1. Two capacitors $C_1$ and $C_2$ are needed with a very accurately defined ratio of $C_1/C_2 = 4.0$, with the constraint that the maximum area that can be used for $C_2$ is 100 ($\mu$m)$^2$. Draw a top view of the layout of the two capacitors. Show the dimensions of the capacitors on your drawing.

- All unit caps: 10$\mu$m x 10$\mu$m
- 4 units for $C_1$
- Common centroid
2. For the two-stage op amp shown below, assume DC biasing places the transistors in the saturation region. (NOTE: The first stage of this op amp is different than the standard two-stage op amp.)

\[
\begin{align*}
\text{\( v_{out}/v_{id} = \frac{2.4 \times 10^6}{10} = \frac{g_{m2}}{2} \cdot 2R_2 \cdot g_{m3} \cdot R_3 \)}
\end{align*}
\]

\[
\begin{align*}
g_{m2} &= \left[ \frac{2(60 \times 10^{-6})(100)(20 \times 10^{-6})}{1} \right]^{\frac{1}{2}} = 4.90 \times 10^{-6} \\
R_2 &= \frac{1}{(0.01 \times 10^{-3})(20 \times 10^{-6})} = 5 \text{M} \Omega \\
g_{m3} &= \left[ \frac{2(190 \times 10^{-6})(100)(100 \times 10^{-6})}{1} \right]^{\frac{1}{2}} = 1900 \times 10^{-6} \\
R_3 &= \frac{1}{(0.02 \times 10^{-3})(100 \times 10^{-6})} = 0.5 \text{M} \Omega
\end{align*}
\]

b) What is the output slew rate? \( SR^+ \)

\[
\max \frac{dV_o}{dt} = \max \frac{I}{C} = \frac{20 \times 10^{-6}}{5 \times 10^{-12}} = 4 \times 10^6 \text{V/s}
\]
Note:

2a) For first stage, if you short its output to get $G_{m_1}$: the resulting circuit is symmetric and $G_{m_1} = \frac{g_{m_2}}{2} = \frac{g_{m_2}}{2}$.

If you set inputs to 0 to compute output resistance of first stage:

$$R_{o_1} = R_{o_2}(1 + g_{m_2} \frac{1}{g_{m_1}}) = 2R_{o_2}$$

So gain of first stage $= \frac{g_{m_2}}{2} \cdot 2R_{o_2}$

- If you use half circuits to get output resistance, you get $R_{o_1} = R_{o_2}$. This is not correct but I gave credit for this answer (and if you used $R_{o_1} = R_{o_2}$ in part c)

[first stage is not symmetric]
c) What is the dominant pole for this op amp?
\[ p_1 = \frac{-21}{\text{sec}} \]

Miller effect:
\[ p_1 \approx \frac{-1}{2F_2 (5pF) (1 + g_m R_o)} \]
\[ = \frac{-1}{(5mV/5pF) \left[ 1 + 1900 \frac{\text{mA}}{V} \cdot 0.5 \text{m} \Omega \right]} \]
\[ = \frac{-21}{\text{sec}} \]

d) Estimate the frequency at which the magnitude of the voltage gain falls to one (ignore the zero and nondominant poles for this calculation). \[ f_u = 8.0 \text{ MHz} \]
\[ a_v(jw) \approx \frac{2.4 \times 10^4}{1 + j \frac{w}{21}} \]

\[ \text{high freq: } |a_v| = \frac{2.4 \times 10^4}{w} \Rightarrow 1 = \frac{2.4 \times 10^4}{\frac{w}{21}} \]

\[ \omega_u = 50 \text{ mrad/ sec} \]
\[ f_u = \frac{\omega_u}{2\pi} = 8.0 \text{ MHz} \]
e) What value of resistor added in series with the 5pF compensation capacitor will move the zero to infinity?
\[ R = 526 \Omega \]
\[ \text{want } R = \frac{1}{g_m} = \frac{1}{1900 \frac{\text{mA}}{V}} = 526 \Omega \]
3. A fully differential one-stage op amp with common-mode feedback is shown. Assume all transistors are saturated, and assume $M1 = M2, M3 = M4, M6 = M7$.

![Diagram of a fully differential one-stage op amp]

- **a)** What is the drain current in $M5$?
  
  $I_{D(M5)} = 50\mu A$

  $3 \times 50\mu A + I_{D5} = 2 \times 100\mu A$

- **b)** What is the differential-mode (DM) voltage gain?

  $v_{out}/v_{id} = \frac{1.6 \times 10^{-5}}{0.1} = g_{m1} \cdot r_{o3} (1 + g_{m3} r_{o1})$

  $g_{m1} = \frac{2(60\mu A)}{V_{dd}} (100)(50\mu A) = 775 \mu A/V$

  $r_{o1} = \frac{1}{(0.01 V^{-1})(50\mu A)} = 200 \Omega$

  $g_{m3} = \frac{2(180 \mu A)}{V_{dd}} (10) 50\mu A = 424 \mu A/V$

  $r_{o3} = \frac{1}{(0.02 V^{-1})(50\mu A)} = 19 \Omega$
c) What is the lower limit of the common-mode input range?

\[ V_{\text{IN(CM)}}^\text{\text{-}} = -3.2V \]

Limit when \( m_1/m_2 \) triode:

\[ V_{\text{IN(CM)}}^\text{\text{-}} = V_{\text{bias}} - V_{\text{GS}} - |V_T| \]

\[ = -1V - \left[ \sqrt{\frac{2 \cdot (50mA)}{180 \nabla^2 V}} + V_{T_N} \right] - 1V \]

\[ = -1V - [0.236V + 1V] - 1V \]

\[ = -3.2V \]

d) Estimate the negative output swing limit of each output.

Negative output swing limit of out+ or out- = \( -2V \)

\[ \text{Neg limit when } m_3/m_4 \text{ triode:} \]

\[ V_{\text{out}+} \text{ or } V_{\text{out}-} = V_{\text{bias}} - V_{T_N} \]

\[ = -1V - 1V = -2V \]
4. Use Blackman's impedance formula to compute the resistance looking into the port labeled below. Compute the return ratios with respect to the $g_m$ controlled source of $M_1$. Assume both transistors are saturated and $(W/L)_{M_1} = (W/L)_{M_2} = 100$.

![Circuit Diagram]

a) What is $R_{(\text{port})}$ when $g_m(M_1) = 0$? 0.5 m\(\Omega\)

\[ R_{0_1} = \frac{1}{(0.02 V^{-1})(100 \mu A)} = 0.5 \text{ m}\Omega \]

b) What is $RR(g_m(M_1) \text{ with the port open})$? 949

\[ RR = R_{0_1} \cdot g_m = (0.5 \text{ m}\Omega)(1900 \mu A) = 949 \]

\[ g_m = \sqrt{2 \left(180 \mu A^2\right)(100)(100 \mu A)} = 1900 \mu A \]
4 (continued)

c) What is $RR(g_m(M1)$ with the port shorted)? 

\[ \text{d) Now applying Blackman's formula, what is } R(\text{port}) \text{ for the feedback circuit? } 526 \Omega \]

\[
R(\text{port}) = 0.5 \, \text{mA} \times \frac{1 + 0}{1 + 949} \\
= 526 \Omega
\]
5. A first-order RC filter that uses only MOS transistors is shown below. Assume that the AC input signal is small compared to the DC input bias voltage. What W/L ratio is needed for M1 to give a -3 dB frequency of 10 MHz? Ignore all junction and overlap capacitances.

\[
\begin{align*}
C &= C_{gs}(m2) = C_{ox} \cdot W \cdot L = 3.5 \text{ fF} / \mu m \cdot 10 \mu m \cdot 10 \mu m \\
&= 350 \text{ fF}
\end{align*}
\]

m1 trade:

\[
R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_T)} \approx \frac{1}{180 \frac{\mu A}{V^2} \left(\frac{W}{L}\right)[5 - 2 - 1] V}
\]

\[
= \frac{2778 \Omega}{\frac{W}{L}}
\]

want \[
\frac{1}{2\pi R_{on} C} = 10 \text{ MHz} \Rightarrow R_{on} = 45.5 \text{ k}\Omega
\]

\[
\Rightarrow \frac{W}{L} = 0.061
\]