EEC 118 HOMEWORK #5
SOLUTIONS

1.1 $F = \overline{A \cdot C + B \cdot (D+E)}$

1.2 Worst case combinations for V_{OL}: AC, BD, BE

1.3 Assume $V_{OL} \approx 0.05 V_{DD} = 0.25V$

For NMOS load:

$$V_{T,n} = V_{T0,n} + \gamma\left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|2\phi_F|}\right) = 1.0V + 0.4V^{1/2}\left(\sqrt{0.6V + 0.25V} - \sqrt{0.6V}\right)$$
$$= 1.06V \approx 1V$$

$V_{GS} = 7V - 0.25V > V_{Tn} = 1.06V \quad V_{DS} = 5V - 0.25V < V_{GS} - V_{T,n}$ linear

$$I_{D,load} = \frac{\mu_n C_{ox}}{2}\left(\frac{5}{5}\right)\left[2(7 - V_{out} - 1)(5 - V_{out}) - (5 - V_{out})^2\right]$$
$$= \frac{\mu_n C_{ox}}{2}\left[35 - 12V_{out} + V_{out}^2\right]$$

For NMOS pulldown, linear since $V_{DS} = 0.25V$ (assumed) and $V_{GS} = 5V$

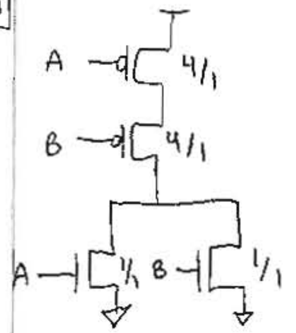
$$I_{D,pulldown} = \frac{\mu_n C_{ox}}{2}\left(\frac{100}{5}\right)\frac{1}{2}\left[2(5-1)V_{out} - V_{out}^2\right] = I_{D,load}$$

$$\frac{\mu_n \cancel{C_{ox}}}{\cancel{2}}\left[35 - 12V_{out} + V_{out}^2\right] = \frac{\mu_n \cancel{C_{ox}}}{\cancel{2}}\left[80V_{out} - 10V_{out}^2\right]$$

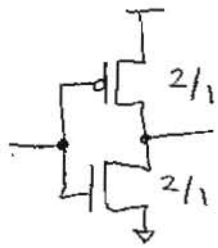
$$9V_{out}^2 - 92V_{out} + 35 = 0 \Rightarrow \boxed{V_{out} = 0.395V = V_{OL}}$$

1.4 No, since pullup and pulldown are both NMOS, k_n cancels out when currents are equated.

$\boxed{2.1}$



Equivalent Inverter (all inputs switching)



$k_p = \left(\frac{W}{L}\right)_p \mu_p C_{ox} = 2 \cdot 20 = 40\ \mu A/V^2$

$k_n = \left(\frac{W}{L}\right)_n \mu_n C_{ox} = 2 \cdot 40 = 80\ \mu A/V^2$

$k_R = \frac{k_n}{k_p} = \frac{80}{40} = 2$

$\boxed{V_{OH} = V_{DD} = 5V}$ $\boxed{V_{OL} = 0V}$

$$V_{IL} = \frac{2V_{out} - V_{TO,p} - V_{DD} + k_R V_{TO,n}}{1 + k_R} \Rightarrow V_{IL} = \frac{2V_{out} + 0.7 - 5 + 1.4}{3}$$

$$\Rightarrow V_{out} = 1.5 V_{IL} + 0.97 \quad (A)$$

$$\frac{k_n}{2}(V_{IL} - V_{TO,n})^2 = \frac{k_p}{2}\left[2(V_{IL} - V_{DD} - V_{TO,p})(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]$$

$$40(V_{IL} - 0.7)^2 = 20\left[2(V_{IL} - 5 + 0.7)(V_{out} - 5) - (V_{out} - 5)^2\right]$$

Plug in (A) for V_{out} $\Rightarrow 1.25 V_{IL}^2 + 6.07 V_{IL} - 17.43 = 0$

$\boxed{V_{IL} = 2.02V}$

$$V_{IH} = \frac{V_{DD} + V_{TO,p} + k_R(2V_{out} + V_{TO,n})}{1 + k_R}$$

$$\frac{k_n}{2}\left[2(V_{IH} - V_{TO,n})(V_{out}) - V_{out}^2\right] = \frac{k_p}{2}(V_{IH} - V_{DD} - V_{TO,p})^2$$

$$V_{IH} = \frac{5 - 0.7 + 2(2V_{out} + 0.7)}{3} \Rightarrow 3V_{IH} = 5.7 + 4V_{out} \Rightarrow V_{out} = 0.75 V_{IH} - 1.425 \quad (B)$$

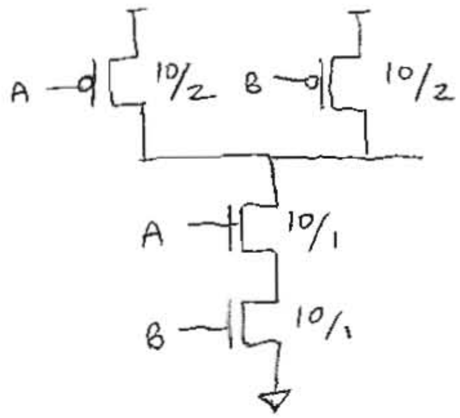
$$40\left[2(V_{IH} - 0.7)V_{out} - V_{out}^2\right] = 20(V_{IH} - 5 + 0.7)^2$$

Plug in (B) for V_{out} $\Rightarrow 0.88 V_{IH}^2 + 5.08 V_{IH} - 18.56 = 0$ $\boxed{V_{IH} = 2.53V}$

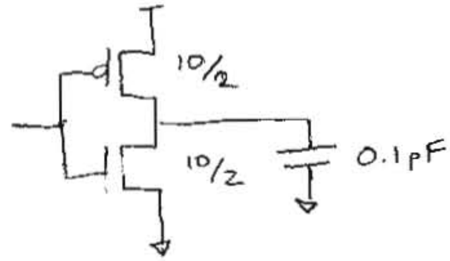
2.1 (cont.)

$$\boxed{\begin{aligned} NM_L &= V_{IL} - V_{OL} = 2.02\,V \\ NM_H &= V_{OH} - V_{IH} = 2.47\,V \end{aligned}}$$

3.1



Equivalent inverter (worst case delays)



$$k_n = k_n' \left(\frac{W}{L}\right)_n = 20\,\mu A/V^2 \left(\frac{10}{2}\right) = 100\,\mu A/V^2$$

$$k_p = k_p' \left(\frac{W}{L}\right)_p = 10\,\mu A/V^2 \left(\frac{10}{2}\right) = 50\,\mu A/V^2$$

Can use several methods. Here, use formulas from lecture:

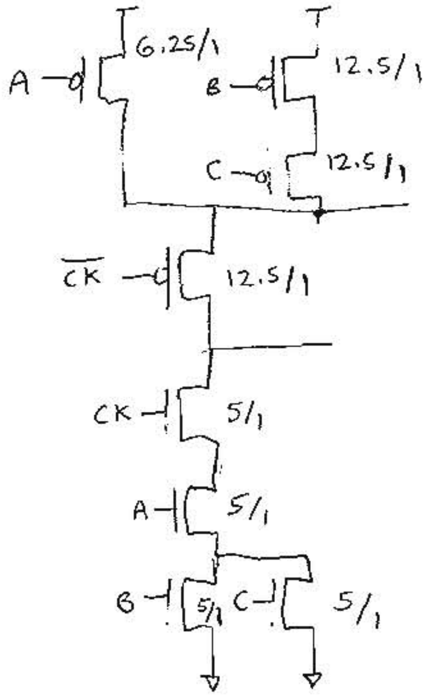
$$t_{pHL} = \frac{C_{load}}{k_n(V_{DD}-V_{T,n})}\left[\frac{2V_{T,n}}{V_{DD}-V_{T,n}} + \ln\left(\frac{4(V_{DD}-V_{T,n})}{V_{DD}} - 1\right)\right]$$

$$t_{pLH} = \frac{C_{load}}{k_p(V_{DD}-|V_{T,p}|)}\left[\frac{2|V_{T,p}|}{V_{DD}-|V_{T,p}|} + \ln\left(\frac{4(V_{DD}-|V_{T,p}|)}{V_{DD}} - 1\right)\right]$$

$$t_{pHL} = \frac{0.1\times10^{-12}}{100\times10^{-6}(4)}\underbrace{\left[\frac{2}{4} + \ln\left(\frac{(4\cdot4)}{5} - 1\right)\right]}_{2.7} = \boxed{0.675\text{ ns}}$$

$$t_{pLH} = 2t_{pHL} = \boxed{1.35\text{ ns}}$$

4.1 $F = A(B + C)$



For $t_{fall} = t_{rise}$, $\mu_n W_n = \mu_p W_p$

$$\Rightarrow W_p = \frac{\mu_n}{\mu_p} W_n = 2.5 W_n$$

Other solutions possible ...