EEC 118 Homework #2 Solutions

Problem 1.1

Assume an abrupt junction (m=0.5). V_D is defined as positive when we forward bias the junction (Rabaey p. 76,82,83,111).

$$\phi_T = \frac{kT}{q} = 0.026V, \quad \phi_0 = \phi_T \ln\left(\frac{N_A N_D}{N_i^2}\right) = 0.026\ln\left(\frac{10^{16}\times 10^{20}}{\left(1.5\times 10^{10}\right)^2}\right) = 0.933V$$

$$C_{j0} = \sqrt{\left(\frac{\varepsilon_{si} q}{2}\frac{N_A N_D}{N_A + N_D}\right)\phi_0^{-1}} = \sqrt{\left(\frac{11.7\times 8.854\times 10^{-14}\times 1.6\times 10^{-19}}{2}\frac{10^{16}\times 10^{20}}{10^{16}+10^{20}}\right)\frac{1}{0.933}}$$
$$= 2.98\times 10^{-8}\, F/cm^2$$

$$C_j = \frac{C_{j0}}{\left(1-\frac{V_D}{\phi_0}\right)^m} = \frac{2.98\times 10^{-8}}{\left(1-\frac{-5}{0.933}\right)^{0.5}} = 1.18\times 10^{-8}\, F/cm^2$$

$$\phi_{0+} = \phi_T \ln\left(\frac{N_{A+} N_D}{N_i^2}\right) = 0.026\ln\left(\frac{10^{19}\times 10^{20}}{\left(1.5\times 10^{10}\right)^2}\right) = 1.111V$$

$$C_{jsw0} = \sqrt{\left(\frac{\varepsilon_{si} q}{2}\frac{N_{A+} N_D}{N_{A+} + N_D}\right)\phi_{0+}^{-1}} = \sqrt{\left(\frac{11.7\times 8.854\times 10^{-14}\times 1.6\times 10^{-19}}{2}\frac{10^{19}\times 10^{20}}{10^{19}+10^{20}}\right)\frac{1}{1.111}}$$
$$= 8.23\times 10^{-7}\, F/cm^2$$

$$C_{jsw} = \frac{C_{jsw0}}{\left(1-\frac{V_D}{\phi_{0+}}\right)^m} = \frac{8.23\times 10^{-7}}{\left(1-\frac{-5}{1.111}\right)^{0.5}} = 3.51\times 10^{-7}\, F/cm^2$$

$$C_{diff} = C_{bottom} + C_{sw} = C_j \times AREA + C_{jsw}\times PERIMETER = C_j L_S W + C_{jsw} x_j \left(2L_S + W\right)$$
$$= 1.18\times 10^{-8}\times 5\times 10^{-4}\times 10\times 10^{-4} + 3.51\times 10^{-7}\times 0.4\times 10^{-4}\left(2\times 5\times 10^{-4} + 10\times 10^{-4}\right)$$
$$= \underline{\underline{34\times 10^{-15} F}}$$

$$V_D = 2.5 \Rightarrow C_{diff} = \underline{\underline{44\times 10^{-15} F}}$$

Problem 1.2

$$C_{OX} = \frac{\varepsilon_{OX}}{t_{OX}} = \frac{3.9\times 8.854\times 10^{-14}}{200\times 10^{-8}} = 1.7\times 10^{-7}\, F/cm^2$$

$$C_{GD}(overlap) = C_{OX} W x_d = \left(1.7\times 10^{-7}\right)\left(10\times 10^{-4}\right)\left(0.25\times 10^{-4}\right) = \underline{\underline{4.31\times 10^{-15} F}}$$

Problem 2.1

For V_{OH}, The load transistor is diode connected and forced to operate in saturation. This requires at least a threshold drop across the transistor, and so the NMOS cannot pull the output all the way up to V_{DD}.

$$V_{T,LOAD} = V_{T0,LOAD} + \gamma\left(\sqrt{|2\phi_F + V_{OH}|} - \sqrt{|2\phi_F|}\right)$$

$$V_{OH} = V_{DD} - V_{T,LOAD} \Rightarrow V_{T,LOAD} = V_{DD} - V_{OH}$$

$$V_{DD} - V_{OH} = V_{T0,LOAD} + \gamma\left(\sqrt{|2\phi_F + V_{OH}|} - \sqrt{|2\phi_F|}\right)$$
$$\Rightarrow 5 - 0.8 = V_{OH} + 0.38\sqrt{0.6 + V_{OH}} - 0.38\sqrt{0.6}$$

$$(4.5 - V_{OH})^2 = \left(0.38\sqrt{0.6 + V_{OH}}\right)^2$$
$$\Rightarrow V_{OH}^{\;2} - 9.132V_{OH} + 20.109 = 0 \Rightarrow V_{OH} = \underline{\underline{3.706V}}$$

For V_{OL}, the load operates in saturation, while the driver is linear.

$$I_{driver} = \frac{\mu C_{OX}}{2}\left(\frac{W}{L}\right)_d \left(2(V_{DD} - V_{T0})V_{OL} - V_{OL}^{\;2}\right)$$

$$I_{load} = \frac{\mu C_{OX}}{2}\left(\frac{W}{L}\right)_l \left(V_{DD} - V_{OL} - V_{T,load}\right)^2$$
$$\Rightarrow I_{load} = \frac{\mu C_{OX}}{2}\left(\frac{W}{L}\right)_l \left(V_{DD} - V_{OL} - V_{T0} - \gamma\left(\sqrt{|2\phi_F + V_{OL}|} - \sqrt{|2\phi_F|}\right)\right)^2$$

$$I_{driver} = I_{load}$$

$$I_{load} = 22.5\left(4.494 - V_{OL} - 0.38\sqrt{0.6 + V_{OL}}\right)^2$$
$$I_D = 1512V_{OL} - 180V_{OL}^{\;2}$$

Using these two equations, we can iteratively solve for V_{OL}.

V_{OL} (V)	I (μA)
1	204.3
0.1373	365.5
0.2491	341.3
0.2346	344.4
0.2342	344.5
0.2343	344.4

$$V_{OL} = \underline{\underline{0.2343V}}$$

Problem 2.2

$$NM_L = V_{IL} - V_{OL} \Rightarrow V_{IL} - 0.23$$
$$NM_H = V_{OH} - V_{IH} \Rightarrow 3.7 - V_{IH}$$

We see that in contrast to a CMOS inverter, we can not get our outputs to either rail. This results in reduced noise margins and robustness of operation.

Because the outputs do not go rail to rail, and the load transistor is always on, a constant DC current flows, resulting in constant power dissipation, unlike a CMOS inverter.

Problem 2.3

When V_{in}=V_{OH} the driver transistor operates in the linear region and the current is the same through both transistors.

$$\begin{aligned} I_{driver} &= \mu_n C_{OX}\left(\frac{W}{L}\right)\left(2(V_{OH} - V_{T0})V_{OL} - V_{OL}{}^2\right) \\ &= \frac{45}{2}\times 8\left(2(3.7-0.8)0.234 - 0.234^2\right) \\ &= \underline{\underline{234\mu A}} \end{aligned}$$

Problem 3.1

$$V_{OH}=V_{DD} \qquad V_{OL}=0 \qquad k_R=\frac{k_n}{k_p}=\frac{\mu_n C_{OX}\left(\frac{W}{L}\right)_n}{\mu_p C_{OX}\left(\frac{W}{L}\right)_p}=\frac{60\times 8}{25\times 12}=1.6$$

$$V_{th}=\frac{V_{th0}+\sqrt{\frac{1}{k_R}}\left(V_{DD}+V_{T0,p}\right)}{1+\sqrt{\frac{1}{k_R}}}=\underline{\underline{1.482V}}$$

To solve for V_{IL}, we First solve for V_{OUT} in terms of V_{IL}…

$$V_{IL}=\frac{2V_{OUT}+V_{T0,p}-V_{DD}+k_R V_{T0,n}}{1+k_R}\Rightarrow V_{OUT}=1.3V_{IL}+1.52$$

Solve KCL with NMOS in saturation and PMOS in linear…

$$\frac{k_n}{2}\left(V_{IN}-V_{T0,n}\right)^2=\frac{k_p}{2}\left(2\left(V_{IN}-V_{DD}-V_{T0,p}\right)\left(V_{OUT}-V_{DD}\right)-\left(V_{OUT}-V_{DD}\right)^2\right)$$

Finally we plug in the equation for V_{OUT} …

$$1.6\left(V_{IL}{}^2-1.2V_{IL}+0.36\right)^2=\left(2\left(V_{IL}-2.6\right)\left(1.3V_{IL}-1.78\right)-\left(1.3V_{IL}+1.78\right)^2\right)$$
$$0.69V_{IL}{}^2+3.77V_{IL}-5.51=0\Rightarrow V_{IL}\cong\underline{1.2V}$$

To solve for V_{IH}, we First solve for V_{OUT} in terms of V_{IH} …

$$V_{IH}=\frac{V_{DD}+V_{T0,p}+k_R\left(2V_{OUT}+V_{T0,n}\right)}{1+k_R}\Rightarrow V_{OUT}=0.81V_{IH}+1.1125$$

Solve KCL with NMOS in linear and PMOS in saturation…

$$\frac{k_n}{2}\left(2\left(V_{IH}-V_{T0,n}\right)V_{OUT}-V_{OUT}{}^2\right)=\frac{k_p}{2}\left(V_{IH}-V_{DD}-V_{T0,p}\right)^2$$

Finally we plug in the equation for V_{OUT} …

$$\left(3.2V_{IH}-0.96\right)\left(0.81V_{IH}+1.1125\right)-\left(0.81V_{IH}+1.1125\right)^2=\left(V_{IH}-2.6\right)^2$$
$$0.94V_{IH}{}^2+2.67V_{IH}-6.93=0\Rightarrow V_{IH}\cong\underline{1.64V}$$

Now we can find the noise margins…

$$NM_L=V_{IL}-V_{OL}\Rightarrow 1.2-0=\underline{\underline{1.2V}}$$
$$NM_H=V_{OH}-V_{IH}\Rightarrow 3.3-1.64=\underline{\underline{1.66V}}$$

Problem 3.2

$$V_{th} = \frac{V_{th0} + \sqrt{\frac{1}{k_R}}\left(V_{DD} + V_{T0,p}\right)}{1 + \sqrt{\frac{1}{k_R}}} \Rightarrow 1.4 = \frac{0.6 + \sqrt{\frac{1}{k_R}}(3.3 - 0.7)}{1 + \sqrt{\frac{1}{k_R}}}$$

$$k_R = \frac{9}{4} \Rightarrow \frac{W_n}{W_p} = \frac{15}{16} = \underline{\underline{0.9375}}$$

Problem 3.3

$$V_{th0,n,\max} = (1.15)(0.6) = 0.69V$$
$$V_{th0,n,\min} = (0.85)(0.6) = 0.51V$$

$$V_{T0,p,\max} = (1.20)(-0.7) = -0.84V$$
$$V_{T0,p,\min} = (0.80)(-0.7) = -0.56V$$

$$V_{th,\max} = \frac{V_{th0,n,\max} + \sqrt{\frac{1}{k_R}}\left(V_{DD} + V_{T0,p,\min}\right)}{1 + \sqrt{\frac{1}{k_R}}}$$

$$\Rightarrow 1.4 = \frac{0.69 + \sqrt{\frac{1}{2.25}}(3.3 - 0.56)}{1 + \sqrt{\frac{1}{2.25}}} = \underline{\underline{1.51V}}$$

$$V_{th,\min} = \frac{V_{th0,n,\min} + \sqrt{\frac{1}{k_R}}\left(V_{DD} + V_{T0,p,\max}\right)}{1 + \sqrt{\frac{1}{k_R}}}$$

$$\Rightarrow 1.4 = \frac{0.51 + \sqrt{\frac{1}{2.25}}(3.3 - 0.84)}{1 + \sqrt{\frac{1}{2.25}}} = \underline{\underline{1.29V}}$$

Problem 4.1

If we assume M3 is off, then we have an unknown voltage at the inverter input. If the voltage is high, then we have a low output, and if it is low, then we have a high output. Either way, according to the previously designed inverter, we have a $V_{GS} > V_{th}$ so M3 is on, and due to the high input impedance of the MOSFETs in the inverter, $I_{MN3} = 0$. Then $V_{DSMN3} = 0$ and we have $V_{in}=V_{out}$ for the inverter. Then as we've seen in the lectures, when $V_{out}=V_{in}=V_{th}=1.4V$

Problem 4.2

From the previous answer, $V_{OUT} = 1.4V$, $V_{GS}(M3)=3.3-1.4=1.9$ is large enough so that M3 is always on. Therefore the specified variation of ±15% has no affect upon the output (V_{OUT}).

Problem 4.3

$$I_D = \frac{k_n}{2}\left(V_{GS} - V_{T0,n}\right)^2 = \frac{60}{2}\times 8(1.4-0.6)^2 = 153.6\mu A$$

$$\pm 15\% \text{ variation causes} \pm 23\% \text{ variation on } I_D$$