# A Beamforming Method for Blind Calibration of Time-Interleaved A/D Converters

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# Outline

- **Motivation**
- Problem Formulation
- Blind Calibration Method
- Convergence
- Simulations and Conclusions

# Motivation

- Applications requiring ADCs operating at high data rates $\Rightarrow$ **Time-interleaved ADCs**.
- Constituent ADCs have gain, offset, timing mismatches that need to be estimated and corrected in the digital domain.
- Correction achieved by digital filter banks operating on ADCs outputs (Johansson and Lowenborg, 2002; Prendergast et al. 2004). Requires 10 to 20 % excess samples.
- Timing offset estimation can be performed either with test signals or blindly. Blind methods do not lower ADC throughput and can adjust to changes online.

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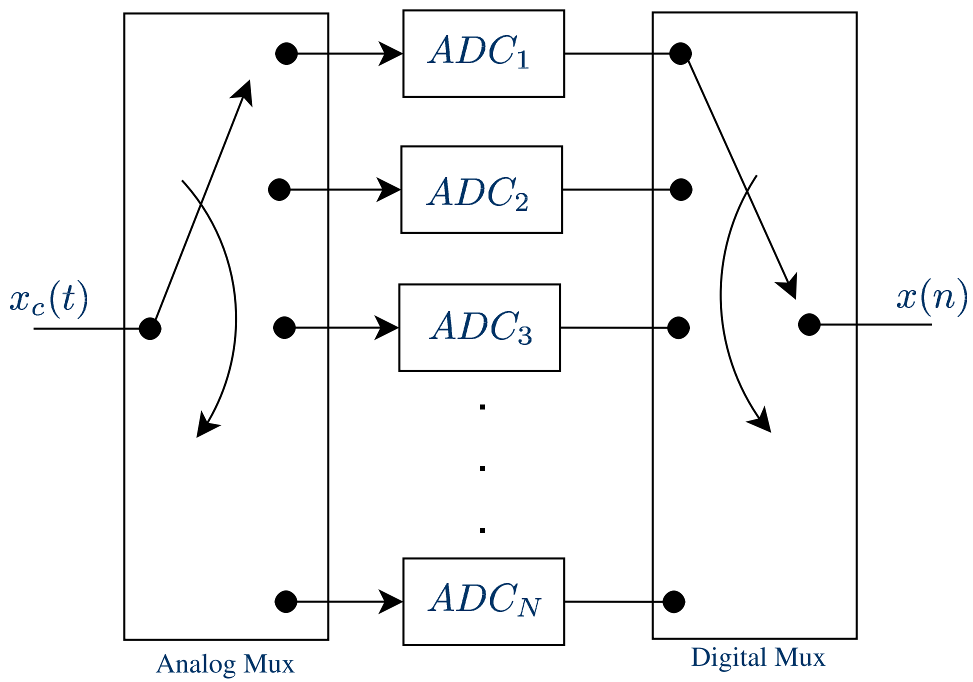
# Problem Formulation

- If $x_c(t)$ is a CT bandlimited signal with bandwidth $B$, can be recovered from its samples $x(n) = x_c(nT_s)$ if $f_s = 1/T_s > f_N = B/\pi$.
- Instead of using a single fast ADC, we employ $N$ slow ADCs operating at $f_s/N$.

# Problem Formulation (cont'd)



# Problem Formulation (cont'd)

- Due to timing offsets, quantization errors and thermal noise, output of $i$-th ADC

$$z_i(m) = x_i(m) + v_i(m)$$

for $1 \leq i \leq N$, where $v_i(m) \sim N(0, N_0/2)$ WGN and

$$x_i(m) = x_c(mT_i + ((i-1) + \delta_{i-1})T_s)$$

where $T_i = NT_s$ = sampling period of slow ADCs, $\delta_{i-1}T_s =$ timing offset of $i$-th ADC measured wrt 1st ADC

- If $F_i(e^{j\omega}) = e^{j\omega(i-1+\delta_{i-1})}$, analysis filter bank model for $N = 4$:



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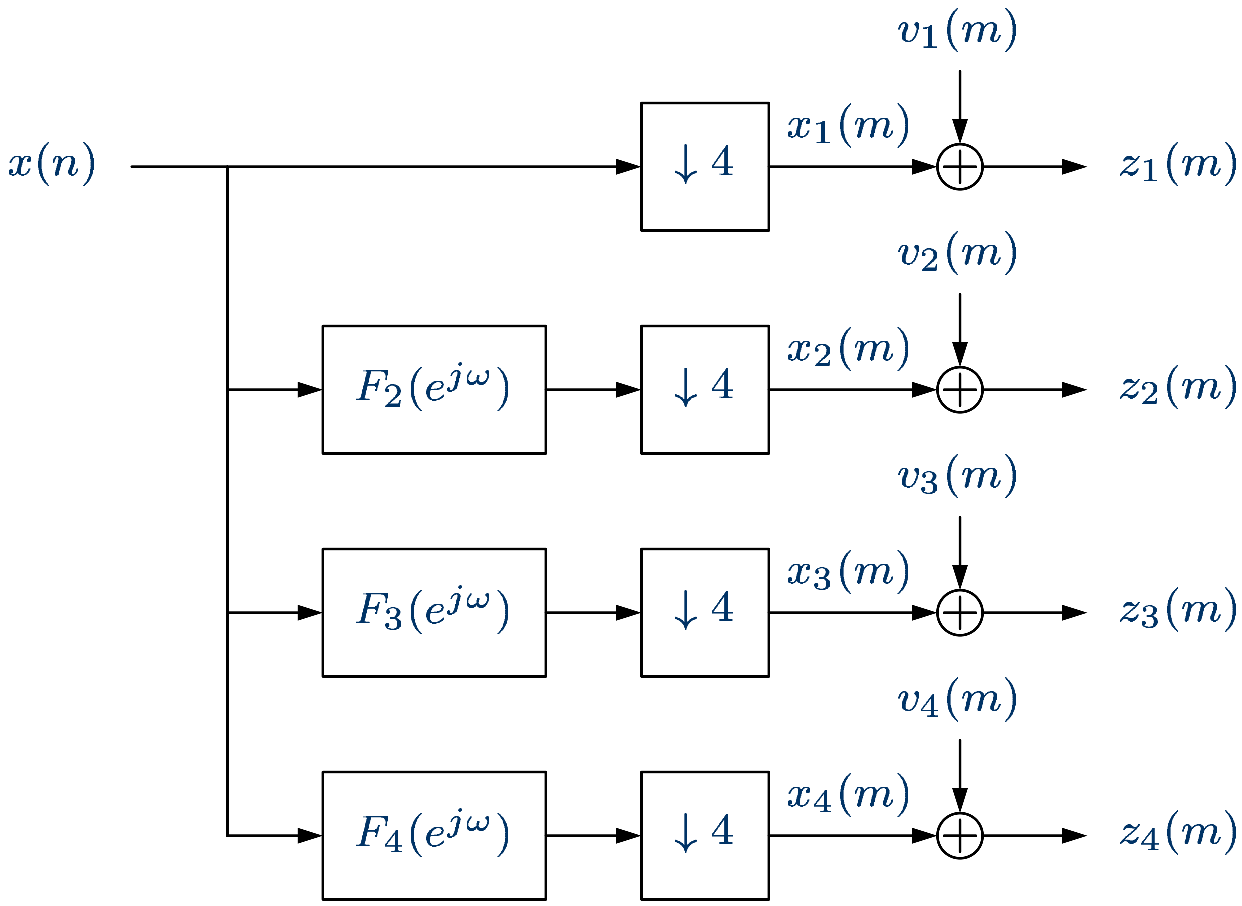
# Problem Formulation (cont'd)



# Problem Formulation (cont'd)

- For $N = 4$, let

$$\mathbf{X}_I(e^{j\omega}) = \begin{bmatrix} X_1(e^{j\omega}) & X_2(e^{j\omega}) & X_3(e^{j\omega}) & X_4(e^{j\omega}) \end{bmatrix}^T$$

$$\mathbf{X}_A(e^{j\omega}) = \begin{bmatrix} X(e^{j\frac{\omega}{4}}) & X(e^{j(\frac{\omega}{4}-\frac{\pi}{2})}) & X(e^{j(\frac{\omega}{4}-\pi)}) & X(e^{j(\frac{\omega}{4}-3\frac{\pi}{2})}) \end{bmatrix}^T$$

= DTFT of exact ADC outputs, and vector of alias components of fast sampled sequence, and

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 \end{bmatrix}^T$$

= vector of timing mismatches.



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# Problem Formulation (cont'd)

- We have

$$\mathbf{X}_I(e^{j\omega}) = \frac{1}{4}\mathbf{M}(e^{j\omega}, \boldsymbol{\delta})\mathbf{X}_A(e^{j\omega}) \; ,$$

where

$$\mathbf{M}(e^{j\omega}, \boldsymbol{\delta}) = \mathbf{D}(e^{j\omega}, \boldsymbol{\delta})\mathbf{V}(\boldsymbol{\delta})$$

$$\mathbf{D}(e^{j\omega}, \boldsymbol{\delta}) \triangleq \text{diag}\,\{e^{j\frac{\omega}{4}(i-1+\delta_{i-1})}\, , \, 1 \leq i \leq 4\}$$

and

$$\mathbf{V}(\boldsymbol{\delta}) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & u_1 & u_1^2 & u_1^3 \\ 1 & u_2 & u_2^2 & u_2^3 \\ 1 & u_3 & u_3^2 & u_3^3 \end{bmatrix}$$

= Vandermonde matrix with

$$u_i = e^{-j\frac{\pi}{2}(i+\delta_i)} \; .$$

# Problem Formulation (cont'd)

- By inverting $\mathbf{V}(\boldsymbol{\delta})$, we can find synthesis filters $G_i(e^{j\omega})$ such that

$$X(e^{j\omega}) = \sum_{i=1}^{N} G_i(e^{j\omega}) X_i(e^{j\omega})$$

  For small $\delta_i$'s, filters $G_i$ admit a closed-form 1st-order Farrow representation.

- Synthesis filter bank for $N = 4$:



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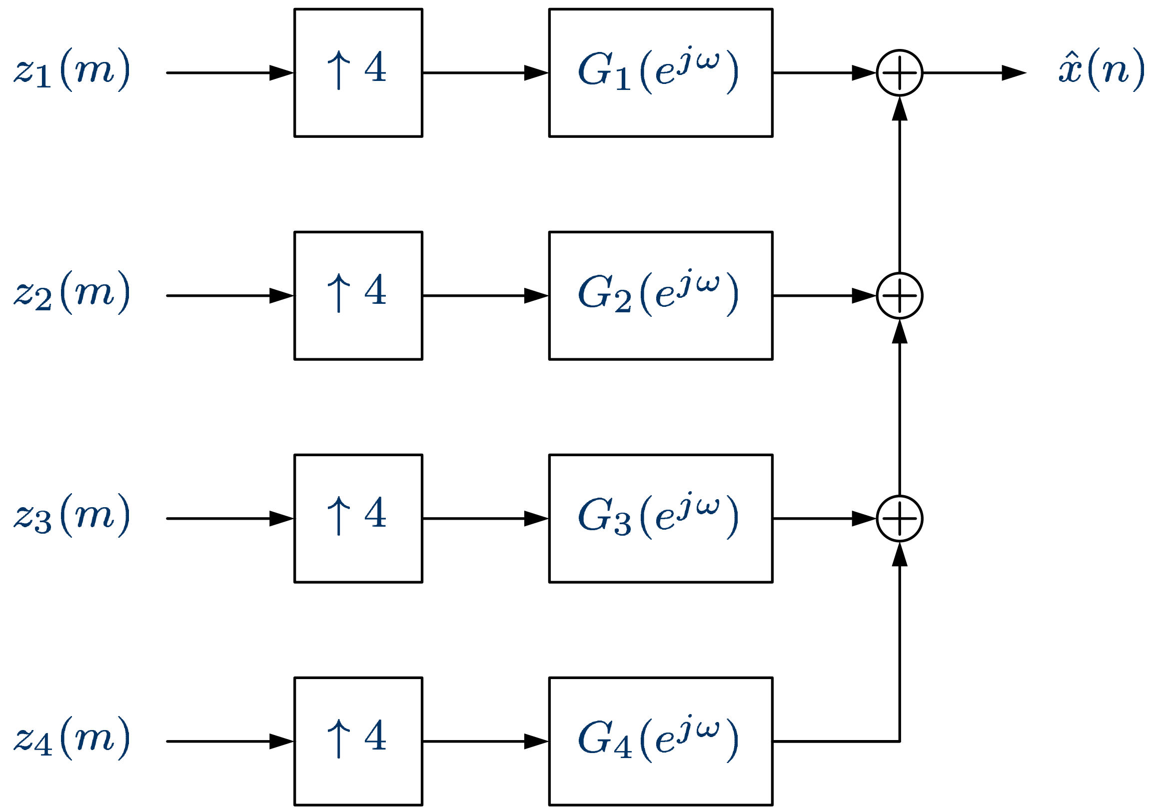
# Problem Formulation (cont'd)



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# Blind Calibration

- Let $N = 4$, if $\alpha = (f_s - f_N)/f_s$ = % of excess samples, the alias matrix is *rank deficient* for $|\omega| < 4\alpha\pi$:

$$\mathbf{X}_I(e^{j\omega}) = \frac{1}{4}\mathbf{M}_R(e^{j\omega}, \boldsymbol{\delta}) \begin{bmatrix} X(e^{j(\frac{\omega}{4}+\frac{\pi}{2})}) \\ X(e^{j\frac{\omega}{4}}) \\ X(e^{j(\frac{\omega}{4}-\frac{\pi}{2})}) \end{bmatrix}$$

with $\mathbf{M}_R(e^{j\omega}, \boldsymbol{\delta}) = \mathbf{D}(e^{j\omega}, \boldsymbol{\delta})\mathbf{V}_R(\boldsymbol{\delta})$, where

$$\mathbf{V}_R(\boldsymbol{\delta}) = \begin{bmatrix} 1 & 1 & 1 \\ u_1^{-1} & 1 & u_1 \\ u_2^{-1} & 1 & u_2 \\ u_3^{-1} & 1 & u_3 \end{bmatrix}$$

is a $4 \times 3$ reduced Vandermonde matrix.



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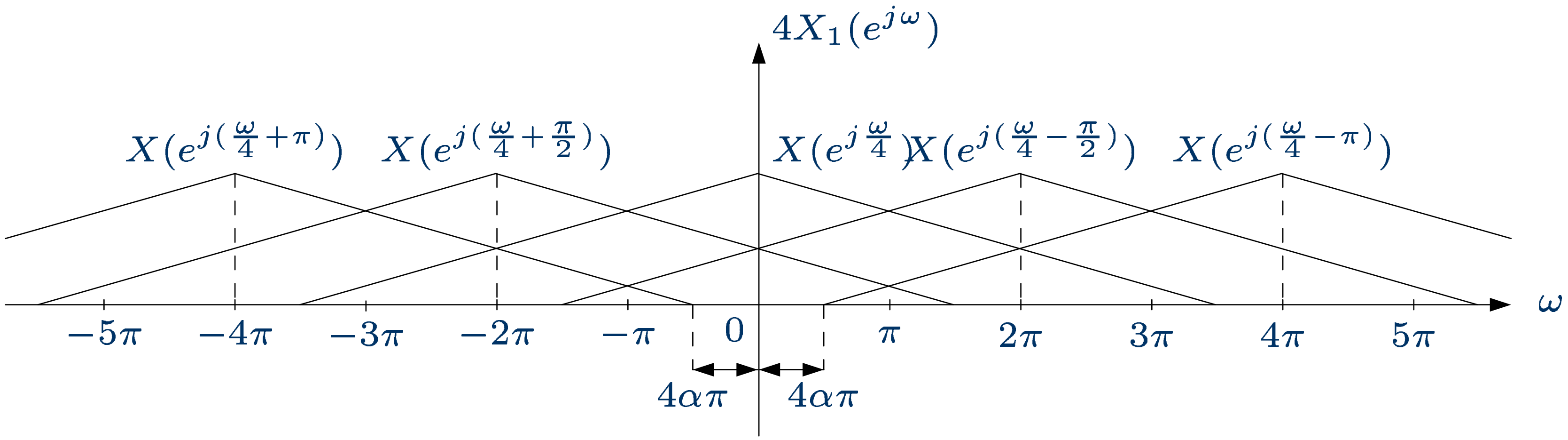
# Blind Calibration (cont'd)




- Can find a nulling filter bank $\mathbf{H}(e^{j\omega}, \boldsymbol{\delta})$ such that

$$\mathbf{H}(e^{j\omega}, \boldsymbol{\delta})\mathbf{X}_I(e^{j\omega}) = 0$$

for $|\omega| < 4\alpha\pi$.



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# Blind Calibration (cont'd)

- Structure of nulling filter bank:

$$\mathbf{H}(e^{j\omega}, \boldsymbol{\delta}) = \mathbf{c}^T(\boldsymbol{\delta})\mathbf{D}^{-1}(e^{j\omega}, \boldsymbol{\delta})$$

for $|\omega| < 4\alpha\pi$, =0 otherwise, where

$$\mathbf{c}^T(\boldsymbol{\delta}) = \begin{bmatrix} c_1(\boldsymbol{\delta}) & c_2(\boldsymbol{\delta}) & c_3(\boldsymbol{\delta}) & c_4(\boldsymbol{\delta}) \end{bmatrix}$$

satisfies

$$\mathbf{c}^T(\boldsymbol{\delta})\mathbf{V}_R(\boldsymbol{\delta}) = 0\ .$$

- Set $c_1 = 1$. Then, for small $\delta_i$s:

$$\begin{aligned} c_2(\boldsymbol{\delta}) &\approx -1 + \frac{\pi}{4}(\delta_2 + \delta_3 - \delta_1) \\ c_3(\boldsymbol{\delta}) &\approx 1 - \frac{\pi}{2}(\delta_3 - \delta_1) \\ c_4(\boldsymbol{\delta}) &\approx -1 + \frac{\pi}{4}(-\delta_2 + \delta_3 - \delta_1) \end{aligned}$$

# Blind Calibration (cont'd)

- Consider the 1st-order Farrow approximation

$$\begin{aligned} K_i(e^{j\omega}, \delta_{i-1}) &= e^{-j\omega(i-1+\delta_{i-1})/4} H_{\mathrm{LP}}(e^{j\omega}) \\ &\approx K_{i0}(e^{j\omega}) + \delta_{i-1} K_{i1}(e^{j\omega}) \ , \end{aligned}$$

where $H_{\mathrm{LP}}(e^{j\omega}) =$ ideal lowpass filter of bandwidth $4\alpha\pi$.

- Let

$$a_i(m) \stackrel{\triangle}{=} k_{i0}(m) * z_i(m) \quad , \quad b_i(m) \stackrel{\triangle}{=} k_{i1}(m) * z_i(m) \ .$$

Consider the adaptive null-steering structure



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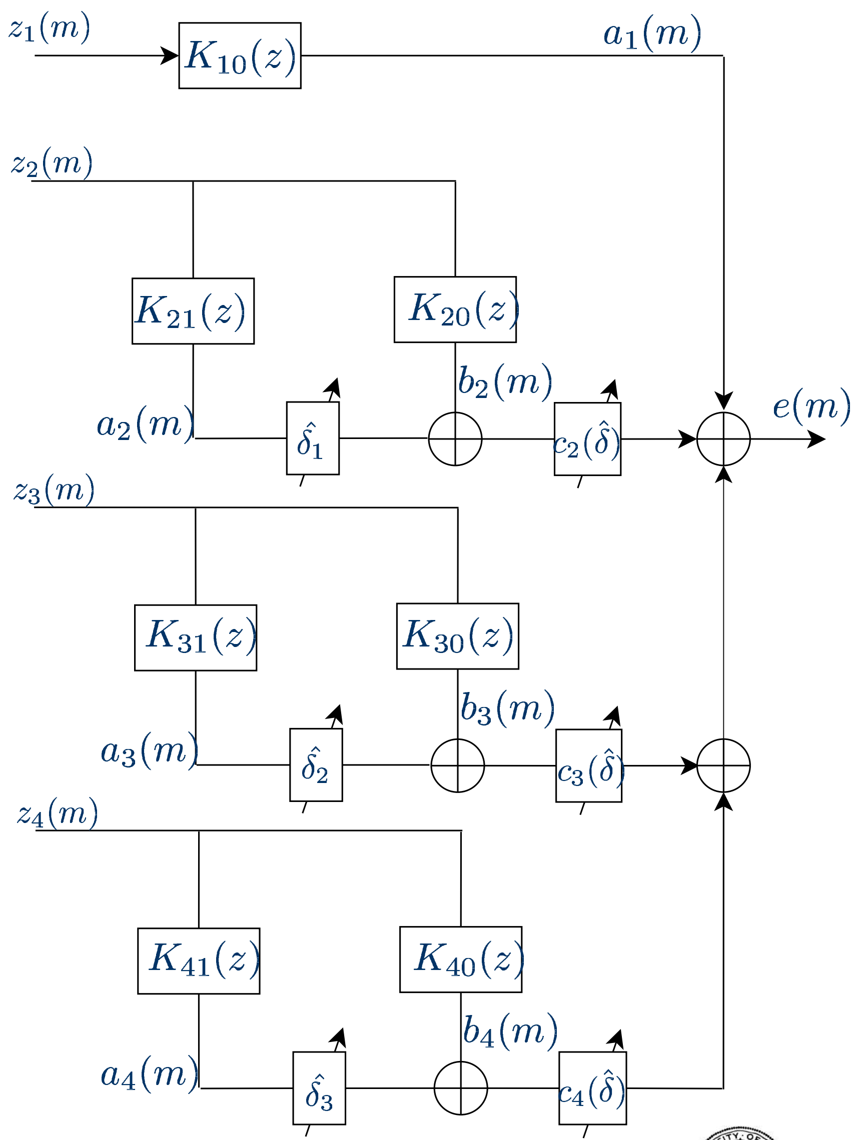
# Blind Calibration (cont'd)



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# Blind Calibration (cont'd)

- Consider the objective function

$$J(\hat{\boldsymbol{\delta}}) = E[e^2(m, \hat{\boldsymbol{\delta}})]/2$$

where

$$e(m, \hat{\boldsymbol{\delta}}) = \sum_{i=1}^{4} c_i(\hat{\boldsymbol{\delta}})(a_i(m) + b_i(m)\hat{\delta}_i)$$

= nulling filter output.

- We have

$$\nabla_{\hat{\boldsymbol{\delta}}} e(m, \hat{\boldsymbol{\delta}}) \approx \begin{bmatrix} -b_2(m) \\ b_3(m) \\ -b_4(m) \end{bmatrix} + \frac{\pi}{4} \begin{bmatrix} -a_2(m) - a_4(m) + 2a_3(m) \\ a_2(m) - a_4(m) \\ a_2(m) + a_4(m) - 2a_3(m) \end{bmatrix}$$



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# Blind Calibration (cont'd)

- $\hat{\boldsymbol{\delta}}(m)$ obtained by *stochastic gradient algorithm*

$$\hat{\boldsymbol{\delta}}(m+1) = \hat{\boldsymbol{\delta}}(m) - \mu e(m, \hat{\boldsymbol{\delta}}(m)) \nabla_{\hat{\boldsymbol{\delta}}} e(m, \hat{\boldsymbol{\delta}}(m))$$

where $\mu =$ step size, with initial condition

$$\hat{\boldsymbol{\delta}}(0) = \mathbf{0} \; .$$



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# Convergence

- Use ODE/stochastic averaging method. Assume $\mu$ small, $x_c(t)$ zero-mean WSS.
- Write adaptive algorithm as

$$\hat{\boldsymbol{\delta}}(m+1) = \hat{\boldsymbol{\delta}}(m) + \mu \mathbf{T}(\hat{\boldsymbol{\delta}}(m), \mathbf{y}(m))$$

where $\mathbf{y}(m) = [\mathbf{a}^T(m)\mathbf{b}^T(m)]^T$. Due to the stochastic gradient structure of the algorithm,

$$t(\hat{\boldsymbol{\delta}}) = E[T(\hat{\boldsymbol{\delta}}, \mathbf{y}(m))] = -\nabla_{\hat{\boldsymbol{\delta}}} J(\hat{\boldsymbol{\delta}}) \ ,$$

so $J(\boldsymbol{\delta})$ = Lyapunov function for ODE

$$\frac{d\hat{\boldsymbol{\delta}}}{dt} = t(\hat{\boldsymbol{\delta}})(t) \ ,$$

so ODE trajectories converge to a minimum of $J(\hat{\boldsymbol{\delta}})$.



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# Convergence (cont'd)

- For small $\boldsymbol{\delta}$ and $\hat{\boldsymbol{\delta}}$

$$J(\hat{\boldsymbol{\delta}}) \approx \frac{1}{2}\Big\{\big[(\delta_2-\delta_1-\delta_3)-(\hat{\delta}_2-\hat{\delta}_1-\hat{\delta}_3)\big]^2 A$$
$$+ \quad \Big[\big((\delta_1-\delta_3)-(\hat{\delta}_1-\hat{\delta}_3)\big)^2+\big(\delta_2-\hat{\delta}_2\big)^2\Big]B+|\mathbf{c}(\hat{\boldsymbol{\delta}})|^2 C\Big\}$$

with

$$A = \frac{1}{2\pi}\int_{-\alpha\pi}^{\alpha\pi}\omega^2 S_x(e^{j\omega})d\omega$$
$$\frac{B}{2} = \frac{1}{2\pi}\int_{\frac{\pi}{2}-\alpha\pi}^{\frac{\pi}{2}+\alpha\pi}\omega^2 S_x(e^{j\omega})d\omega$$
$$C = 2\alpha N_0$$

where we neglect cubic terms.



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# Convergence (cont'd)

- For noiseless case, unique minimum of $J$ for small offset and offset estimates is $\hat{\boldsymbol{\delta}} = \boldsymbol{\delta}$ if $A > 0$ and $B > 0$, so $x(n)$ must have power in band $[-\alpha\pi, \alpha\pi]$ and $[\pm\pi/2 - \alpha\pi, \pm\pi/2 + \alpha\pi]$.
- Ensures that as $m \to \infty$

$$\hat{\boldsymbol{\delta}}(m) \sim N(\boldsymbol{\delta}, \mu P)$$

where $P$ = positive definite matrix.

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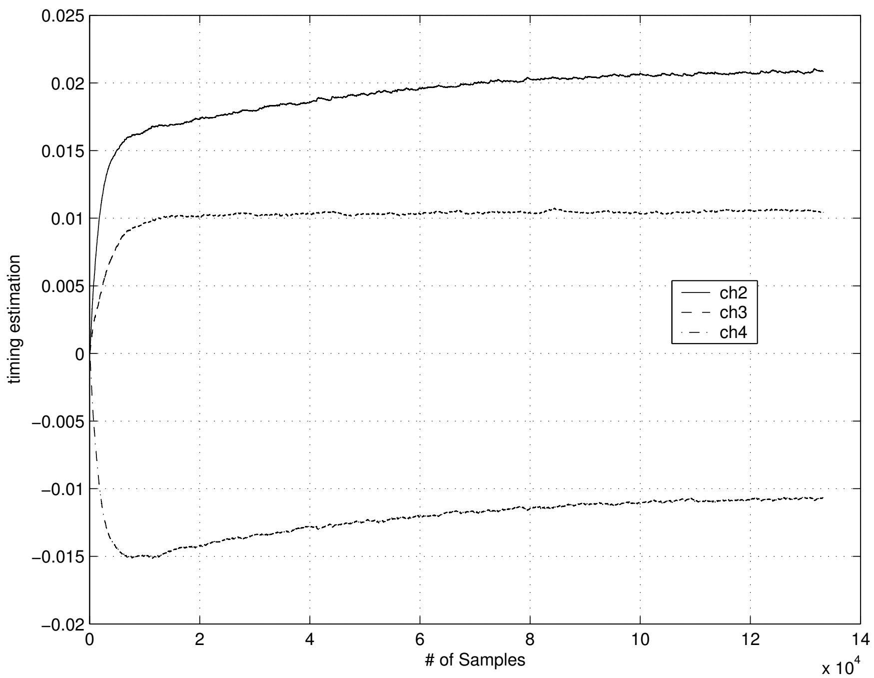
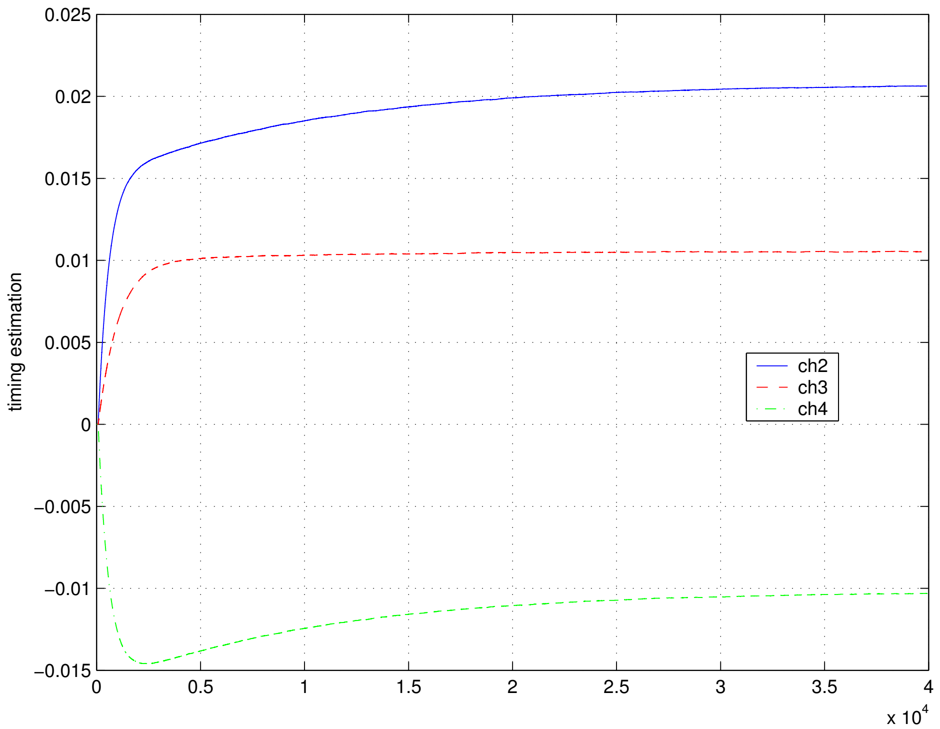
# Bandlimited WGN input




Simulation parameters

- Signal bandwidth: $[-0.75\pi, 0.75\pi]$
- $\delta_1 = 0.02$, $\delta_2 = 0.01$, $\delta_3 = -0.01$.
- $\mu = 2\,10^{-4}$.
- design $\alpha = 0.225$.

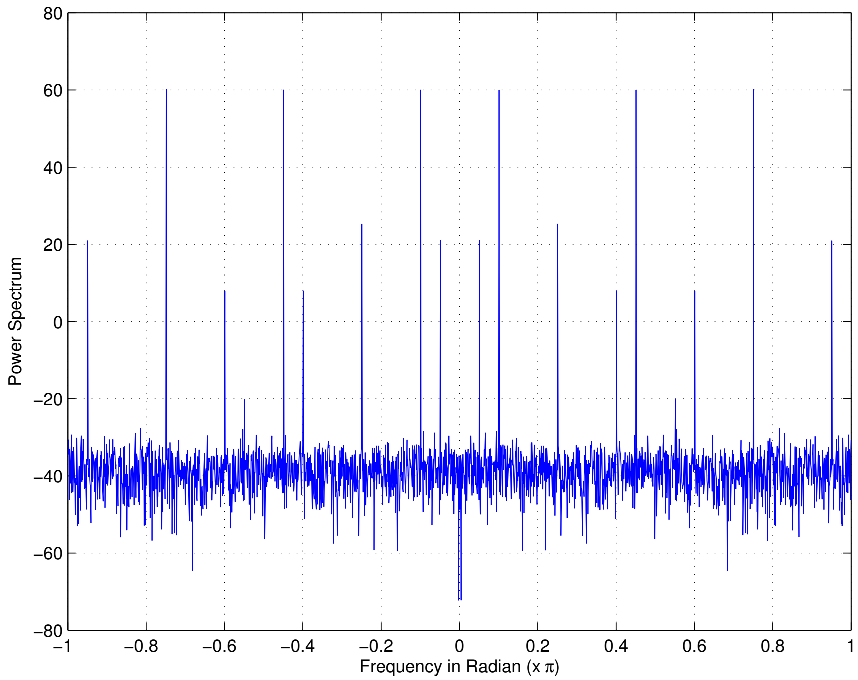
# Multitone sinusoidal input



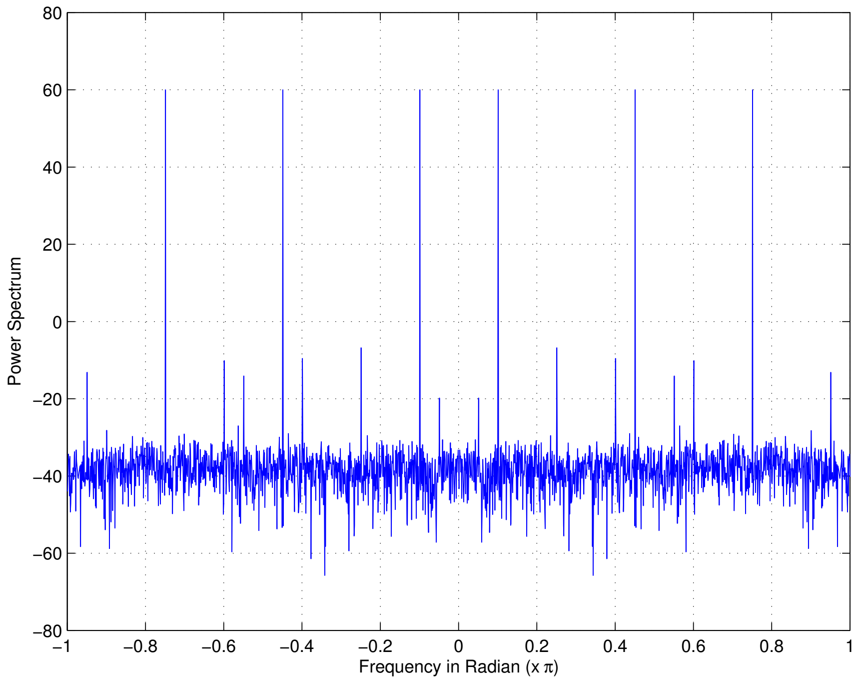

Simulation parameters

- Input frequencies: $0.1\pi$, $0.45\pi$, $0.75\pi$.
- $\delta_1 = 0.02$, $\delta_2 = 0.01$,
- $\mu = 10^{-3}$.
- design $\alpha = 0.15$.

# Calibrated ADC output




Uncalibrated ADC, 70dB SNR




Calibrated ADC, 70dB SNR



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# Conclusion

- Blind calibration of time-interleaved ADCs presented, requires 10 to 20 % oversampling and intermittent excitation of certain frequency bands.
- Simulated for up to 16 channels, but 2 or 4 channels primary interest for today's ADC technology.
- Postprocessing of analog circuits with mismatched components source of interesting adaptive signal processing problems.



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**Thank you!!**