PROBLEM No.1 (15 points)

Reduce the following state table to a minimum number of states:

Present State Sn	Next State Sn+1		Output Z	
	X=0	X=1	X=0	X=1
a	h	c	1	0
b	c	d	0	1
c	h	b	0	0
d	f	h	0	0
e	c	f	0	1
f	f	g	0	0
g	g	c	1	0
h	a	c	1	0

a) [10] Write the resulting state table. Determine the equivalent states - show them !

b) [3] Draw the resulting state diagram with all states, transitions and outputs !

c) [2] Mark the transitions that are producing Z=1 as an output on the state diagram (circle them !)

	a	b	c	d	e	f	g
b	X						
c	X	X					
d	X	X	~~f-h b-h~~				
e	X	~~d-f~~	X	X			
f	X	X	X	h-g	X		
g	h-g	X	X	X	X	X	
h	h-a	X	X	X	X	X	a-g

self-implying

equivalent states:

$a \equiv g \equiv h$

$d \equiv f$

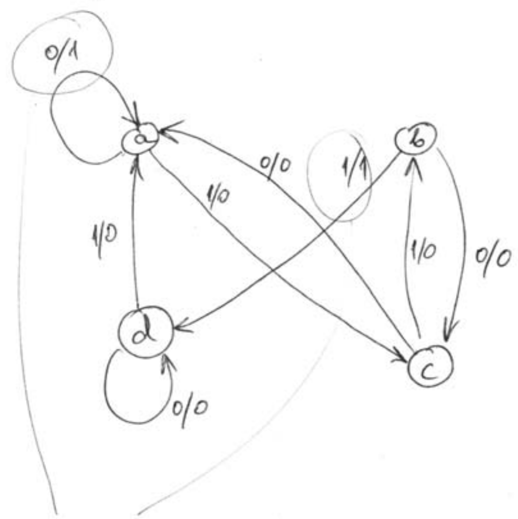
$e \equiv b$

(extra space for Problem 1)

new table:

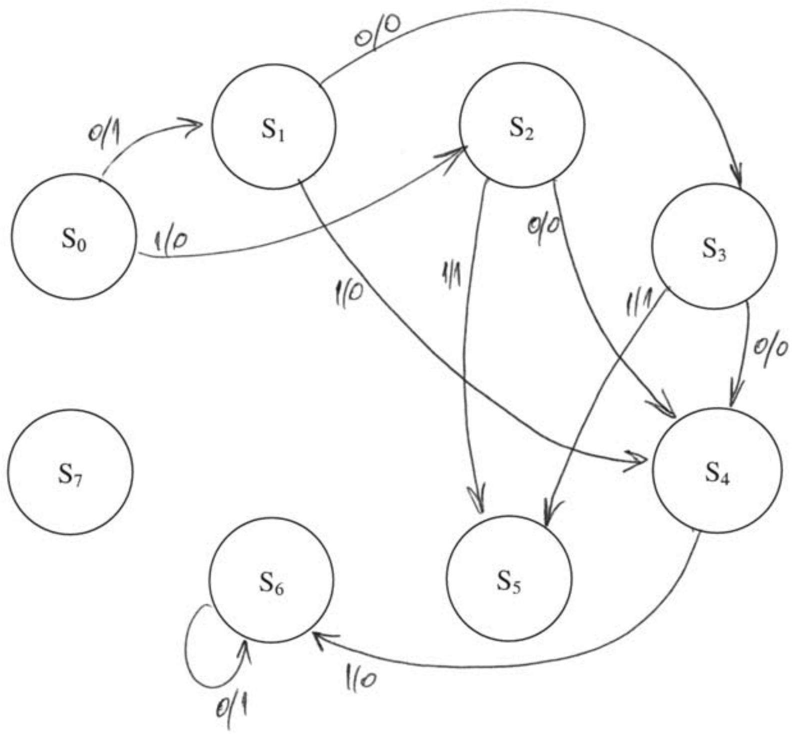
S_n	S_{n+1}		z	
	$X=0$	$X=1$	$X=0$	$X=1$
a	a	c	1	0
b	c	d	0	1
c	a	b	0	0
d	d	a	0	0

state diagram:

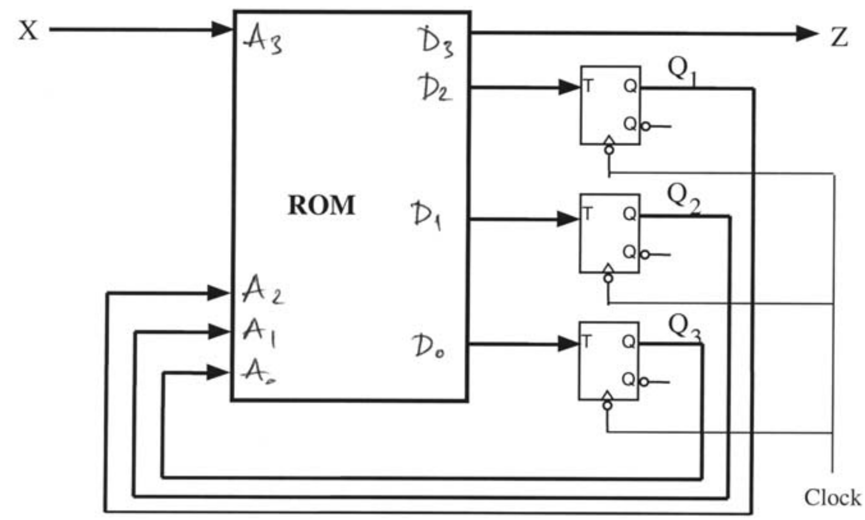


these transitions produce $z=1$

Problem 2.
correction: There is also a transition from S5 to S0 for X/Z = 0/0 and 1/0. Also, there is a transition from S4 to S5 for X/Z = 0/1.

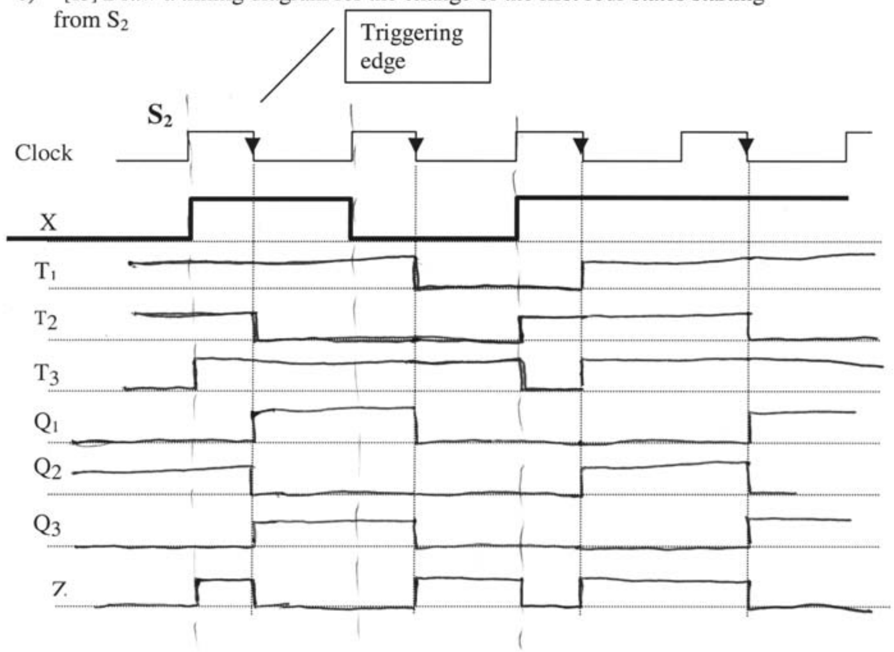


- Mealy FSM
- problems:
  - S_6 - can't get out from S_6
  - S_7 - after power-up ⇒ undefined transition



Address A3 A2 A1 A0	Content of ROM D3 D2 ~~D2~~ D1 D0	next state
0000	1 0 0 1	0 0 1
0001	0 0 1 0	0 1 1
0010	0 1 1 0	1 0 0
0011	0 1 1 1	1 0 0
0100	1 0 0 1	1 0 1
0101	0 1 0 1	0 0 0
0110	1 0 0 0	1 1 0
0111	x x x x	x x x
1000	0 0 1 0	0 1 0
1001	0 1 0 1	1 0 0
1010	1 1 1 1	1 0 1
1011	1 1 1 0	1 0 1
1100	0 0 1 0	1 1 0
1101	0 1 0 1	0 0 0
1110	x x x x	x x x
1111	x x x x	x x x
$X\ Q_1\ Q_2\ Q_3$	$Z\ T_1\ T_2\ T_3$	$Q_1^+ Q_2^+ Q_3^+$

c) [15] Draw a timing diagram for the change of the first four states starting from S_2



d) Suppose the ROM in part a) is to be replaced by a PLA:

- [6] Write **minimal form** equations for the inputs to the T flip-flops, called respectively T_1, T_2 and T_3?

- [2] Write an equation for the output Z that can be realized as part of the PLA.

$Q_2Q_3 \backslash XQ_1$	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	1	X	X	1
10	1	0	X	1

$$\Rightarrow T_1 = \bar{Q}_1 Q_2 + Q_1 Q_3 + X Q_3$$

$Q_2Q_3 \backslash XQ_1$	00	01	11	10
00	0	0	1	1
01	1	0	0	0
11	1	X	X	1
10	1	0	X	1

$$T_2 = \bar{Q}_1 Q_2 + X \bar{Q}_3 + \bar{X} \bar{Q}_1 Q_3$$

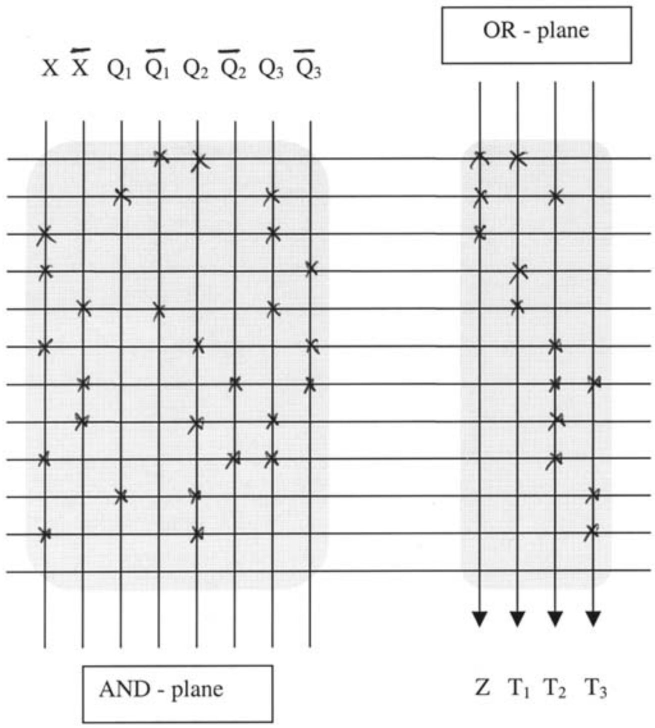
$Q_2Q_3 \backslash XQ_1$	00	01	11	10
00	1	1	0	0
01	0	1	1	1
11	1	X	X	0
10	0	0	X	1

$Q_2Q_3 \backslash XQ_1$	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	X	X	1
10	0	1	X	1

$$T_3 = Q_1 Q_3 + \bar{X} \bar{Q}_2 \bar{Q}_3 + X Q_2 \bar{Q}_3 + X \bar{Q}_2 Q_3 + \bar{X} Q_2 Q_3$$

$$Z = Q_1 Q_2 + X Q_2 + \bar{X} \bar{Q}_2 \bar{Q}_3$$

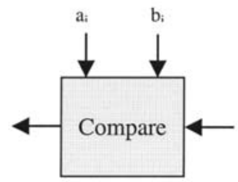
e) [3] Complete the "stick-diagram" PLA representation of the part d).



correction: In the last row of the table (for $G_i = A_i = B_i = 1$), G_{i+1} should be 1 instead of 0.

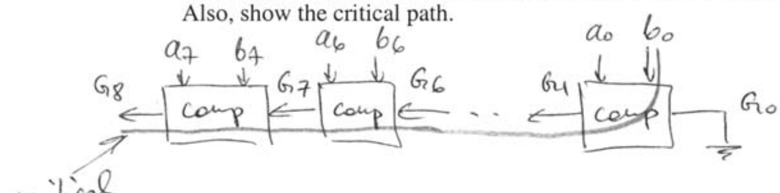
PROBLEM No. 3 (15 points)

Design an iterative network for comparing two unsigned binary numbers. These numbers are designated as: $a = a_n \ldots\ldots\ldots a_0$, $b = b_n \ldots\ldots\ldots b_0$. The comparator checks if a > b! A typical cell for the comparator is shown:



Note that there is a signal propagating between the cells. What is the function of that signal ? How would you use it ?

(a.) [5] Draw a schematic of a comparator comparing two 8-bit words using these cells. Explain how it works and connect any signals that need to be connected. Also, show the critical path.



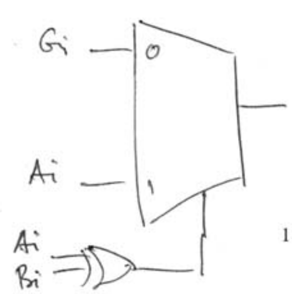
$$G_{i+1} = 1 \Leftrightarrow a_i a_{i-1} \cdots a_0 > b_i b_{i-1} \cdots b_0$$

(b.) [10] Derive the logic inside this cell and write equations for this logic. Show the most efficient logic implementation for this comparator cell (a cell can be realized using one MUX and one logic gate).

G_i	A_i	B_i	G_{i+1}
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

B_i \ G_iA_i	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$$\Rightarrow G_{i+1} = G_iA_i + G_i\bar{B_i} + A_i\bar{B_i}$$



Problem 4.

(extra space for Problem 4)

A B C	J_A K_A	J_B K_B	J_C K_C	A^+ B^+ C^+
0 0 0	1 0	1 0	1 1	1 1 1
0 0 1	0 1	0 1	1 1	0 0 0
0 1 0	1 1	1 0	0 0	1 1 0
0 1 1	0 0	0 1	0 0	0 0 1
1 0 0	1 0	1 0	1 0	1 1 1
1 0 1	0 1	1 1	1 0	0 1 1
1 1 0	1 1	1 0	1 0	0 1 1
1 1 1	0 0	1 1	1 0	1 0 1

C \ AB	00	01	11	10
0	1	1	0	1
1	0	0	1	0

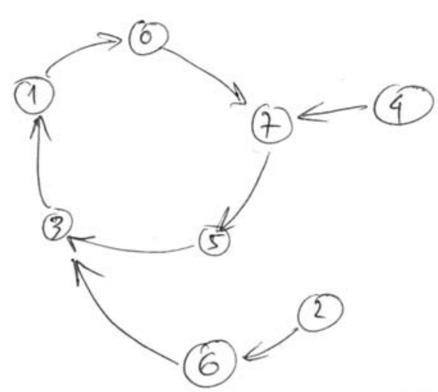
$A^+ = \bar{A}\bar{C} + \bar{B}\bar{C} + ABC$

C \ AB	00	01	11	10
0	1	1	1	1
1	0	0	0	1

$B^+ = \bar{C} + A\bar{B}$

C \ AB	00	01	11	10
0	1	0	1	1
1	0	1	1	1

$C^+ = A + \bar{B}\bar{C} + BC$



→ self-starting