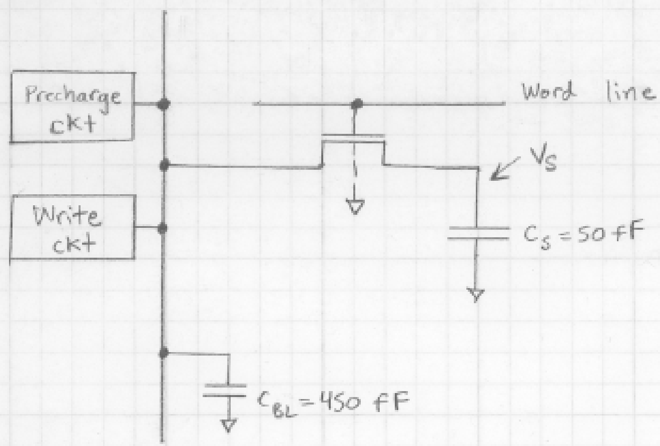
EEC116 Problem Set #6 Solution

1. Single Transistor DRAM

Problem 1



a.) Bitline $BL = V_{DD} = 5V$, Wordline $= V_{DD} = 5V$

Final voltage $V_S = V_{DD} - V_{T,n}$, but $V_{SB} = V_S \Rightarrow$

$$V_S = V_{DD} - V_{T_0} - \gamma\left(\sqrt{|2\phi_F| + V_S} - \sqrt{|2\phi_F|}\right)$$

$$= 5 - 1.0 - 0.3\left(\sqrt{0.6 + V_S} - \sqrt{0.6}\right)$$

Solve by iteration (or graphically) $\boxed{V_S = 3.62\ V}$

b.) During a READ-1, $V_S = 3.62\ V$ and $BL = V_{DD}/2$. Capacitors C_{BL} and C_S short together and share their charge.

$$Q_{BL} = C_{BL}\frac{V_{DD}}{2} = (450\ fF)(2.5\ V)$$

$$Q_S = C_S V_S = (50\ fF)(3.62V)$$

$$V_{BL}(\text{final}) = \frac{Q_{TOT}}{C_{TOT}} = \frac{Q_{BL} + Q_S}{C_{BL} + C_S} = \frac{(450\ fF)(2.5V) + (50\ fF)(3.62V)}{500\ fF}$$

$$\boxed{V_{BL}(\text{final}) = 2.61\ V}$$

Note using 10.1 in K+L p.420, $V_{BL}(\text{final}) = \frac{V_{DD}}{2} + \Delta V = 2.5V + \frac{50\ fF}{500\ fF}2.5V$

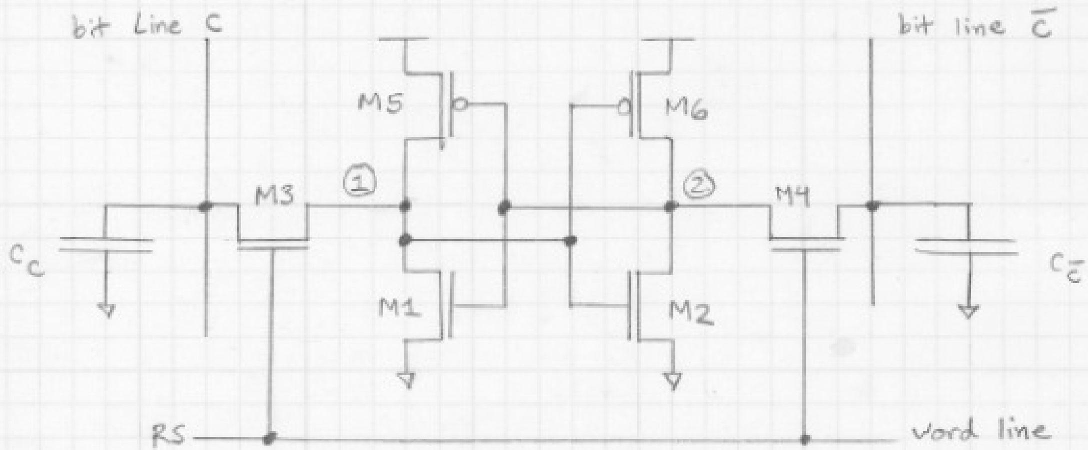
$= 2.75\ V$ is incorrect!

The book equation assumed $V_S = V_{DD}$!

2. SRAM Cell Design

Problem 2

To change the state of the cell for $V_C \le 0.5\,V$, the voltage at node ① must be V_{DD} (if it were 0V then the cell wouldn't change state).



For M1, M2, $W/L = 4/4$
For M3, M4, $W/L = 2/4$
} must size M5, M6 assuming $(W/L)_5 = (W/L)_6$

Suppose node ① is initially at V_{DD}. To switch the SRAM state, node ① voltage must go below the M6, M2 inverter switching threshold:

$$\text{Voltage @ node ①} = V_1 \le V_{TH} = \frac{V_{TO,n} + \sqrt{\frac{1}{K_R}}\,(V_{DD} - |V_{TO,p}|)}{\left(1 + \sqrt{\frac{1}{K_R}}\right)}$$

$$K_R = \frac{k_n}{k_p} = \frac{k_n'(W/L)_2}{k_p'(W/L)_6} = \frac{2\,(4/4)}{(W/L)_6} = 2\left(\frac{L}{W}\right)_6$$

If node ① initially at V_{DD}, node ② is initially 0V. Assuming $V_{TH} \approx V_{DD}/2$, then M5 and M3 are in linear region and M1 is cut off.

$$I_{DS,M3} = I_{DS,M5} \Rightarrow$$

$$\frac{k_n'(W/L)_3}{2}\left[2(V_{DD} - V_{T,n3})(V_1 - V_C) - (V_1 - V_C)^2\right] = \frac{k_p'(W/L)_5}{2}\left[2(-V_{DD} - V_{T,p3})(V_1 - V_{DD}) - (V_1 - V_{DD})^2\right]$$

Assume $V_{DD} = 5\text{ V}$. $V_{TO,p} = -0.7\text{ V}$

For $V_{T,n3}$: $V_{SB} = 0.5\text{ V} \Rightarrow V_{T,n3} = V_{TO} + \gamma\left(\sqrt{|2\phi_F| + V_{SB}} - \sqrt{|2\phi_F|}\right)$

$$= 0.7\text{ V} + 0.4\left(\sqrt{0.6+0.5} - \sqrt{0.6}\right)\text{V} = 0.810\text{ V}$$

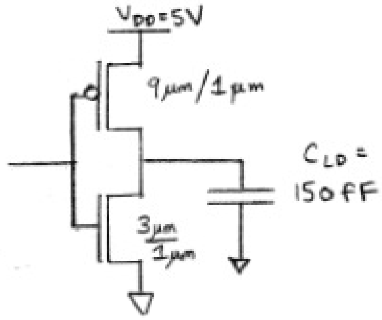
Let $x = (W/L)_6$, $V_1 = V_{TH} = \dfrac{0.7 + \sqrt{x/2}\,(5 - 0.7)}{\left(1 + \sqrt{x/2}\right)} = \dfrac{0.7 + \sqrt{x/2}\,(4.3)}{1 + \sqrt{x/2}}$.

Plug $V_1(x)$ into the current equations and solve numerically...

$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 \cong \boxed{0.5}$ $V_{TH} = 1.9\text{ V}$, therefore linear assumption was good.

3. Voltage Scaling

(3.1)



$t_{pLH} = t_{pHL}$ for balanced inverter.

$$t_{pHL} = \frac{C\Delta V}{I_{avg}} = \frac{C_{LD}(V_{DD}/2)}{I_{avg}} = \frac{C_{LD}(V_{DD}/2)}{\frac{1}{2}\left(\underbrace{I_{DS}\Big|_{V_{GS}=5V,\ V_{DS}=5V}}_{\text{sat}} + \underbrace{I_{DS}\Big|_{V_{GS}=5V,\ V_{DS}=2.5V}}_{\text{linear}}\right)}$$

$$= \frac{(150fF)(2.5V)}{\frac{1}{2}\left(\frac{300\mu A/V^2}{2}\left(\frac{3}{1}\right)(5V-0.8V)^2 + 300\mu A/V^2\left(\frac{3}{1}\right)\left[(5V-0.8V)(2.5V) + (2.5V)^2/2\right]\right)}$$

$$= \frac{(150fF)(2.5V)}{7.288\,mA} = \boxed{51.45\,ps}$$

(3.2) $P_{max} = C_{LD}V_{DD}^2 f_{max} = \frac{C_{LD}V_{DD}^2}{\frac{1}{2}(t_{pLH}+t_{pHL})} = \boxed{72.88\,mW}$

(3.3) $\boxed{V_{DD\,min} = 1.5093V}$

$$P_{max} = C_{LD}V_{DD,min}^2\left(\frac{1}{500ps}\right) = \boxed{683\,\mu W}$$