EEC118 HOMEWORK 4 SOLUTIONS

1.1 a) $\frac{W_n}{W_p} = ?$ $V_{th} = V_{in} = V_{out} = 2.2$ $V_{out} = V_{10\%} = 0.5v$

$\tau_{rise} = 5ns$ $V_{90\%} = 4.5v$

$$I_{avg} = \frac{1}{2}\left[I(V_{in} = V_{OL} \to 0, V_{out} = 0.5v) + I(V_{in} = V_{OL} \to 0, V_{out} = 4.5v)\right]$$

$|V_{DS}| \overset{?}{>} |V_{GS} - V_{th}|$ $|V_{DS}| \overset{?}{>} |V_{GS} - V_{th}|$

$4.5 > 5 - 1$ $0.5 > 5 - 1$

sat for PMOS linear region

$$I_{avg} = \frac{1}{2}\left[\frac{1}{2}k_p(V_{in} - V_{Tp})^2 + \frac{1}{2}k_p\left[2(V_{in} - V_{T,p})V_{out} - V_{out}^2\right]\right]$$

$$= \frac{1}{4}k_p\left[16 + 2(-5+1)(-0.5) - (-0.5)^2\right] = \frac{W_p}{L_p} \times 98.75\mu A$$

$$\tau_{rise} = \frac{C \cdot \Delta V}{I_{avg}} \Rightarrow 5\times10^{-9} = \frac{2\times10^{-12}\times 4}{\frac{W_p}{L_p}\times 98.75\times10^{-6}} \Rightarrow \frac{W_p}{L_p} \simeq 16.2 \qquad W_p \simeq 16.2\mu m$$

$$V_{th} = \frac{V_{T,n} + \sqrt{\frac{1}{k_R}}(V_{DD} + V_{T,p})}{\left(1 + \sqrt{1/k_R}\right)} \Rightarrow 2.2 = \frac{0.8 + \sqrt{\frac{1}{k_R}}(5-1)}{\left(1 + \sqrt{\frac{1}{k_R}}\right)} \Rightarrow k_R = 1.65$$

$$\frac{W_n}{W_p} = 0.66 \Rightarrow W_n = 0.66 \times 16 \simeq 10.58\mu m$$

1.2 $\tau_p = ?$

$$\tau_{PHL} = \frac{C_{load}}{k_n(V_{DD} - V_{T,n})}\left[\frac{2V_{T,n}}{V_{DD} - V_{T,n}} + \ln\left(\frac{4(V_{DD} - V_{T,n})}{V_{DD}} - 1\right)\right]$$

$$\tau_{PLH} = \frac{C_{load}}{k_p(V_{DD} - |V_{T,p}|)}\left[\frac{2|V_{T,p}|}{V_{DD} - |V_{T,p}|} + \ln\left(\frac{4(V_{DD} - |V_{T,p}|)}{V_{DD}} - 1\right)\right]$$

$$\tau_{PHL} = \frac{2\times10^{-12}}{50\times10^{-6}\times 10.58\times 4.2}\left[\frac{1.6}{4.2} + \ln\left(\frac{4(5-0.8)}{5} - 1\right)\right] = 1.115ns$$

$$\frac{2\times10^{-12}}{20\times10^{-6}\times16.2\times4}\left[\frac{2}{4} + \ln\left(\frac{4\times4}{5} - 1\right)\right] = 1.988ns$$

$$\tau_p = \frac{\tau_{PHL} + \tau_{PLH}}{2} = \frac{1.115 + 1.988}{2} = 1.551ns$$

1.3 c)

If V_{DD} goes from 5v to 3.3v, then
τ_p will increase.
V_{th} will decrease.

$V_{th} = 1.417v$ (Old one was 2.2v)

$\tau_{PLH} = 3.893ns$
$\tau_{PHL} = 2.038ns$
$\tau_p = 2.965ns$ ($\tau_{P_{old}}$ was 1.551ns)

2.1 d)

$\tau_{PHL} < 0.825ns$

$C_{out} = 500fF + C_{db,n} + C_{db,p}$
$C_{db,n} = (100 + 9W_n)fF$
$C_{db,p} = (80 + 7W_p)fF$

$0.825 \times 10^{-9} = \frac{C_{out}}{W_n \times 50 \times 10^{-6} \times 4.2} \times 1.239 \Rightarrow \frac{C_{out}}{W_n} = 1.398 \times 10^{-13}$

$(680 + 9W_n + 7W_p) \times 10^{-15} = W_n \times 1.398 \times 10^{-13} \Rightarrow \boxed{680 + 9W_n + 7W_p = 139.8 W_n}$

$2.4 = \frac{0.8 + \sqrt{1/k_R}\,(5-1)}{(1 + \sqrt{1/k_R})} \Rightarrow k_R = 1 \Rightarrow \frac{W_n}{W_p} = \frac{2}{5}$ $\boxed{W_p = \frac{5}{2} W_n}$

$680 + 9W_n + \frac{35}{2} W_n = 139.8 W_n \Rightarrow \boxed{W_n = 6\mu m}$ $\boxed{W_p = 15\mu m}$

2.2 c)

$\tau_{rise} = \frac{C \cdot \Delta V}{I_{avg_1}} = \frac{250 \times 10^{-15} \times 4}{I_{avg_1}}$ $\tau_{fall} = \frac{C \cdot \Delta V}{I_{avg_2}} = \frac{250 \times 10^{-15} \times 4}{I_{avg_2}}$

$I_{avg_1} = \frac{1}{2}\left[\frac{1}{2} k_p (V_{in} - V_{Tp})^2 + \frac{1}{2} k_p \left[2(V_{in} - V_{T,p}) V_{DS_p} - V_{DS_p}^2\right]\right] = 1.481mA$

$\tau_{rise} = \frac{250 \times 10^{-15} \times 4}{1.481 \times 10^{-3}} = 0.675\ ns$

$I_{avg_2} = \frac{1}{2}\left[\frac{1}{2} k_n (V_{in} - V_{Tn})^2 + \frac{1}{2} k_n \left[2(V_{in} - V_{T,n}) V_{out} - V_{out}^2\right]\right] = 1.619mA$

$\tau_{fall} = 0.617ns$

3.10

a) i) use exact method (diff. equations) for rise & fall time

τ_{fall}

$$C\frac{dV_{out}}{dt} = -\frac{1}{2}k_n(V_{in}-V_{T,n})^2$$

nMOS in sat. for $4v \le V_{out} \le 4.5v$

$$\frac{dV_{out}}{dt} = -2.4\times10^9 \text{ V/s}$$

$$\int_{t=0}^{t=t_{sat}} dt = -\frac{1}{2.4\times10^9}\int_{V_{out}=4.5}^{4} dV_{out}$$

$$t_{sat} = \frac{0.5}{2.4\times10^9} = \underline{0.2083 \text{ ns}}$$

nMOS in linear reg. for $0.5v \le V_{out} \le 4v$

$$C.\frac{dV_{out}}{dt} = -\frac{1}{2}k_n\left[2(V_{in}-V_{T,n})V_{out} - V_{out}^2\right]$$

$$\int_{t=t_{sat}}^{t=t_{delay}} dt = -6.\bar{6}\times10^{-9}\int_{4}^{0.5}\frac{dV_{out}}{8V_{out}-V_{out}^2}$$

$$t_{delay}-t_{sat} = \underline{2.23 \text{ns}}$$

$$t_{fall} = 2.23 + 0.2083 = \underline{2.4383 \text{ns}}$$

τ_{rise}

$$C\frac{dV_{out}}{dt} = \frac{1}{2}k_p(V_{in}-V_{DD}-V_{T,p})^2$$

pMOS in sat for $0.5v \le V_{out} \le 1.2v$

$$\frac{dV_{out}}{dt} = 2.406\times10^9 \text{ V/s}$$

$$\int_{t=0}^{t=t_{sat}} dt = \frac{1}{2.406\times10^9}\int_{V_{out}=0.5}^{1.2} dV_{out}$$

$$t_{sat} = \frac{0.7}{2.406\times10^9} = \underline{0.2908 \text{ ns}}$$

pMOS in linear reg. for $1.2v \le V_{out} \le 4.5v$

$$C.\frac{dV_{out}}{dt} = \frac{1}{2}k_p\left[2(V_{in}-V_{DD}-V_{T,p})(V_{out}-V_{DD}) - (V_{out}-V_{DD})^2\right]$$

$$\int_{t=t_{sat}}^{t=t_{delay}} dt = 5.\bar{9}\times10^{-9}\int_{1.2}^{4.5}\frac{dV_{out}}{38-7.6V_{out}-(5-V_{out})^2}$$

$$t_{delay}-t_{sat} = \underline{2.059 \text{ ns}}$$

$$t_{rise} = 2.059 + 0.2908 = \underline{2.3498 \text{ns}}$$

ii) use average current method

τ_{fall}

$$\tau_{fall} = \frac{C.\Delta V}{I_{avg}} = \frac{1.5\times10^{-12}\times4}{I_{avg}}$$

$$I_{avg} = \frac{1}{2}\left[k_n(V_{in}-V_{Tn})^2 + \frac{1}{2}\left[2(V_{in}-V_{Tn})V_{out}-V_{out}^2\right]\right]$$

$$= 2.22\text{mA}$$

$$\tau_{fall} = \underline{2.7\text{ns}}$$

τ_{rise}

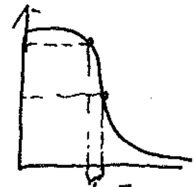
$$\tau_{rise} = \frac{C.\Delta V}{I_{avg}} = \frac{1.5\times10^{-12}\times4}{I_{avg}}$$

$$I_{avg} = \frac{1}{2}\left[\frac{1}{2}k_p(V_{in}-V_{Tp})^2 + \frac{1}{2}k_p\left[2(V_{in}-V_{Tp})V_{DSp}-V_{DSp}^2\right]\right]$$

$$= 2.248\text{mA}$$

$$\tau_{rise} = \underline{2.668\text{ns}}$$

3.2 b) $f = \frac{1}{2\tau_p} \Rightarrow \tau_p = \tau_{PHL} + \tau_{PLH}$ Notice:



$\hookrightarrow \tau_{PHL}$ only spans from $V_{DD}-V_{tn}$ to $\frac{V_{DD}}{2}$.
But we asked ~~for~~ the time period for full logic swing.
Thus $f = \frac{1}{\tau_{rise}+\tau_{fall}} = \frac{1}{2(\tau_{PHL}+\tau_{PLH})}$

$$\tau_{PHL} = \frac{1.5\times10^{-12}}{45\times10^{-6}\times10\times(5-1)}\left[\frac{2}{4} + \ln\left(\frac{4\times4}{5}-1\right)\right] = 1.07\,ns$$

$$\tau_{PLH} = \frac{1.5\times10^{-12}}{25\times10^{-6}\times20\times3.8}\left[\frac{2.4}{3.8} + \ln\left(\frac{4\times3.8}{5}-1\right)\right] = 2.11\,ns$$

$\tau_p = 3.18\,ns$ $\quad f = \frac{1}{6.36\times10^{-9}} \simeq \underline{157\,MHz}$

3.3 $Power = C_{load}\cdot V_{DD}^2 \cdot f = 1.5\times10^{-12}\times5^2\times157\times10^6 \simeq \underline{5.91\,mW}$

3.4 $\tau_{PHL} = \frac{m}{\frac{W_n}{L_n}} \Rightarrow 0.75\,\tau_{PHL} = 0.75\cdot\frac{m}{\frac{W_n}{L_n}} \Rightarrow 0.75\,\tau_{PHL} = \frac{m}{\boxed{\frac{W_n}{L_n}\times\frac{1}{0.75}}}$

this means $\left(\frac{W_n}{L_n}\right)_{new} = \frac{10}{0.75} = \underline{13.3}$

same procedure for $\left(\frac{W_p}{L_p}\right) \Longrightarrow \left(\frac{W_p}{L_p}\right)_{new} = \frac{20}{0.75} = \underline{26.6}$

V_{TH} will not change because k_R does not change here.