CRLB for Blind Timing Offset Estimation of a Two-Channel Time-Interleaved A/D Converter

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Outline

- **Motivation**
- Blind Calibration Method
- Large Sample CRLB
- Simulations
- Conclusion

Motivation

- Applications requiring ADCs operating at high data rates $\Rightarrow$ **Time-interleaved ADCs**.
- Constituent ADCs have gain, offset, timing mismatches that need to be estimated and corrected in the digital domain.
- Correction achieved by digital filters operating on ADCs outputs.
- Timing offset estimation can be performed either with test signals or blindly. Blind methods do not lower ADC throughput and can adjust to changes online.

Motivation (cont'd)

- Several blind calibration methods have been proposed (Jamal et al. 2004, Elbornsson et al, 2004, Huang and Levy, 2006).
- Method of Huang and Levy requires slightly oversampled signals (10 to 20%), formulates blind calibration as a time delay estimation problem.
- What is the performance of this approach? $\Rightarrow$ **CRLB analysis**.

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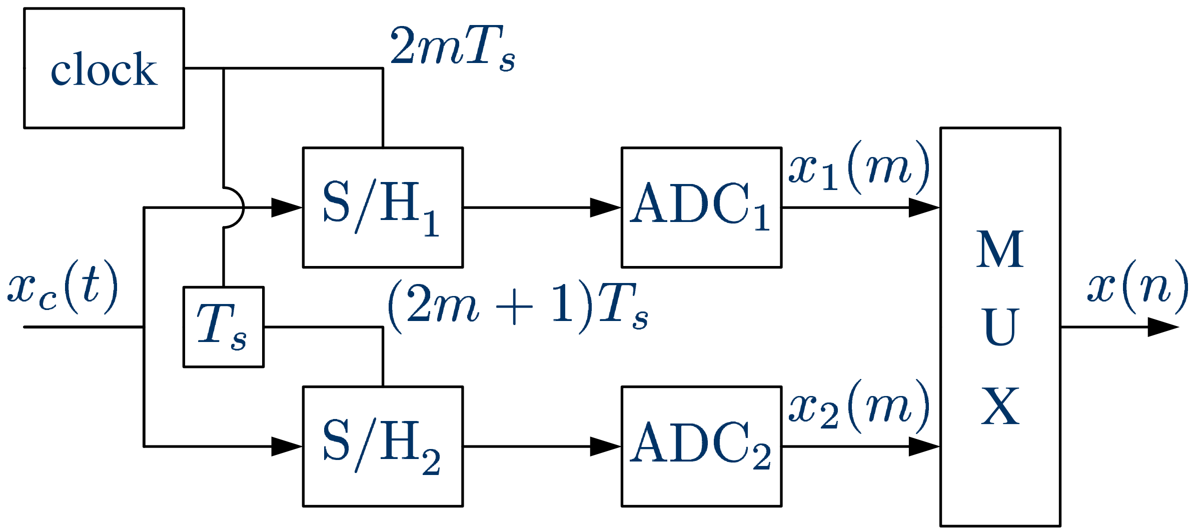
Blind Calibration Method

- A CT bandlimited signal $x_c(t)$ with bandwidth B, can be recovered from its samples $x(n) = x_c(nT_s)$ if $f_s = 1/T_s > f_N = B/\pi$.
- For 2-channel time-interleaved ADC, instead of one fast ADC, can use 2 slow ADCs operating at $f_s/2$.



Blind Calibration Method (cont'd)

- Due to timing offsets, quantization errors and thermal noise, ADC outputs given by

$$\begin{aligned} z_1(m) &= x_1(m) + v_1(m) = x(2m) + v_1(m) \\ z_2(m) &= x_2(m) + v_2(m) = (f_2 * x)(2m) + v_2(m) \end{aligned}$$

where $v_i(m) \sim N(0, N_0/2)$, $i = 1,\ 2$ independt WGNs.

- If δ = timing offset between two channels,

$$f_2(n) = \frac{\sin(\pi(n+1+\delta))}{\pi(n+1+\delta)}$$

= impulse response of delay filter $F_2(e^{j\omega}) = e^{j\omega(1+\delta)}$.

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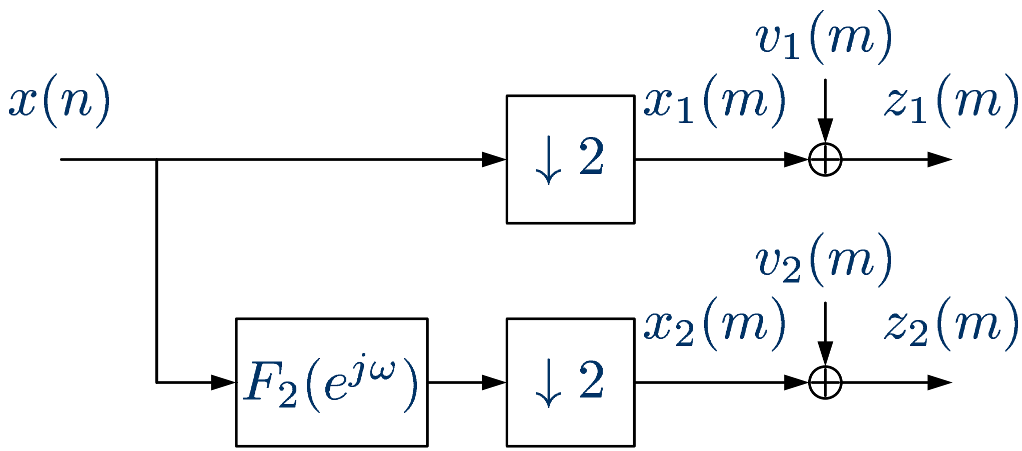
Blind Calibration Method (cont'd)

Analysis filter bank model of 2-channel ADC:



Blind Calibration Method (cont'd)

- If $x_c(t)$ is oversampled with $\alpha = (f_s - f_N)/f_s$ = oversampling ratio, $x_1(m)$ and $x_2(m)$ are **alias-free** over $[0, 2\alpha\pi]$.

- Lowpass filtering the ADC outputs with

$$H_{\mathrm{LP}}(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq 2\alpha\pi \\ 0 & 2\alpha\pi < |\omega| \leq \pi \end{cases}$$

gives

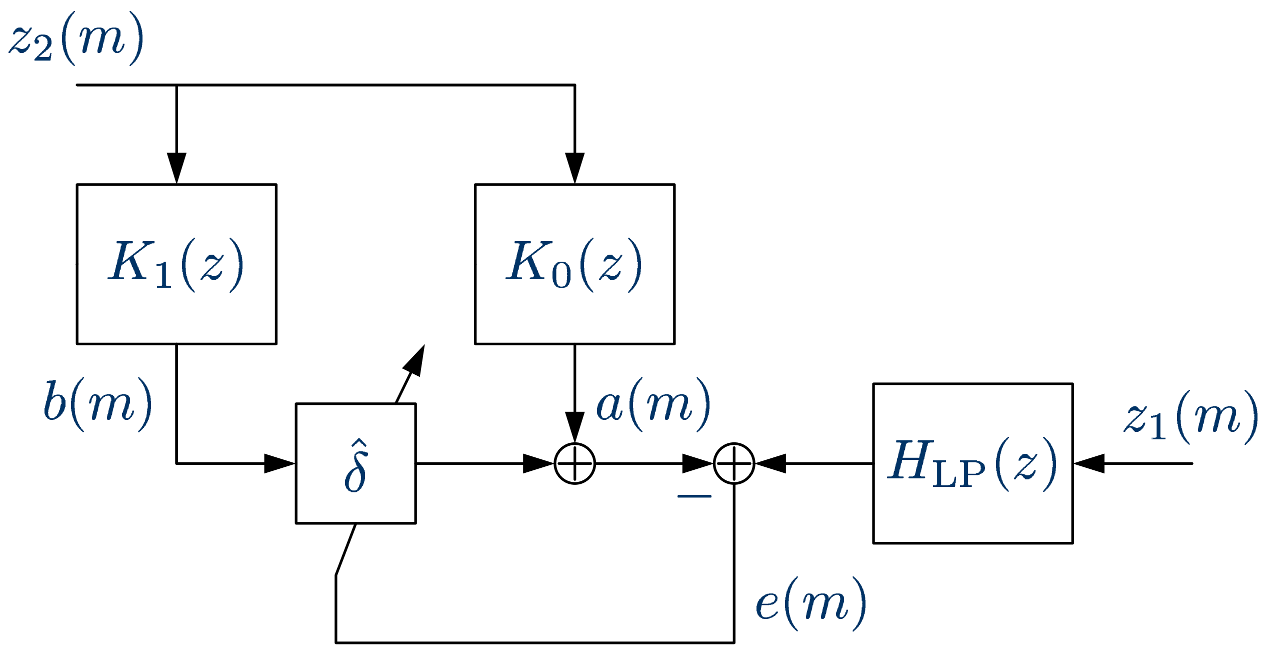
$$\begin{aligned} y_1(m) &= (h_{\mathrm{LP}} * z_1)(m) = s(m - (1+\delta)/2) + w_1(m) \\ y_2(m) &= (h_{\mathrm{LP}} * z_1)(m) = s(m) + w_2(m)\ . \end{aligned}$$

- **Time-delay estimation problem!**

Blind Calibration Method (cont'd)

Adaptive time-delay filter uses Farrow approximation with

$$K_0(e^{j\omega}) = \begin{cases} e^{-j\omega/2} & |\omega| \leq 2\alpha\pi \\ 0 & 2\alpha\pi < |\omega| \leq \pi \end{cases} , \quad K_1(e^{j\omega}) = -j\frac{\omega}{2}K_0(e^{j\omega}) \, .$$



Blind Calibration Method (cont'd)

- Uses 3 fixed FIR filters, only one adjustable coefficient.
- Time delay estimated by ETDE algorithm

$$\hat{\delta}(m+1) = \hat{\delta}(m) + \mu b(m)e(m)\ .$$

- Estimate converges and is approximately unbiased.

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Large Sample CRLB

- Given observations

$$\mathbf{z}(m) = \begin{bmatrix} z_1(m) \\ z_2(m) \end{bmatrix}$$

for $0 \leq m \leq M-1$, seek to evaluate CRLB for unbiased estimates of δ.

- Assume M large and $x(n)$ zero-mean WSS with PSD $S_x(e^{j\omega})$.
- Interleaved samples vector

$$\mathbf{x}_I(m) = \begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}$$

has matrix PSD

$$\mathbf{S}_I(e^{j\omega}) = \mathbf{F}_A(e^{j\omega}, \delta)\mathbf{S}_A(e^{j\omega})\mathbf{F}_A^H(e^{j\omega}, \delta)$$

Large Sample CRLB (cont'd)

with

$$\mathbf{F}_A(e^{j\omega},\delta) = \begin{bmatrix} 1 & 1 \\ e^{j\frac{\omega}{2}(1+\delta)} & e^{j(\frac{\omega}{2}-\pi)(1+\delta)} \end{bmatrix}$$

$$\mathbf{S}_A(e^{j\omega}) = \frac{1}{2}\begin{bmatrix} S_x(e^{j\frac{\omega}{2}}) & 0 \\ 0 & S_x(e^{j(\frac{\omega}{2}-\pi)}) \end{bmatrix}.$$

- Log-likelihood of observations can be expressed as

$$\ln p(\mathbf{z}|\delta) = -M\ln(2\pi) - \frac{M}{4\pi}\int_{-\pi}^{\pi} \ln\det \mathbf{S}_{\mathbf{z}}(e^{j\omega},\delta)d\omega - \frac{1}{4\pi}\int_{-\pi}^{\pi} \mathbf{Z}^H(e^{j\omega})\mathbf{S}_{\mathbf{z}}^{-1}(e^{j\omega},\delta)\mathbf{Z}(e^{j\omega})d\omega \quad (1)$$

Large Sample CRLB (cont'd)

with

$$\mathbf{S}_{\mathbf{z}} = \mathbf{S}_I(e^{j\omega}, \delta) + \frac{N_0}{2}\mathbf{I}_2$$

and

$$\mathbf{Z}^T(e^{j\omega}) = \left[\begin{array}{cc} Z_1(e^{j\omega}) & Z_2(e^{j\omega}) \end{array} \right],$$

where $Z_i(e^{j\omega})$ = DTFT of sequence $z_i(m)$ for $0 \leq m \leq M-1$, $i = 1,\ 2.$

Fisher information: Identity (1) gives

$$\begin{aligned} I(\delta) &= E[-\frac{\partial^2}{\partial \delta^2} \ln p(\mathbf{z}|\delta)] \\ &= \frac{M}{4\pi} \int_{-\pi}^{\pi} Q(e^{j\omega}) d\omega \end{aligned} \quad (2)$$

Large Sample CRLB (cont'd)

with

$$Q(e^{j\omega}) = \frac{N(e^{j\omega})}{D(e^{j\omega})} ,$$

where

$$N(e^{j\omega}) = \Big[\big(\frac{\omega}{2}\big)^2 S_x(e^{j\frac{\omega}{2}}) - \big(\frac{\omega}{2} - \pi\big)^2 S_x(e^{j(\frac{\omega}{2}-\pi)})\Big] \cdot \Big[S_x(e^{j\frac{\omega}{2}}) - S_x(e^{j(\frac{\omega}{2}-\pi)})\Big] + \frac{\pi^2}{2} S_x(e^{j\frac{\omega}{2}}) S_x(e^{j(\frac{\omega}{2}-\pi)})$$

and

$$D(e^{j\omega}) = \big(\frac{N_0}{2} + S_x(e^{j\frac{\omega}{2}})\big)\big(\frac{N_0}{2} + S_x(e^{j(\frac{\omega}{2}-\pi)})\big) .$$

Large Sample CRLB (cont'd)

Fisher information decomposition: We have

$$I = I_{\rm AF} + I_{\rm AL}$$

where

$$\begin{aligned} I_{\rm AF} &= \frac{M}{4\pi}\int_{-2\alpha\pi}^{2\alpha\pi} Q(e^{j\omega})d\omega \\ &= \frac{M}{2\pi}\int_{-\alpha\pi}^{\alpha\pi} \frac{\theta^2}{N_0/2} S_x(e^{j\theta})d\theta \end{aligned}$$

= information in the alias-free band = Fisher information for time-delay estimation, and

$$I_{\rm AL} = \Big(\int_{-\pi}^{-2\alpha\pi} + \int_{2\alpha\pi}^{\pi}\Big)Q(e^{j\omega})d\omega$$

= information in the aliased band.

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Large Sample CRLB (cont'd)

CRLB for time-delay blind calibration:

$$E[(\delta - \hat{\delta})^2] \geq I_{\mathrm{AF}}^{-1} > I^{-1}$$

achieved by ETDE for large sample high SNR case.

CRLB for all blind calibration algorithms:

$$E[(\delta - \hat{\delta})^2] \geq I^{-1}$$

achieved by ML estimate (huge complexity) for large sample case.

Question: When are I and I_{AF} close to each other, or equivalently

$$I_{\mathrm{AL}} \ll I_{\mathrm{AF}}\ ?$$

Intuitively, the aliased band does not contain much useful information (cannot separate aliased signal components).

Large Sample CRLB (cont'd)

Bandlimited WGN analysis: If

$$S_x(e^{j\omega}) = \begin{cases} P_x & 0 \le |\omega| \le \pi(1-\alpha) \\ 0 & \pi(1-\alpha) < |\omega| \le \pi \, , \end{cases}$$

we have

$$\sigma_x^2 = E[x^2(n)] = P_x(1-\alpha)$$

and

$$\begin{aligned} I_{\mathrm{AF}} &= M \frac{\sigma_x^2}{N_0/2} \frac{\alpha^3 \pi^2}{3(1-\alpha)} \\ I_{\mathrm{AL}} &= M \frac{\pi^2}{4} (1-2\alpha) \, . \end{aligned}$$

Large Sample CRLB (cont'd)

Neglecting thermal noise

$$\frac{N_0}{2} \approx 2^{-2(b-1)} \sigma_x^2$$

= quantization noise, where $b = \#$ quantization bits.

Result:

$$R = \frac{I_{\mathrm{AF}}}{I_{\mathrm{AL}}} = 2^{2b} \frac{\alpha^3}{3(1-\alpha)(1-2\alpha)} \,.$$

Can be large if b large enough and α not too small: $R = 30$ for $b = 8$, $\alpha = 0.1$.

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Simulations

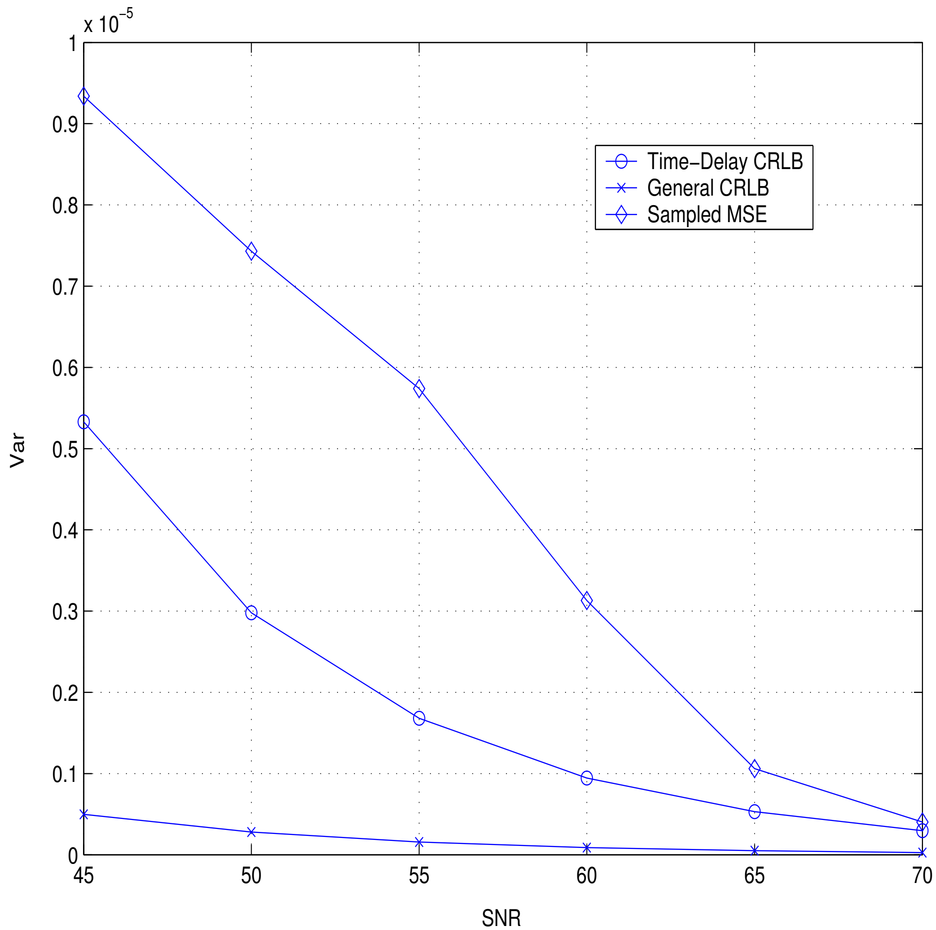
- Compare sampled MSE

$$\widehat{MSE} = \frac{1}{K}\sum_{k=1}^{K}(\delta - \hat{\delta}_k)^2$$

for Monte Carlo simulation with $K = 200$ runs against two CRLBs I_{AF}^{-1} and I^{-1}.

- Set $\delta = -0.01$ (1% timing error) and $x(n)$= bandlimited WGN over $[-0.75\pi, 0.75\pi]$.
- Use 21 taps filters H_{LP} K_0, K_1 designed with $\alpha = 0.15$ (need to include a transition region).
- Set $\mu = 10^{-5}$ to keep excess MSE low, $2\,10^5$ samples per run, SNR varies from 45dB to 70dB.

Simulations (cont'd)



- $\widehat{MSE}$ approaches I_{AF}^{-1}
- Both close to I^{-1} as SNR reaches 70dB

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Conclusion

- Performed a CRLB analysis of blind timing mismatch estimation for two-channel time-interleaved ADCs.
- Two CRLBs derived: one general, other for algorithms based on alias-free band of lowpass filtered ADC outputs.
- For wideband WGN input, obtained a condition describing when the two CRLBs are close to each other.
- Simulations show that time delay blind calibration method of Huang and Levy (2006) is nearly efficient.

Thank you!!