Problem 1 (20 points)

Using algebraic manipulations (without using Karnaugh maps) prove that:

$$F = \underbrace{(\overline{A}+\overline{B}+\overline{D})(\overline{A}+B+\overline{D})}(B+C+D)\underbrace{(A+\overline{C})(A+\overline{C}+D)} = \overline{A}\,\overline{C}D+AC\overline{D}+B\overline{C}\,\overline{D}$$

$$= (\overline{A}+\overline{D})(B+C+D)(A+\overline{C}) =$$

$$= (\overline{A}+\overline{D})(A+\overline{C})(B+C+D) =$$

$$= (\overline{A}\,\overline{C} + A\overline{D} + \overline{D}\,\overline{C})(B+C+D) =$$

$$= \cancel{\overline{A}B\overline{C}} + \overline{A}\,\overline{C}D + \cancel{AB\overline{D}} + AC\overline{D} + B\overline{C}\,\overline{D} =$$

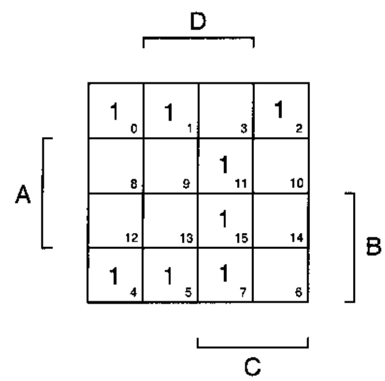
cons.

cons.

$$= \overline{A}\,\overline{C}D + AC\overline{D} + B\overline{C}\,\overline{D}$$

Problem 2 (25 points)

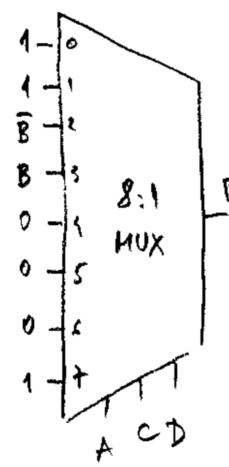
Realize the given function F = F(A,B,C,D):



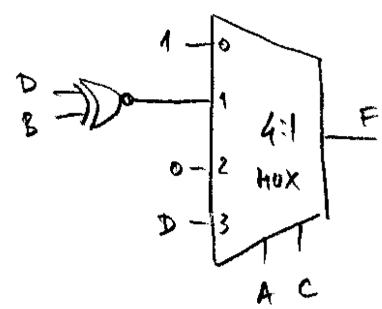
a) using one 8:1 MUX with control inputs A, C and D

b) using one 4:1 MUX. Select the control inputs to minimize the number of added gates.

a)



b)



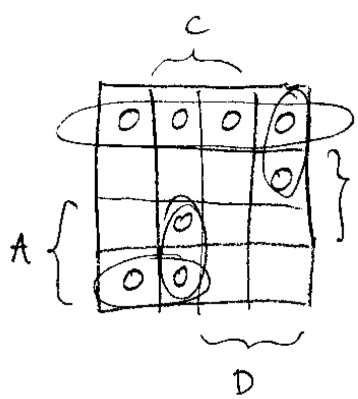
Problem 3 (30 points)

Show that:

F = ABD+BCD+ABC+ABD reduces to F = BCD+AD+ABC

You can use **Karnaugh map** (if you wish) – however you are allowed to use only **product of sums** (i.e. you can only minimize them using zeros).

List the **essential prime implicants** for this expression.



$$F = (A+B)(B+D)(A+C+\bar{D})(\bar{A}+\bar{C}+D)$$

$$F = (AD+B)(\bar{A}C + \bar{A}\bar{D} + A\bar{C} + \bar{C}\bar{D} + AD + CD) =$$

$$= (AD+B)(\bar{A}(C+\bar{D}) + A(\bar{C}+D)) =$$

$$= \bar{A}B(C+\bar{D}) + AD(\bar{C}+D) + AB(\bar{C}+D) =$$

$$= \bar{A}BC + \underline{\bar{A}B\bar{D}} + AD + \cancel{ABD} + \underline{AB\bar{C}}$$

$$= \bar{A}BC + AD + B\bar{C}\bar{D}$$

EPI : AD