B		A B			
		00	01	11	10
C D	00	1	1	1	1
	01	X	0	0	X
	11	X	0	0	1*
	10	X	1*	0	1*

$$F = \underline{A'D'} + \underline{B'} + \underline{C'D'}$$

(*) Indicates a minterm that makes the corresponding prime implicant essential.

$C'D' \rightarrow m_{12}$; $A'D' \rightarrow m_6$; $B' \rightarrow m_{10}$ or m_{11}

5.8 (a)

C D \ A B		A B			
		00	01	11	10
C D	00	0	1	0	0
	01	0	1	1	1
	11	X	X	X	0
	10	1	0	X	1

$$F = (C+D')(B'+C')(A+B+C)(A'+C+D)$$

C D \ A B		A B			
		00	01	11	10
C D	00	0	1	0	0
	01	0	1	1	1
	11	X	X	X	0
	10	1	0	X	1

$$F = A'B'C + A'CD + B'CD'$$

5.8 (b)

C D \ A B		A B			
		00	01	11	10
C D	00	0	1	X	X
	01	1	0	0	0
	11	1	X	X	1
	10	X	0	X	0

$$F = (A+C)(B'+D')(B+D)(C+D)$$

Alt: $F = (A+C)(B'+D')(B+D)(B'+C)$

C D \ A B		A B			
		00	01	11	10
C D	00	0	1	X	X
	01	1	0	0	0
	11	1	X	X	1
	10	X	0	X	0

$$F = A'B'D + B'CD' + CD$$

5.19 (a)

C D		A B			
		00	01	11	10
00	00	1		1	
	01		1	1	
11	00	1	1		1
	01	1	1		

$$F = ABC' + B'CD + A'C + A'BD' + A'BD$$

Alt: $F = ABC' + B'CD + A'C + A'BD' + B'CD$

5.19 (b)

C D		A B			
		00	01	11	10
00	00	X	1		1
	01				
11	00	X	X		
	01				
10	00	1			
	01				

$$F = A'CD' + B'CD' + A'BD'$$

Alt: $F = A'CD' + B'CD' + A'BC$

5.19 (c)

C D		A B			
		00	01	11	10
00	00		X		X
	01	1	1	1	
11	00		1		
	01		1		

$$F = A'CD + A'B + B'CD$$

5.19 (d)

Y Z		W X			
		00	01	11	10
00	00	1		1	1
	01	X	1	1	1
11	00	1	1		X
	01		X	1	

$$F = X'Y' + W'Z + YZ + W'Z'$$

Alt: $\begin{cases} F = X'Y' + W'Y' + W'Z + W'Z' \\ F = X'Y' + W'Y' + W'Z + WX' \end{cases}$

5.19 (e)

C D		A B			
		00	01	11	10
00	00	0	X	1	1
	01	0	0	X	0
11	00	1	0	1	0
	01		1	1	
10	00	0	1	1	X
	01				

$$F = AB'CD + B'D' + AD' + AB$$

7.1 (a)

c d		a b			
		00	01	11	10
c d	00	0	1	0	1
	01	0	0	0	1
	11	0	1	0	0
	10	0	1	0	1

$$f = a'b'd' + a'b'c' + a'bc + a'b'd'$$

Sum of products solution requires 5 gates, 16 inputs

c d		a b			
		00	01	11	10
c d	00	0	1	0	1
	01	0	0	0	1
	11	0	1	0	0
	10	0	1	0	1

$$f = (a'+b')(a+b)(a+c+d')(b+c'+d')$$

$$f = (a'+b')(a+b)(b+c'+d')(b'+c+d')$$

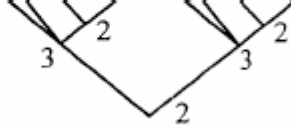
$$f = (a'+b')(a+b)(a+c+d')(a'+c'+d')$$

$$f = (a'+b')(a+b)(b'+c+d')(a'+c'+d')$$

Product of sums solution requires 5 gates, 14 inputs, so product of sums solution is minimum.

7.1 (b) Beginning with the minimum sum of products solution, we can get

$$f = a'b(c+d') + ab'(c'+d')$$

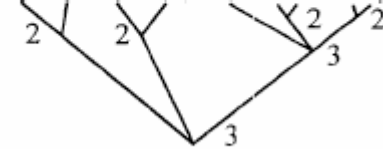


5 gates, 12 inputs

So sum of products solution is minimum.

Beginning with a minimum product of sums solution, we can get

$$f = (a+b)(a'+b')(d'+ac'+a'c)$$



6 gates, 14 inputs

7.2 (a) $AC'D + ADE' + BE' + BC' + A'D'E'$
 $= E'(AD + B) + A'D'E' + C'(AD + B)$

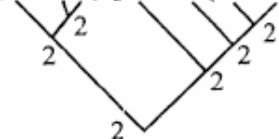
$$F = (AD + B)(E' + C') + A'D'E'$$



4 levels, 6 gates, 13 inputs

7.2 (b) $AE + BDE + BCE + BCFG + BDFG + AFG$
 $= AE + AFG + BE(C + D) + BFG(C + D)$

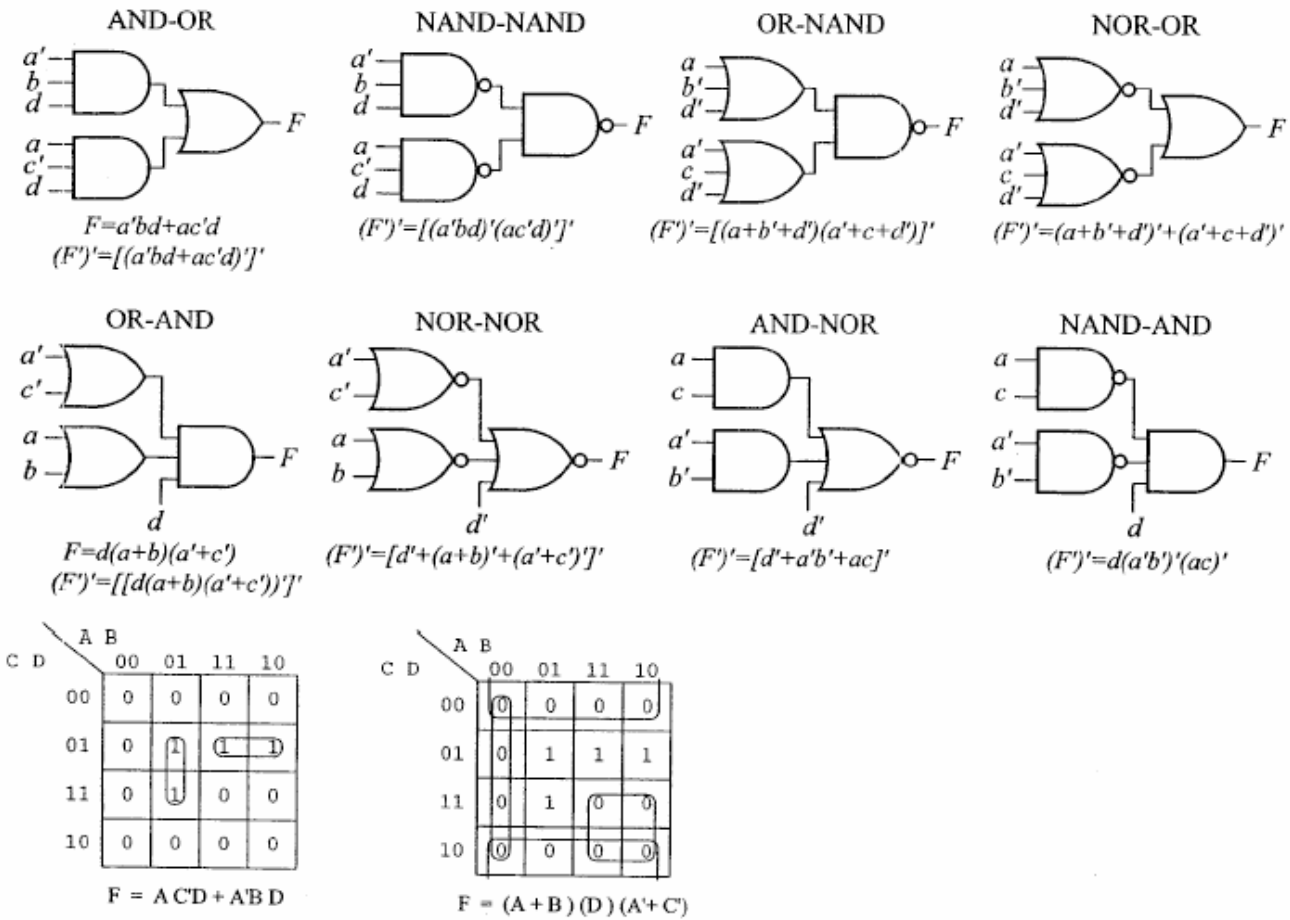
$$F = (E + FG)[A + B(C + D)]$$



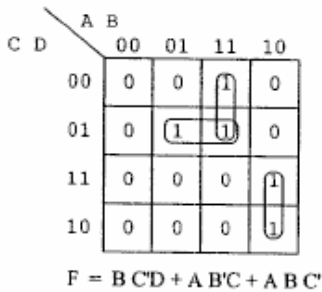
4 levels, 6 gates, 12 inputs

7.3 $F(a, b, c, d) = a'bd + ac'd$ or $d(a'b + ac') = d(a+b)(a'+c')$

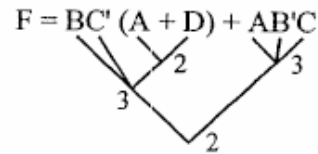
You can obtain this equation in the product of sums form using a Karnaugh map, as shown below:



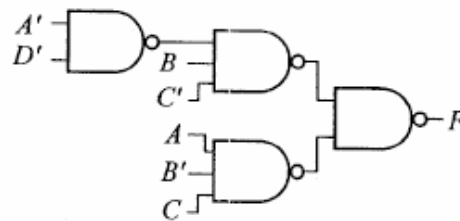
7.4 $F(A, B, C, D) = \sum m(5, 10, 11, 12, 13)$



$$F = ABC' + BC'D + A'BC = BC'(A + D) + A'BC$$



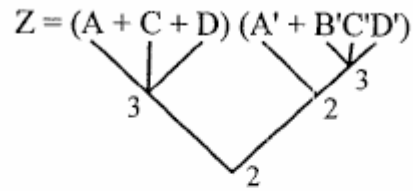
4 gates, 10 inputs



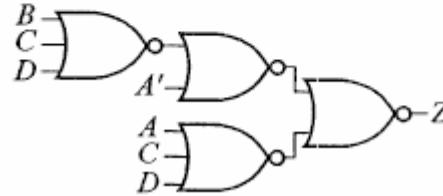
7.5

C D		A B			
		00	01	11	10
00	0	0	0	1	
01	1	1	0	0	
11	1	1	0	0	
10	1	1	0	0	

$$Z = (A + C + D)(A' + D')(A' + C')(A' + B')$$



4 gates, 10 inputs



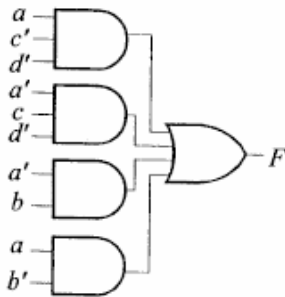
7.14 (a)

c d		a b			
		00	01	11	10
00	0	1	0	1	
01	0	1	0	1	
11	0	1	0	1	
10	0	1	0	1	

$$f = (a + b + c)(a + b + d')(a' + b' + d')(a' + b' + c')$$

5 gates, 16 inputs

and $f = a'b + ab' + b'cd' + ac'd'$
 $f = a'b + ab' + a'cd' + bc'd'$
 (two other minimum solutions)
 5 gates, 14 inputs *minimal*



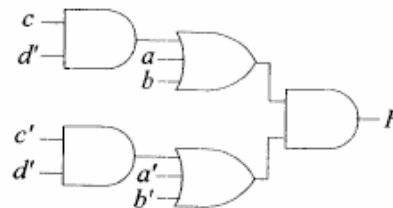
7.14 (b) Beginning with the sum of products solution, we get

$$\begin{aligned} f &= a'b + ab' + d'(a'c + ac') \\ &= a'b + ab' + d'(a' + c')(a + c) \end{aligned}$$

6 gates, 14 inputs

But, beginning with the product of sums solution above, we get

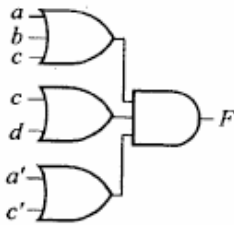
$$f = (a + b + cd')(a' + b' + c'd') \text{ — 5 gates, 12 inputs, which is minimum}$$



7.15 (a) From K-maps:

$$F = a'c + bc'd + ac'd \text{ --- 4 gates, 11 inputs}$$

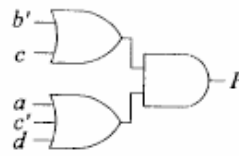
$$F = (a + b + c)(c + d)(a' + c') \text{ --- 4 gates, 10 inputs, minimal}$$



7.15 (b) From K-maps:

$$F = cd + ac + b'c' \text{ --- 4 gates, 9 inputs}$$

$$F = (b' + c)(a + c' + d) \text{ --- 3 gates, 7 inputs, minimal}$$

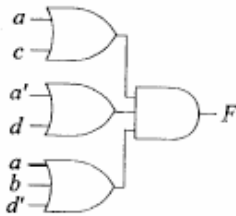


7.15 (c) From K-maps:

$$F = ad + a'cd' + bcd$$

$$= ad + a'cd' + a'bc \text{ --- 4 gates, 11 inputs}$$

$$F = (a + c)(a' + d)(a + b + d') \text{ --- 4 gates, 10 inputs, minimal}$$

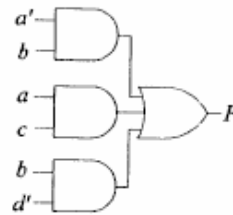


7.15 (d) From K-maps:

$$F = a'b + ac + bd' \text{ --- 4 gates, 9 inputs, minimal}$$

$$F = (a + b)(a' + c + d')(a' + b + c)$$

$$= (a + b)(a' + c + d')(b + c + d) \text{ --- 4 gates, 11 inputs}$$

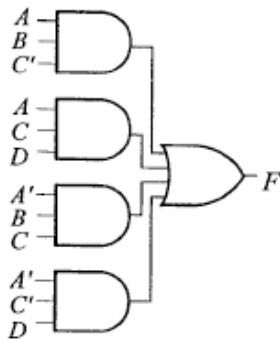


7.16 (a) In this case, multi-level circuits do not improve the solution. From K-maps:

$$F = ABC' + ACD + A'BC + A'C'D \text{ --- 5 gates, 16 inputs, minimal}$$

$$F = (A' + B + C)(A + C + D)(A' + C' + D)(A + B + C') \text{ --- 5 gates, 16 inputs, also minimal}$$

Either answer is correct.

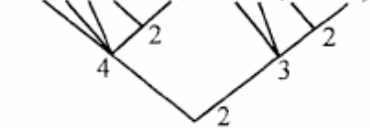


7.16 (b) Too many variables to use a K-map; use algebra. Add ACE by consensus, then use $X + XY = X$

$$\overbrace{ABCE + ABEF + ACD' + ABEG + ACDE + ACE}$$

$$= ABEF + ACD' + ABEG + ACE$$

$$F = ABE(F + G) + AC(D' + E)$$



5 gates, 13 inputs, minimal

