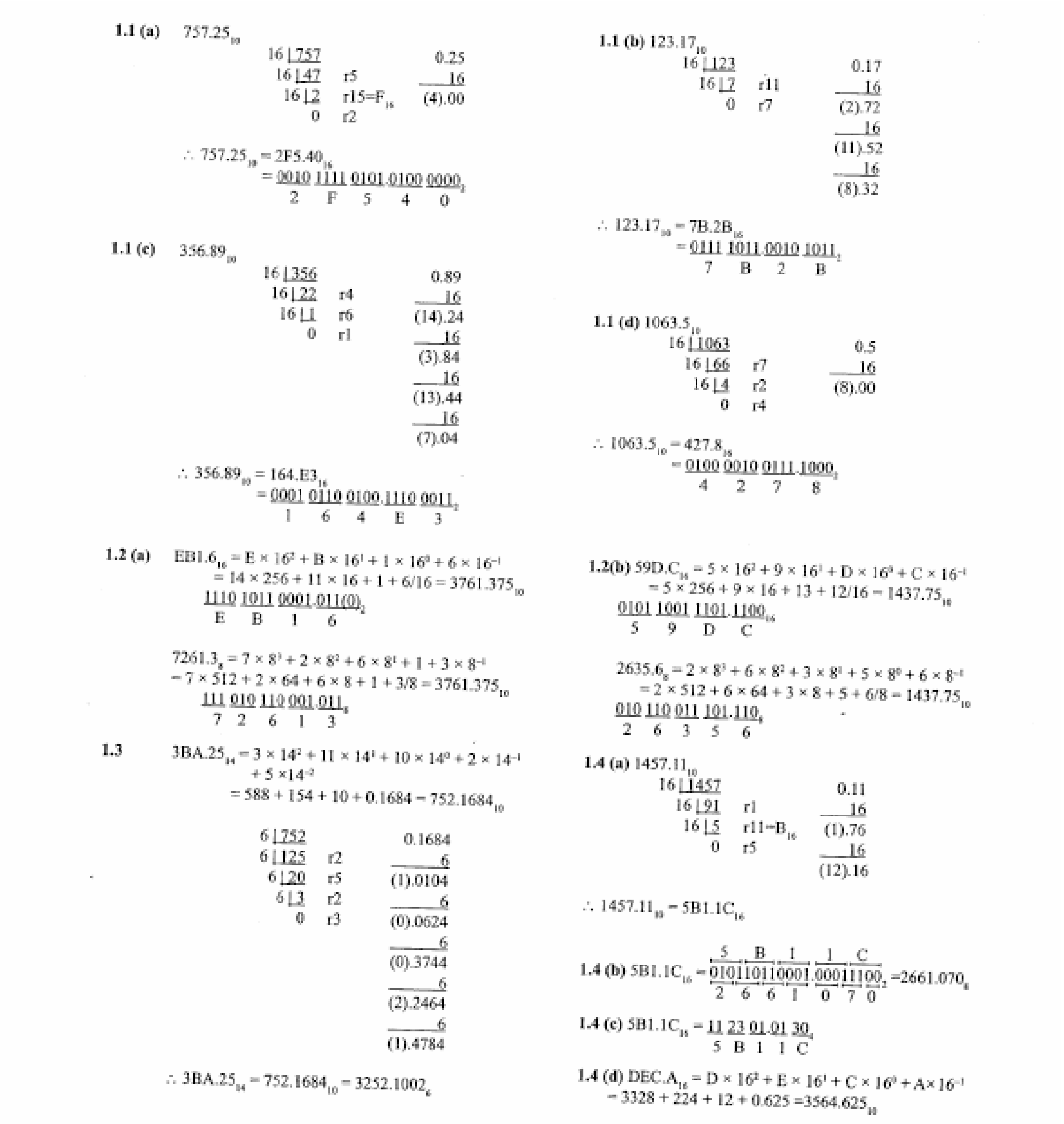
EEC180A DIGITAL SYSTEMS I Winter, 2006.

Solutions for Homework # 1



1.1 (a) 757.25_{10}

16 | 757
16 | 47 r5
16 | 2 r15=F_{16}
0 r2

0.25
× 16
(4).00

$\therefore 757.25_{10} = 2F5.40_{16}$
$= \underbrace{0010}_{2}\,\underbrace{1111}_{F}\,\underbrace{0101}_{5}.\underbrace{0100}_{4}\,\underbrace{0000}_{0}{}_2$

1.1 (c) 356.89_{10}

16 | 356
16 | 22 r4
16 | 1 r6
0 r1

0.89
× 16
(14).24
× 16
(3).84
× 16
(13).44
× 16
(7).04

$\therefore 356.89_{10} = 164.E3_{16}$
$= \underbrace{0001}_{1}\,\underbrace{0110}_{6}\,\underbrace{0100}_{4}.\underbrace{1110}_{E}\,\underbrace{0011}_{3}{}_2$

1.2 (a) $EB1.6_{16} = E \times 16^2 + B \times 16^1 + 1 \times 16^0 + 6 \times 16^{-1}$
$= 14 \times 256 + 11 \times 16 + 1 + 6/16 = 3761.375_{10}$
$\underbrace{1110}_{E}\,\underbrace{1011}_{B}\,\underbrace{0001}_{1}.\underbrace{011(0)}_{6}{}_2$

$7261.3_8 = 7 \times 8^3 + 2 \times 8^2 + 6 \times 8^1 + 1 + 3 \times 8^{-1}$
$= 7 \times 512 + 2 \times 64 + 6 \times 8 + 1 + 3/8 = 3761.375_{10}$
$\underbrace{111}_{7}\,\underbrace{010}_{2}\,\underbrace{110}_{6}\,\underbrace{001}_{1}.\underbrace{011}_{3}{}_8$

1.3 $3BA.25_{14} = 3 \times 14^2 + 11 \times 14^1 + 10 \times 14^0 + 2 \times 14^{-1}$
$+ 5 \times 14^{-2}$
$= 588 + 154 + 10 + 0.1684 = 752.1684_{10}$

6 | 752
6 | 125 r2
6 | 20 r5
6 | 3 r2
0 r3

0.1684
× 6
(1).0104
× 6
(0).0624
× 6
(0).3744
× 6
(2).2464
× 6
(1).4784

$\therefore 3BA.25_{14} = 752.1684_{10} = 3252.1002_6$

1.1 (b) 123.17_{10}

16 | 123
16 | 7 r11
0 r7

0.17
× 16
(2).72
× 16
(11).52
× 16
(8).32

$\therefore 123.17_{10} = 7B.2B_{16}$
$= \underbrace{0111}_{7}\,\underbrace{1011}_{B}.\underbrace{0010}_{2}\,\underbrace{1011}_{B}{}_2$

1.1 (d) 1063.5_{10}

16 | 1063
16 | 66 r7
16 | 4 r2
0 r4

0.5
× 16
(8).00

$\therefore 1063.5_{10} = 427.8_{16}$
$= \underbrace{0100}_{4}\,\underbrace{0010}_{2}\,\underbrace{0111}_{7}.\underbrace{1000}_{8}{}_2$

1.2(b) $59D.C_{16} = 5 \times 16^2 + 9 \times 16^1 + D \times 16^0 + C \times 16^{-1}$
$= 5 \times 256 + 9 \times 16 + 13 + 12/16 = 1437.75_{10}$
$\underbrace{0101}_{5}\,\underbrace{1001}_{9}\,\underbrace{1101}_{D}.\underbrace{1100}_{C}{}_{16}$

$2635.6_8 = 2 \times 8^3 + 6 \times 8^2 + 3 \times 8^1 + 5 \times 8^0 + 6 \times 8^{-1}$
$= 2 \times 512 + 6 \times 64 + 3 \times 8 + 5 + 6/8 = 1437.75_{10}$
$\underbrace{010}_{2}\,\underbrace{110}_{6}\,\underbrace{011}_{3}\,\underbrace{101}_{5}.\underbrace{110}_{6}{}_8$

1.4 (a) 1457.11_{10}

16 | 1457
16 | 91 r1
16 | 5 r11=B_{16}
0 r5

0.11
× 16
(1).76
× 16
(12).16

$\therefore 1457.11_{10} = 5B1.1C_{16}$

1.4 (b) $5B1.1C_{16} = \overbrace{0101}^{5}\overbrace{1011}^{B}\overbrace{0001}^{1}.\overbrace{0001}^{1}\overbrace{1100}^{C}{}_2 = 2661.070_8$
$= \underbrace{010}_{2}\underbrace{110}_{6}\underbrace{110}_{6}\underbrace{001}_{1}.\underbrace{000}_{0}\underbrace{111}_{7}\underbrace{00}_{0}{}_2$

1.4 (c) $5B1.1C_{16} = \underbrace{11}_{5}\,\underbrace{23}_{B}\,\underbrace{01}_{1}.\underbrace{01}_{1}\,\underbrace{30}_{C}{}_4$

1.4 (d) $DEC.A_{16} = D \times 16^2 + E \times 16^1 + C \times 16^0 + A \times 16^{-1}$
$= 3328 + 224 + 12 + 0.625 = 3564.625_{10}$

1.5 (a)

$$\begin{array}{r} 111 \\ 1111 \\ +1010 \\ \hline 11001 \end{array} \quad \text{(Add)} \qquad \begin{array}{r} 1111 \\ -1010 \\ \hline 0101 \end{array} \quad \text{(Sub)} \qquad \begin{array}{r} 1111 \\ \times 1010 \\ \hline 0000 \\ 1111 \\ \hline 11110 \\ 0000 \\ \hline 011110 \\ 1111 \\ \hline 10010110 \end{array} \quad \text{(Multiply)}$$

1.5 (b, c) *See FLD p. 625 for solution.*

1.6 (a)
$$\begin{array}{r} 1111 \\ 11110100 \\ -\ 1000111 \\ \hline 10101101 \end{array}$$

(b)
$$\begin{array}{r} 111\ \ \ 1 \\ 1110110 \\ -\ 111101 \\ \hline 0111001 \end{array}$$

(c)
$$\begin{array}{r} 11111\ 1 \\ 10110010 \\ -\ \ 111101 \\ \hline 01110101 \end{array}$$

1.7 2's complement:

(a)
$$\begin{array}{r} 010101 \\ +\ 001011 \\ \hline 100000 \end{array}$$

OVERFLOW!

(b)
$$\begin{array}{r} 110010 \\ +\ 100000 \\ \hline (1)\ 010010 \end{array}$$

OVERFLOW!

(c)
$$\begin{array}{r} 100111 \\ +\ 010010 \\ \hline 111001 \end{array}$$

(d)
$$\begin{array}{r} 110100 \\ +\ 001101 \\ \hline (1)\ 000001 \end{array}$$

(e)
$$\begin{array}{r} 110101 \\ +\ 101011 \\ \hline (1)\ 100000 \end{array}$$

1's complement:

(a)
$$\begin{array}{r} 010101 \\ +\ 001011 \\ \hline 100000 \end{array}$$

OVERFLOW!

(b) not assigned
because –32 cannot
be represented
in 6 bits

(c)
$$\begin{array}{r} 100110 \\ +\ 010010 \\ \hline 111000 \end{array}$$

(d)
$$\begin{array}{r} 110011 \\ +\ 001101 \\ \hline (1)\ 000000 \\ +\qquad\quad 1 \\ \hline 000001 \end{array}$$

(e)
$$\begin{array}{r} 110100 \\ +\ 101010 \\ \hline (1)\ 011110 \\ +\qquad\quad 1 \\ \hline 011111 \end{array}$$

OVERFLOW!

1.8 For a word length of N, the range of 2's complement numbers that can be represented is -2^{N-1} to $2^{N-1} - 1$.

So, for a word length of 8, the range is -2^7 to $2^7 - 1$, or *-128 to 127*. Because 1's complement has a "negative zero" (11111111) in addition to zero (00000000), the values that can be represented range from $-(2^7 - 1)$ to $2^7 - 1$, or *-127 to 127*.

1.10 (a) 1305.375_{10}

$$\begin{array}{rll} 16\,\underline{|\,1305} & & 0.375\\ 16\,\underline{|\,81} & r9 & \underline{\quad\ 16}\\ 5 & r1 & (6).000 \end{array}$$

$$\therefore 1305.375_{10} = 519.600_{16}$$
$$= \underset{5}{\underline{0101}}\ \underset{1}{\underline{0001}}\ \underset{9}{\underline{1001}}.\underset{6}{\underline{0110}}\ \underset{0}{\underline{0000}}\ \underset{0}{\underline{0000}}_2$$

1.10 (c) 301.12_{10}

$$\begin{array}{rll} 16\,\underline{|\,301} & & 0.12\\ 16\,\underline{|\,18} & r13 & \underline{\quad\ 16}\\ 1 & r2 & (1).92\\ & & \underline{\quad\ 16}\\ & & (14).72 \end{array}$$

$$\therefore 301.12_{10} = 12D.1E_{16}$$
$$= \underset{1}{\underline{0001}}\ \underset{2}{\underline{0010}}\ \underset{D}{\underline{1101}}.\underset{1}{\underline{0001}}\ \underset{E}{\underline{1110}}_2$$

1.11 (a) $101\ 111\ 010\ 100.101_2 = 5724.5_8$

$= 5 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 4 \times 8^0 + 5 \times 8^{-1}$

$= 5 \times 512 + 7 \times 64 + 2 \times 8 + 4 + 5/8$

$= 3028.625_{10}$

$1011\ 1101\ 0100.1010_2 = BD4.A_{16}$

$B \times 16^2 + D \times 16^1 + 4 \times 16^0 + A \times 16^{-1}$

$11 \times 256 + 13 \times 16 + 4 + 10/16$

$= 3028.625_{10}$

1.12 (a) $375.54_8 = 3 \times 64 + 7 \times 8 + 5 + 5/8 + 4/64 = 253.6875_{10}$

$$\begin{array}{rll} 3\,\underline{|\,253} & & 0.69\\ 3\,\underline{|\,84} & r1 & \underline{\quad\ 3}\\ 3\,\underline{|\,28} & r0 & (2).07\\ 3\,\underline{|\,9} & r1 & \underline{\quad\ 3}\\ 3\,\underline{|\,3} & r0 & (0).21\\ 3\,\underline{|\,1} & r0 & 3\\ 0 & r1 & (0).63\\ & & 3\\ & & (1).89 \end{array}$$

$$\therefore 375.54_8 = 100101.2001_3$$

1.10 (b) 111.33_{10}

$$\begin{array}{rll} 16\,\underline{|\,111} & & 0.33\\ 6 & r15 = F_{16} & \underline{\quad\ 16}\\ & & (5).28\\ & & \underline{\quad\ 16}\\ & & (4).48 \end{array}$$

$$\therefore 111.33_{10} = 6F.54_{16}$$
$$= \underset{6}{\underline{0110}}\ \underset{F}{\underline{1111}}.\underset{5}{\underline{0101}}\ \underset{4}{\underline{0100}}_2$$

1.10 (d) 1644.875_{10}

$$\begin{array}{rll} 16\,\underline{|\,1644} & & 0.875\\ 16\,\underline{|\,102} & r12 & \underline{\quad\ 16}\\ 6 & r6 & (14).000 \end{array}$$

$$\therefore 1644.875_{10} = 66C.E00_{16}$$
$$= \underset{6}{\underline{0110}}\ \underset{6}{\underline{0110}}\ \underset{C}{\underline{1100}}.\underset{E}{\underline{1110}}\ \underset{0}{\underline{0000}}\ \underset{0}{\underline{0000}}_2$$

1.11 (b) $100\ 001\ 101\ 111.010_2 = 4157.2_8$

$= 4 \times 8^3 + 1 \times 8^2\ \ 5 \times 8^1 + 7 \times 8^0 + 2 \times 8^{-1}$

$= 4 \times 512 + 1 \times 64 + 5 \times 8 + 7 + 2/8$

$= 2159.25_{10}$

$1000\ 0110\ 1111.0100_2 = 86F.4_{16}$

$= 8 \times 16^2 + 6 \times 16^1 + F \times 16^0 \times 4 \times 16^{-1}$

$= 8 \times 256 + 6 \times 16 + 15 + 4/16$

$= 2159.25_{10}$

1.12 (b) 384.74_{10}

$$\begin{array}{rll} 4\,\underline{|\,384} & & 0.74\\ 4\,\underline{|\,96} & r0 & \underline{\quad\ 4}\\ 4\,\underline{|\,24} & r0 & (2).96\\ 4\,\underline{|\,6} & r0 & \underline{\quad\ 4}\\ 4\,\underline{|\,1} & r2 & (3).84\\ 0 & r1 & 4\\ & & (3).36 \end{array}$$

$$\therefore 384.74_{10} = 12000.233113_4\ldots$$

1.12 (c) $A52.A4_{11} = 10 \times 121 + 5 \times 11 + 2 + 10/11 + 4/121$

$= 1267.94_{10}$

$$\begin{array}{rll} 9\,\underline{|\,1267} & & 0.94\\ 9\,\underline{|\,140} & r7 & \underline{\quad\ 9}\\ 9\,\underline{|\,15} & r5 & (8).46\\ 9\,\underline{|\,1} & r6 & \underline{\quad\ 9}\\ 0 & r1 & (4).14 \end{array}$$

$$\therefore A52.A4_{11} = 1267.94_{10} = 1657.8427_9\ldots$$