

Problem 1.13 The voltage of an electromagnetic wave traveling on a transmission line is given by $v(z, t) = 5e^{-\alpha z} \sin(4\pi \times 10^9 t - 20\pi z)$ (V), where z is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At $z = 2$ m, the amplitude of the wave was measured to be 2 V. Find α .

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 4\pi \times 10^9$ rad/s and $\beta = 20\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 2 \times 10^9$ Hz = 2 GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.1$ m. From Eq. (1.30),

$$u_p = \omega/\beta = 2 \times 10^8 \text{ m/s.}$$

(b) Using just the amplitude of the wave,

$$2 = 5e^{-\alpha z}, \quad \alpha = \frac{-1}{2 \text{ m}} \ln \left(\frac{2}{5} \right) = 0.46 \text{ Np/m.}$$

Problem 2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad (\text{distortionless line}).$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R' \sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \quad \beta = \omega \sqrt{L'C'}, \quad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned} \gamma &= \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{G'}{C'} + j\omega\right)} \\ &= \sqrt{L'C'} \sqrt{\left(\frac{R'}{L'} + j\omega\right) \left(\frac{R'}{L'} + j\omega\right)} \\ &= \sqrt{L'C'} \left(\frac{R'}{L'} + j\omega\right) = R' \sqrt{\frac{C'}{L'}} + j\omega \sqrt{L'C'}. \end{aligned}$$

Hence,

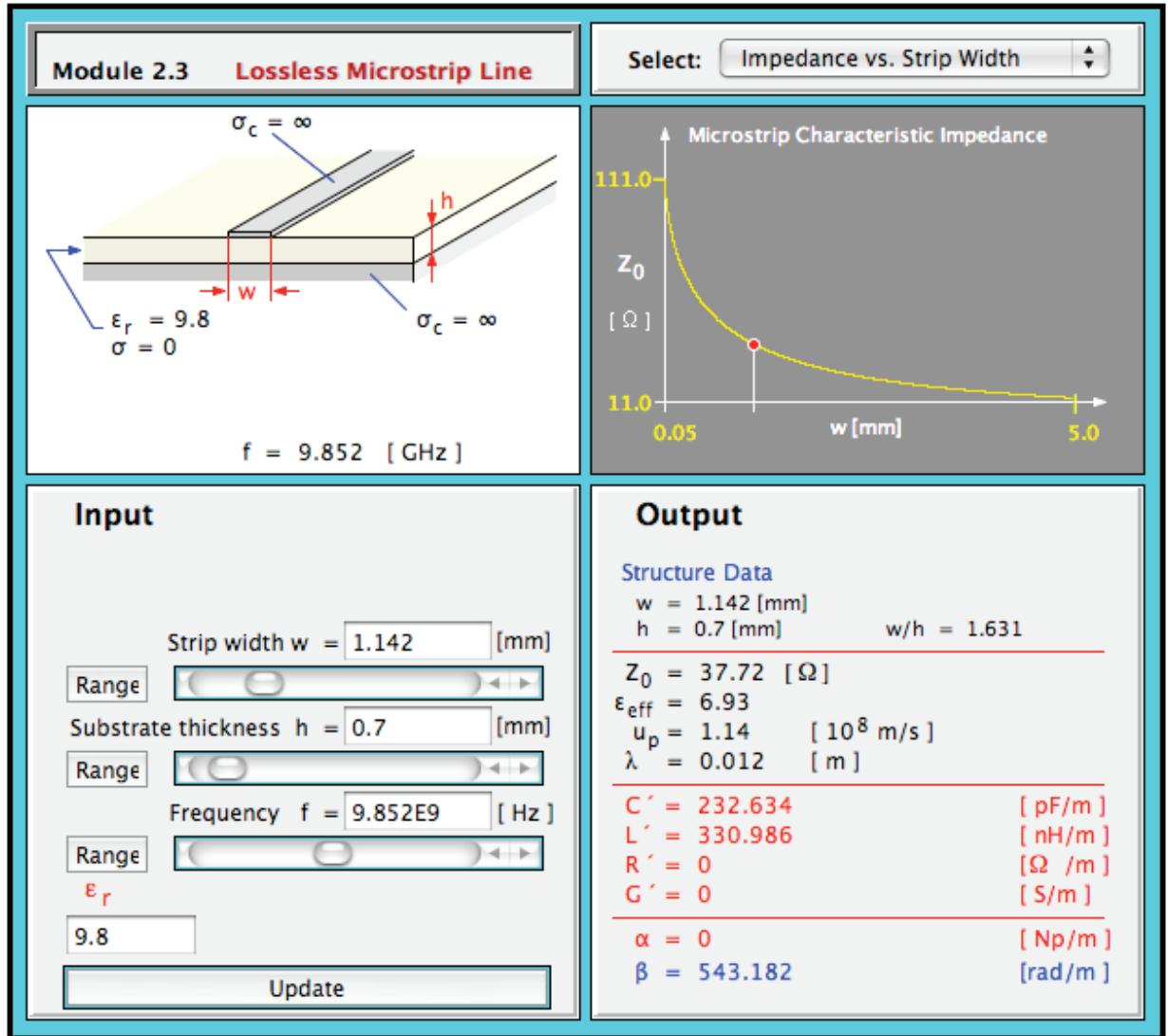
$$\alpha = \Re(\gamma) = R' \sqrt{\frac{C'}{L'}}, \quad \beta = \Im(\gamma) = \omega \sqrt{L'C'}, \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}} \sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.12 Generate a plot of Z_0 as a function of strip width w , over the range from 0.05 mm to 5 mm, for a microstrip line fabricated on a 0.7-mm-thick substrate with $\epsilon_r = 9.8$.

Solution:



Problem 2.33 Two half-wave dipole antennas, each with an impedance of 75Ω , are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

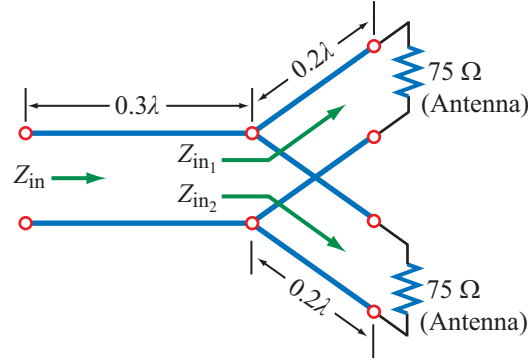


Figure P2.33: Circuit for Problem 2.33.

All lines are 50Ω and lossless.

- (a) Calculate Z_{in1} , the input impedance of the antenna-terminated line, at the parallel juncture.
- (b) Combine Z_{in1} and Z_{in2} in parallel to obtain Z'_L , the effective load impedance of the feedline.
- (c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$\begin{aligned}
 Z_{in1} &= Z_0 \left[\frac{Z_{L1} + jZ_0 \tan \beta l_1}{Z_0 + jZ_{L1} \tan \beta l_1} \right] \\
 &= 50 \left\{ \frac{75 + j50 \tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75 \tan[(2\pi/\lambda)(0.2\lambda)]} \right\} = (35.20 - j8.62) \Omega.
 \end{aligned}$$

(b)

$$Z'_L = \frac{Z_{in1} Z_{in2}}{Z_{in1} + Z_{in2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31) \Omega.$$

(c)

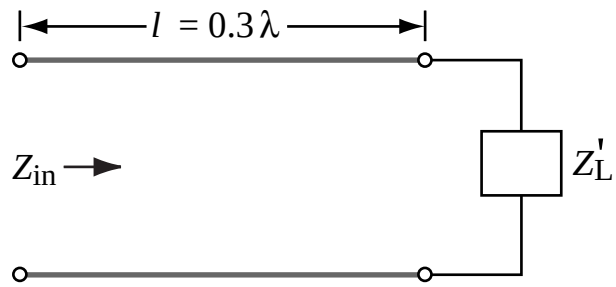


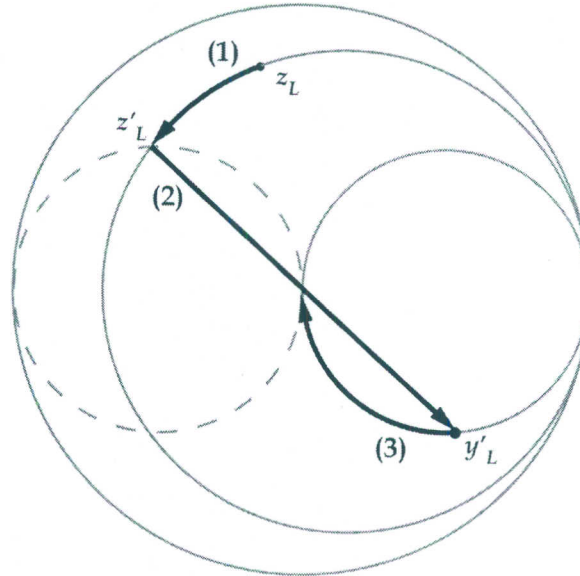
Figure P2.33: (b) Equivalent circuit.

$$Z_{\text{in}} = 50 \left\{ \frac{(17.60 - j4.31) + j50 \tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31) \tan[(2\pi/\lambda)(0.3\lambda)]} \right\} = (107.57 - j56.7) \, \Omega.$$

Problem 5

Solution:

- (a) One possible solution looks like this.



Step (1): The series reactive element jx moves the load along a constant- r circle until it hit the constant- $g=1$ circle, which is simply a mirror of the constant- $r=1$ circle.

Step (2): Find the corresponding admittance y'_L . Since the constant- $g=1$ circle and the constant- $r=1$ circle are mirror of each other, y'_L should sit on the constant- $r=1$ circle.

Step (3): The shunt reactive element jb moves y'_L along the constant- $r=1$ circle to the origin.

- (b) The Smith chart solution is given on the next page. You could also solve the problem mathematically.

The total normalized input admittance of the circuit should be 1.

$$y_{in} = \frac{1}{0.2 + j(1 + x)} + jb = 1$$

Solving the above equation gives $x=0.6$, $b=2$.

- (c) Shown below. If the z_L lies inside the constant- $r=1$ circle, then you can never move to the constant- $g=1$ circle no matter what x value you have.

The Complete Smith Chart

Black Magic Design

