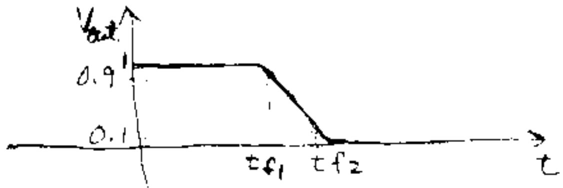
EEC 118

Quiz solution



fall time

KCL at output

nMOS saturated

$$C_L \frac{V_{out}}{dt} + \frac{\beta_n}{2}(V_{GS}-V_{tn})^2 = 0 \quad , \quad V_{GS} = V_{DD}$$

$$t_{f1} = 2\frac{C_L}{\beta_n (V_{DD}-V_{tn})^2}\int_{V_{DD}-V_{tn}}^{0.9V_{DD}} dV_{out} = \frac{2C_L}{\beta_n (V_{DD}-V_{tn})^2} V_{out}\Big|_{V_{DD}-V_{tn}}^{0.9V_{DD}} =$$

$$\frac{2C_L}{\beta_n (V_{DD}-V_{tn})^2}(0.9V_{DD}-(V_{DD}-V_{tn})) = \frac{2C_L}{\beta_n (V_{DD}-V_{tn})^2}(V_{tn}-0.1V_{DD})$$

KCL at output

nMOS linear

$$C_L \frac{dV_{out}}{dt} + \beta_n\left[(V_{GS}-V_{tn})V_{DS} - \frac{V_{DS}^2}{2}\right] = 0 \quad , \quad V_{DD} = V_{GS} \;,\; V_{DS} = V_{out}$$

$$t_{f2} = \frac{C_L}{\beta_n (V_{DD}-V_{tn})}\int_{0.1V_{DD}}^{V_{DD}-V_{tn}} \left[\frac{V_{out}^2}{2(V_{DD}-V_{tn})} - V_{out}\right]^{-1} dV_{out}$$

in order to solve this integral, need to use partial faction expansion and let $2(V_{DD}-V_{tn}) = x$

$$\left[\frac{V_{out}^2}{x} - V_{out}\right]^{-1} = \frac{x}{V_{out}(V_{out}-x)}$$

$$\frac{x}{V_{out}(V_{out}-x)} = \frac{A}{V_{out}-x} + \frac{B}{V_{out}} \longrightarrow \text{solve} \quad A = 1,\; B = -1$$

$$\int \frac{dV_{out}}{V_{out}-x} - \int \frac{dV_{out}}{V_{out}} = \ln(V_{out}-x) - \ln(V_{out})$$

$$\ln(V_{out}-x) - \ln(V_{out})\Big|_{0.1V_{DD}}^{V_{DD}-V_{tn}} =$$

$$\ln(V_{DD}-V_{tn}-2(V_{DD}-V_{tn})) - \ln(V_{DD}-V_{tn}) - [\ln(0.1V_{DD}-2V_{DD}-V_{tn}) - \ln(0.1V_{DD})] =$$

$$\ln\left(\frac{19V_{DD}-20V_{tn}}{V_{DD}}\right) \cdot \frac{C_L}{\beta_n (V_{DD}-V_{tn})};$$

constant term

Let $n = \frac{V_{tn}}{V_{DD}}$

$$t_f = t_{f2} - t_{f1} = \frac{2C_L}{\beta_n V_{DD}(1-n)}\left[\frac{(n-0.1)}{(1-n)} + \frac{1}{2}\ln(19-20n)\right]$$