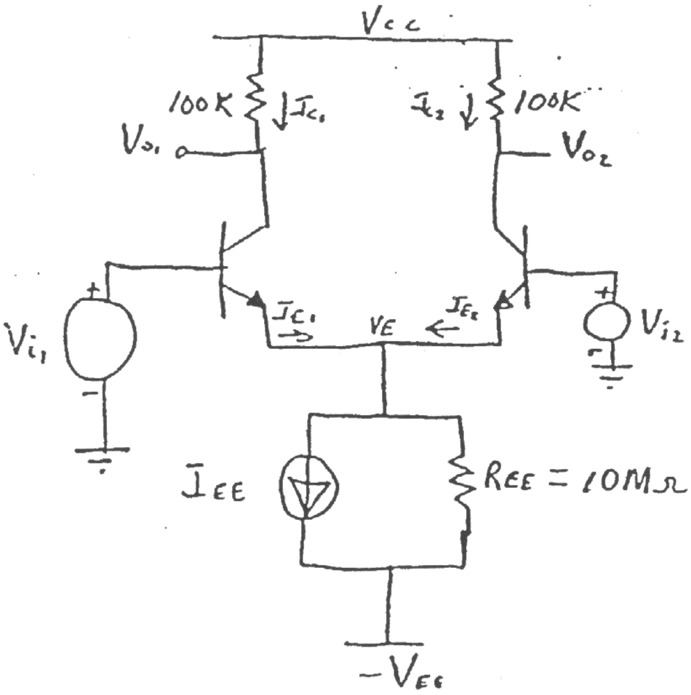
EEC PS#4 Solution

1) Find Diff. mode gain, Common mode gain, differential mode input resistance, Common mode input resistance. Find CMRR.

$I_{EE} = 20\mu A$ $R_C = 100k\Omega$ Neglect r_o, r_μ, r_b.

$R_{EE} = 10M\Omega$ $V_{EE} = V_{CC} = 5V$



D.C CALCULATIONS

Assume $V_E = 0V$ Because of SYMMETRY $I_{EE} = 20\mu A + \frac{5V}{10M\Omega} = 20.5\mu A \cong 20\mu A$

$\therefore I_{C_1} = I_{C2} = 10\mu A$ $g_m = \frac{10\mu A}{26mV} = 385\mu A/V$

1 (cont.)

Differential Mode Gain

$$A_{dm} = -g_m R_C = \underline{-38.5}$$

Common Mode gain

$$A_{cm} = \frac{-g_m R_C}{1+2g_m R_{EE}(1+\frac{1}{\beta_o})} = \frac{-38.5}{1+(38.5\times10^{-3})(2)(10^7)(1+1/\beta_o)} = \underline{-4.97\times10^{-3}}$$

$\beta_o = 200$

Differential Mode input Resistance

$$R_{id} = 2r_\pi = \frac{2\beta_o}{g_m} = \frac{400}{38.5\times10^{-3}} = \underline{1.04 M\Omega}$$

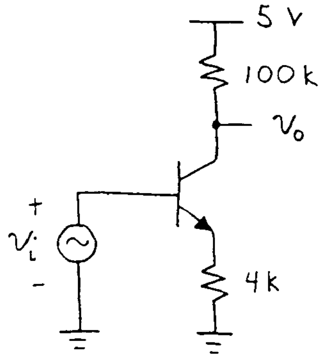
Common Mode Input Resistance

$$R_{ic} = r_\pi + 2R_{EE}(1+\beta_o) = 520K + 2(10M\Omega)(1+200)$$
$$= 520K + 4{,}020 M\Omega$$
$$\cong \underline{4.020\times10^9 \Omega}$$

CMRR

$$CMRR = 1+2g_m R_{EE}(1+\frac{1}{\beta_o})$$
$$= 1+2(38.5\times10^{-3})10^7(1+\frac{1}{200})$$
$$= \underline{7.73\times10^3}$$

2) Looking at the half ckt



$\Rightarrow$ CE w/ degeneration

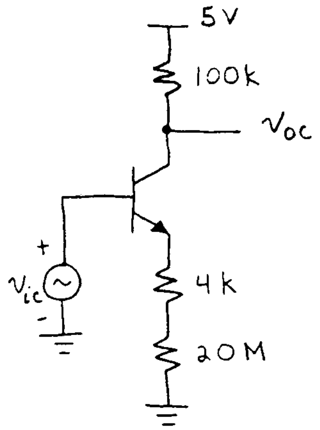
$$A_{dm} = \frac{-g_m}{1 + g_m R_E} R_C$$

$$= \frac{-0.385\ mA/v}{1 + (0.385\ mA/v)(4k)} [100k]$$

$$\boxed{A_{dm} \simeq -15}$$

$$g_{m1} = g_{m2} = \frac{10\mu A}{26 mV} = 0.385\ mA/V$$

For Common mode ckt



$$\Rightarrow A_{cm} = \frac{-g_m}{1 + g_m R_E} [100k]$$

$$= \frac{-0.385 mA/v}{1 + (0.385 mA/v)(20M)} 100k$$

$$\boxed{A_{cm} = -0.005}$$

$$\Rightarrow CMRR = \frac{A_{dm}}{A_{cm}} = 3000 = \boxed{69 dB}$$

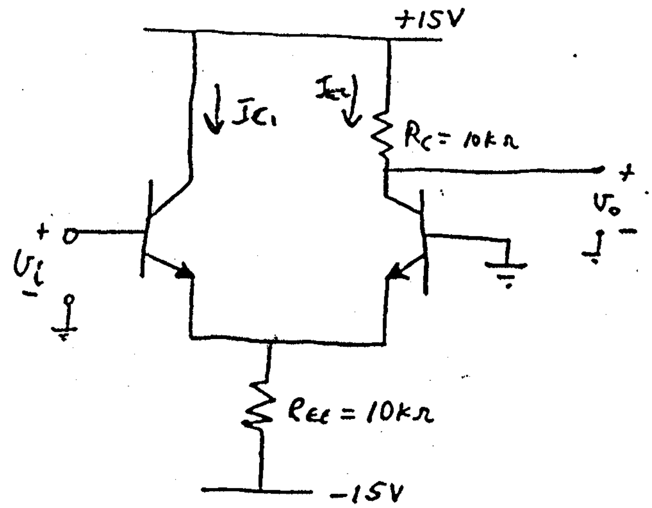
$$\Rightarrow R_{id} = 2(r_\pi + (\beta + 1) 4k)$$

$$= \boxed{2.12\ M\Omega}$$

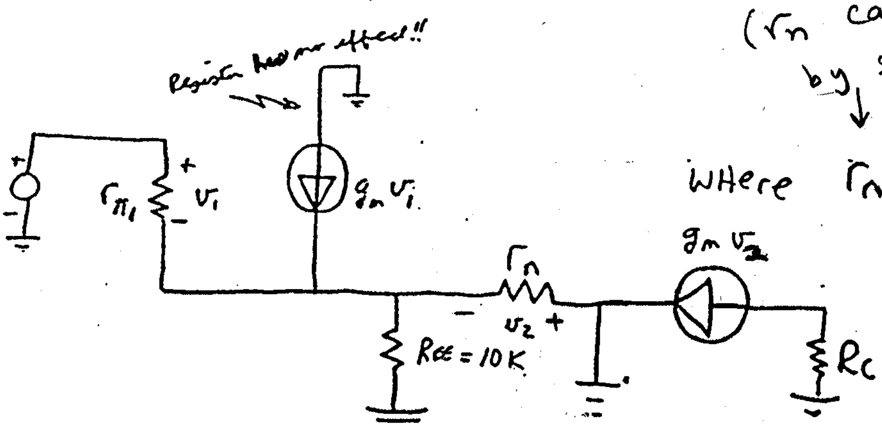
$$\Rightarrow R_{ic} = r_\pi + (\beta + 1)(20M + 4k)$$

$$= \boxed{4.02 \times 10^9\ \Omega}$$

3. R_{IN}, A_V, R_o



$2v_{ic} = v_{id} = v_i$



(r_n called r_e by some)

where $r_n = \frac{\alpha_F}{g_m} = \frac{\frac{200}{201}}{27.5mA} = 36.2\Omega$

d.c. Analysis $-(I_{E_1} + I_{E_2}) = \frac{15V - .7V}{10K} = \underline{1.43mA} = \frac{I_{E_1}}{2}$

$I_{C_1} = I_{C_2} = \frac{I_{E1}}{2} = \frac{1.43mA}{2} = \underline{.715mA}$

$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{.715mA}{26mV} = 27.5mA/V = g_{m2}$ since $I_{C_1} = I_{C_2}$

$\beta_o = 200$ from model data in Chap 2.

$r_{\pi_1} = r_{\pi_2} = \frac{200}{27.5mA/V} = \underline{\underline{7.273k\Omega}}$

so $R_{IN} = r_{\pi_1} + (\beta+1)(R_{EE} \| r_n)$

$= 7.273K + (201)(10K \| 36.2\Omega)$

$\boxed{R_{IN} = \underline{14.53K}}$

3. con't

$R_{OUT} = R_C \,//\, \text{current source}$

$\boxed{R_{OUT} = R_C = \underline{10K\Omega}}$

$v_{OUT} = v_{o2} = -\frac{v_{od}}{2} + v_{oc} \triangleq -A_{dm}\frac{v_{id}}{2} + A_{cm} v_{ic} = -\frac{A_{dm}}{2}(v_i) + A_{cm}\left(\frac{v_i}{2}\right)$

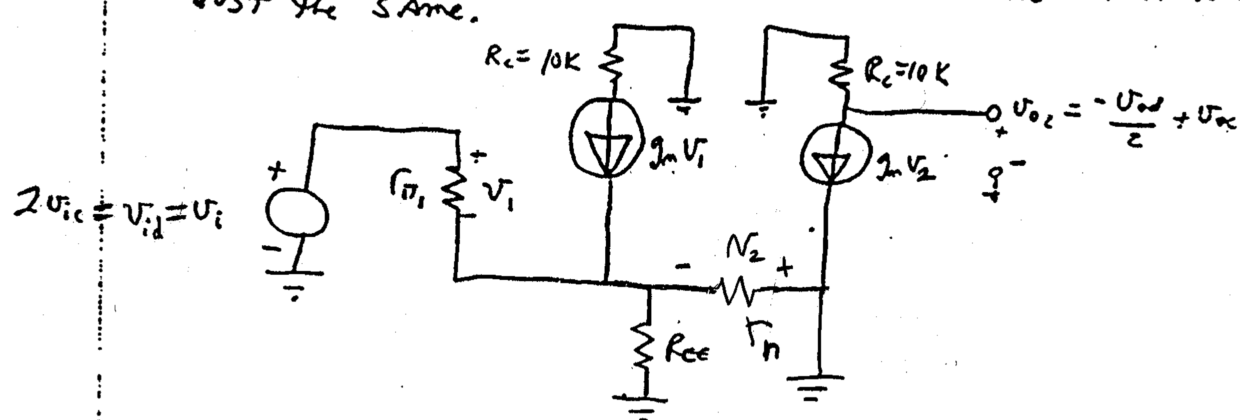
BUT $A_{dm} = -g_m R_C = (27.5\,mA/V)(10K) = \underline{-275}$

$A_{cm} = \frac{-g_m R_C}{1 + 2 g_m R_E\left(1+\frac{1}{\beta_o}\right)} = \frac{-275}{1+550\left(1+\frac{1}{200}\right)} = \underline{-.4966}$

$v_o = \frac{275}{2}(v_i) - \frac{.4966}{2} v_i$

$\frac{v_o}{v_i} = \underline{137.25}$

Now If we put A Resistor R_C the ANALYSIS is JUST the SAME.



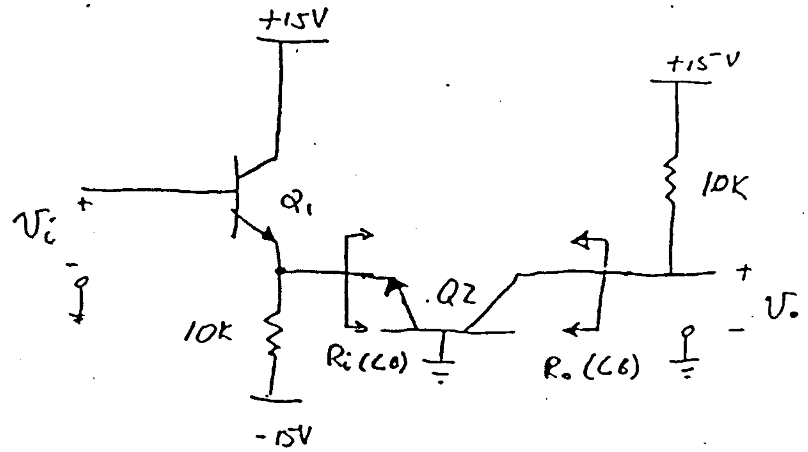
$R_{IN} = r_{\pi 1} + (\beta+1)(R_{EE} \,//\, r_\pi)$

$= \underline{1453k\Omega}$

$R_{OUT} \cong R_C = \underline{10K\Omega}$

$A_v = \frac{-A_{dm}}{2} + \frac{A_{cm}}{2} = \frac{275}{2} - \frac{.4966}{2} = \underline{137.25}$

3. (b) TO ANALYZE AS A CC − CB REDRAW:



NOW: Q_1 C.C. Q_2 C.B.

$$R_{IN_{TOTAL}} = R_i(\text{C.C. } Q_1) = \overbrace{r_{\pi_1} + (\beta+1)(15K \,//\, R_i\ \text{C.B.})}^{\text{USE HANDOUT}}$$

BUT $R_i(\text{C.B}) \cong \dfrac{1}{g_{m_2}}$

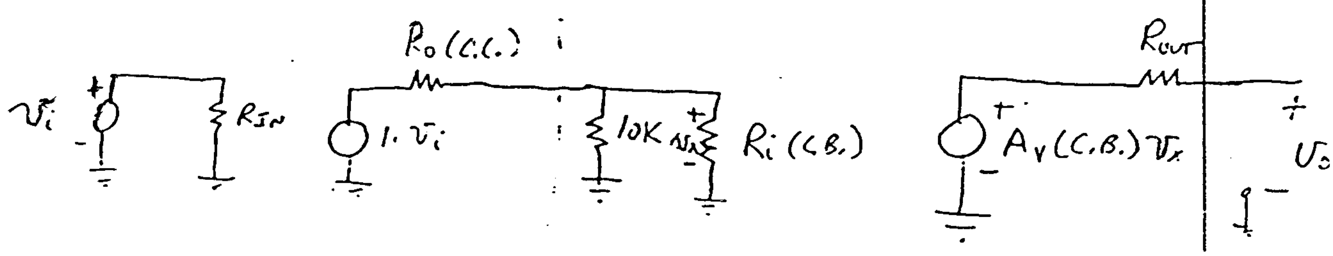
$$g_{m_1} = g_{m_2} = \frac{.71mA}{26mV} = \underline{\underline{27.3mA/V}}$$

$$r_{\pi_1} = \frac{200}{g_{m_1}} = \frac{200}{27.3mA/V} = \underline{\underline{7.324K}}$$

$$R_{IN_{TOTAL}} = 7.324K + (201)(10K // 36.6\Omega)$$

$$= \underline{\underline{14.69K}}$$

$$R_{out} = R_o(\text{C.B.}) \,//\, 10K\Omega \cong 10K\Omega$$



$$v_o = A_v(\text{C.B.})\, v_x = \left[1 \cdot v_i \frac{R_i(\text{CB}) // 10K}{R_o(\text{CC}) + R_i(\text{CB}) // 10K\Omega} \right] A_v(\text{CB})$$

$$V_o \cong g_{m2}\, 10k \cdot \left[V_i \frac{1/g_{m2}}{1/g_{m1} + 1/g_{m2}} \right] = g_{m2}\, 10k \frac{V_i}{2} = 136\, V_i$$

$$\boxed{\text{So } \frac{V_o}{V_{in}} = 136}$$

SPICE COMPARISON

	CC-CB			diff pair		
	R_i [Ω]	R_o [Ω]	A_v [V/V]	R_i [Ω]	R_o [Ω]	A_v [V/V]
SPICE	14.58k	10k	136.7	14.58k	10k	136.7
Calculated	14.69k	10k	136	14.53k	10k	137.25

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*******          *******  SPICE 2G.5 (          )  *******21:16:17*****



****     OPERATING POINT INFORMATION       TEMPERATURE =    27.000 DEG

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CC - CB

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**** BIPOLAR JUNCTION TRANSISTORS


              Q1          Q2

MODEL       XTER        XTER
IB          3.54E-06    3.54E-06
IC          7.08E-04    7.08E-04
VBE            0.765       0.765
VBC          -15.000      -7.918
VCE           15.765       8.683
BETADC       200.001     200.000
GM          2.74E-02    2.74E-02
RPI         7.30E+03    7.30E+03
RX          0.00E+00    0.00E+00
RO          1.00E+12    1.00E+12
CPI         0.00E+00    0.00E+00
CMU         0.00E+00    0.00E+00
CBX         0.00E+00    0.00E+00
CCS         0.00E+00    0.00E+00
BETAAC       200.000     200.000
FT          4.36E+17    4.36E+17



****     SMALL-SIGNAL CHARACTERISTICS

     V(6)/VIN                                       =  1.367D+02

     INPUT RESISTANCE AT VIN                        =  1.458D+04

     OUTPUT RESISTANCE AT V(6)                      =  1.000D+04

        JOB CONCLUDED

        TIME                      PAGE     DIRECT     BUFFERED
   CPU            ELAPSED        FAULTS     I/O         I/O
0: 0: 0.58      0: 0: 1.61         36         7            1
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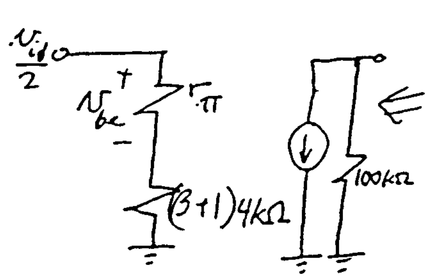
4 Recall from Lecture #1 that the linear approximation holds only for $v_{be} \ll V_T$ (all quadratic or higher-order terms in v_{be}/V_T were neglected). The approximation is accurate to within 50% for $v_{be} < V_T$. ∴ Find maximum input values V_A and V_B such that small-signal $v_{be} < V_T$

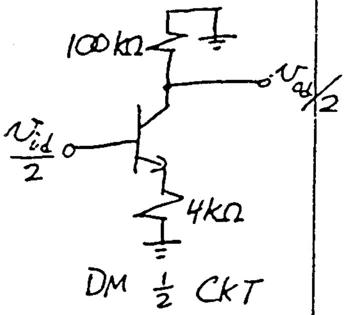
⟶ Differential Mode (determine V_A):

Neglect r_b, r_μ, r_o:

By inspection,

$$v_{be} = \frac{r_\pi}{r_\pi + (\beta+1)(4k)} \frac{v_{id}}{2}$$





$r_\pi = 520k\Omega$

Set $v_{be} = V_T$ for $v_{id} = V_A$

$$v_{id}\Big|_{v_{be}=V_T} = V_A = \frac{(2)(V_T)(r_\pi + (\beta+1)4k)}{r_\pi} = 132.4\,mV \text{ for } V_T = 26mV \Rightarrow \boxed{V_A = 132mV = 0.132\,V}$$

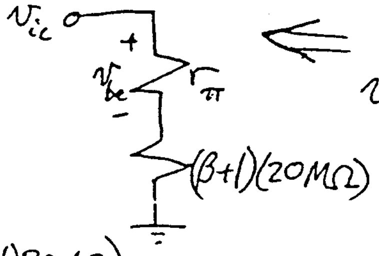
-Answer-

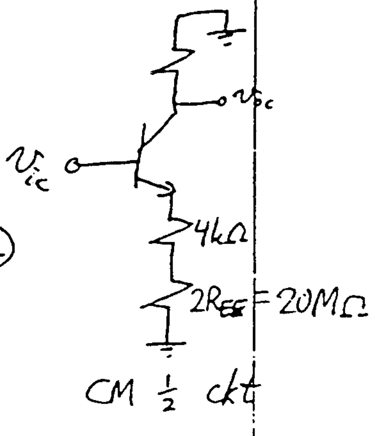
Maximum Differential Input

⟶ Common Mode (determine V_B)

Neglect 4kΩ compared to 20MΩ:

$$v_{be} = \frac{r_\pi}{r_\pi + (\beta+1)(20M\Omega)} v_{ic}$$





Set $v_{be} = V_T$ to find $v_{ic} = V_B$

$$v_{ic}\Big|_{v_{be}=V_T} = V_B = \frac{(V_T)(r_\pi + (\beta+1)20M\Omega)}{r_\pi}$$

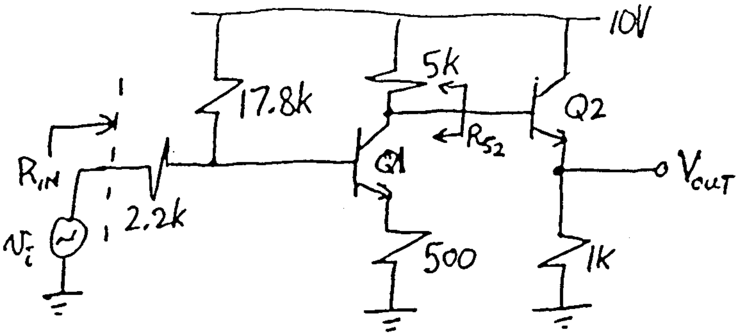
$$= \frac{(26mV)(520k + 4020M\Omega)}{520k} = 201\,V \text{ for } \beta = 200,\ V_T = 26mV$$

$\boxed{V_B = 201\,V}$ -Answer-

Maximum CM Input for ss. linearity

Note that other factors (e.g. entering sat., supplies and breakdowns) will prevent the circuit from violating this CM linearity limit (V_B).

5.



Neglecting base current in Q1, the DC Base Voltage of Q1 is

$$V_{B1} = \frac{2.2k}{2.2k+17.8k}(10V) = 1.1V$$

$\boxed{KVL}$ $V_{B1} - V_{BE1} - (I_{E1})(500\Omega) = 0$

$$V_{BE1} \approx 1.1V - (I_{C1})(500\Omega) \quad \text{for } I_C \approx I_E$$

Neglecting Early Effect $\quad V_{BE1} = V_T \ln \frac{I_{C1}}{I_{S1}} = 1.1V - (I_{C1})(0.5k\Omega)$ $\quad I_S = 5\times10^{-15}A$ from fig 2.23

$$(0.026V)\ln \frac{I_{C1}}{5\times10^{-12}mA} = 1.1V - I_{C1}(0.5k\Omega) \leftarrow \text{Iterate to find } I_{C1}$$

$$I_{C1} = 0.855\,mA = 855\mu A \quad \boxed{\text{HSPICE}: I_{C1} = 848\mu A}$$

Note that $V_{BE1} = 0.7V$ was NOT used to determine I_{C1} because the 500Ω only has about 0.4 V across it, and a 100 mV discrepancy in V_{BE} is significant (in this case, $V_{BE} = 0.67V$).

Note also that at $\boxed{\beta = 200,}$ $I_{B1} \approx 4.3\mu A \ll \frac{10V}{20k\Omega} = 500\mu A$, so the voltage divider approximation for V_{B1} is good!

$\underline{I_{C1} = 855\mu A} \longrightarrow g_{m1} = \frac{855\mu A}{26mV} = \underline{32.9\,m\mho} \rightarrow r_{\pi 1} = \frac{200}{32.9m\mho} = \underline{6.08\,k\Omega}$

$V_{B2} = V_{C1} = 10V - (5k\Omega)(855\mu A) = 5.73V \quad r_{o1} = \frac{130V}{855\mu A} = 152\,k\Omega$

$\therefore V_{OUT} \approx V_{B2} - 0.7V \approx 5.0\,VDC \quad \boxed{\text{HSPICE}: I_{C2} = 4.9mA}$

$I_{C2} \approx \frac{V_{OUT}}{1k\Omega} = \underline{5.0\,mA} \longrightarrow g_{m2} = \frac{5.0mA}{26mV} = 192m\mho \rightarrow r_{\pi 2} = \frac{200}{192m\mho} = \underline{1.04}$

$r_{o2} = \frac{130V}{5mA} = 26k\Omega$

For CEWD, plus by inspection of above diagram:

$$R_{IN} = 2.2k\Omega + (17.8k\Omega) \| (r_{\pi 1} + (\beta+1)R_E)$$

$$= 2.2k + (17.8k) \| (6.08k + (201)(500\Omega)) = 17.5k\Omega \Rightarrow \boxed{R_{IN} = 17.5k\Omega}$$

- Answer -

HSPICE $R_{in} = 17.47k\Omega$

$\underline{R_{OUT}}$ is that for a CC stage (see Handout)

$$R_{OUT} = (1k\Omega) // (R_{o2}) \qquad R_{o2} \approx \frac{1}{g_{m_2}} + \frac{R_{S2}}{\beta+1}$$ where R_S is that looking back into the CEWD

$$R_{o1}' = R_{S2} = 5k\Omega // R_{o1} \approx (5k\Omega) // \left[r_{o1}\left(1 + g_{m1} R_{E1} \frac{r_{\pi 1}}{r_{\pi 1} + R_{S1} + R_{E1}}\right)\right]$$

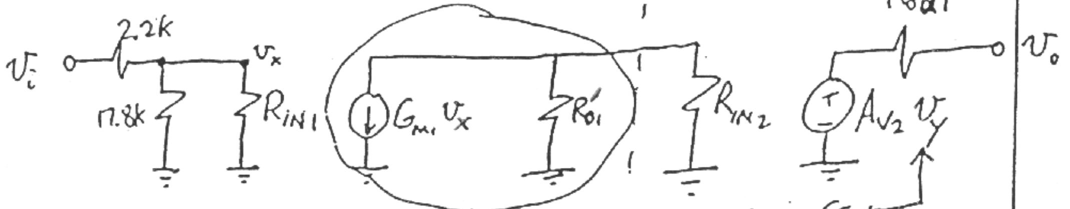
$$= 5k\Omega // \left[(152 k\Omega)\left(1 + (32.9 m\mho)(0.5k\Omega) \frac{6.08 k\Omega}{6.08k + 0.5k + \underbrace{2.2k//17.8k}_{R_{S1}}}\right)\right]$$

$$= 5k\Omega // 1.93 M\Omega \approx 5k\Omega$$

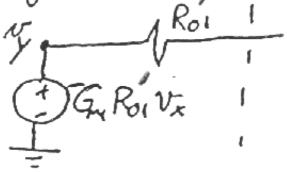
$$R_{OUT} = (1k\Omega) // \left(\frac{1}{192 m\mho} + \frac{5k\Omega}{201}\right) = 1k\Omega // 30.1\Omega$$

$$\boxed{R_{OUT} \approx 29\Omega}$$ -Answer- $\boxed{\text{HSPICE } R_{out} = 31.5\Omega}$

For A_v:



v_y is defined after Thevenizing input to CEWD



from Single Xstr Handout:

$$R_{IN1} = r_{\pi 1} + (\beta+1) R_{E1} = \underline{106.6 k\Omega}$$

$$G_{m1} \approx \frac{g_{m1}}{1 + g_{m1} R_{E1}} = \frac{32.9 m\mho}{1 + (32.9 m\mho)(0.5k\Omega)} = \underline{1.89 m\mho}$$

$$R_{IN2} = r_{\pi 2} + (\beta+1)(R_{L2} // r_{o2}) = 1.04 k\Omega + (201)(1k\Omega // 26k\Omega) = 195 k\Omega$$

$$A_V = \left(\frac{R_{IN1} // 17.8k}{R_{IN1} // 17.8k + 2.2k}\right)(G_{m1} R_{o1}')(A_{v2})$$

$$A_{v2} \approx \frac{R_{L2}}{R_{L2} + R_{o2}} = \frac{1k\Omega}{1k\Omega + 30.1\Omega} = 0.971$$

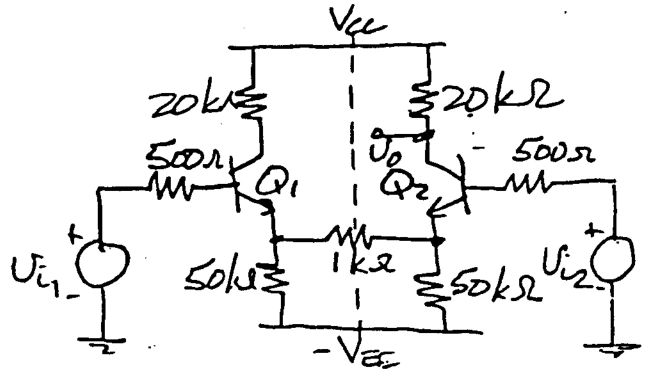
$$A_V = \left(\frac{106.6k // 17.8k}{106.6k // 17.8k + 2.2k}\right)\left(-(1.89 m\mho)(5k\Omega)\right)(0.971)$$

$$\boxed{A_V = -8.00}$$ -Answer- $\boxed{\text{HSPICE} \Rightarrow A_V = -7.96 \text{ at } 1 kHz}$

6) Compute $V_o(t)$ for the circuit below. Use $\frac{1}{2}$ ckt analysis technique. Assume all xtrs are FAR with $I_C = 1mA$

$V_{i1} = 4mV \sin 10t$

$V_{i2} = 6mV \sin 10t$



$V_{id} = V_{i1} - V_{i2}$

$= \underline{-2mV \sin 10t}$

$V_{ic} = \dfrac{V_{i1} + V_{i2}}{2}$

$= \underline{5mV \sin 10t}$

the dotted line in the ckt is the line of symmetry. Can use ½ ckt analysis.

$V_o(t) = -\dfrac{V_{od}}{2} + V_{oc}$

$= -\dfrac{A_{dm} V_{id}}{2} + A_{cm} V_{ic}$

$I_C = 1mA$

$g_{m1} = g_{m2} = \dfrac{I_C}{V_T} = \dfrac{1mA}{26mV}$

$= \underline{38.5m℧}$

$r_{\pi1} = r_{\pi2} = \dfrac{\beta}{g_m} = \dfrac{200}{38.5m℧}$

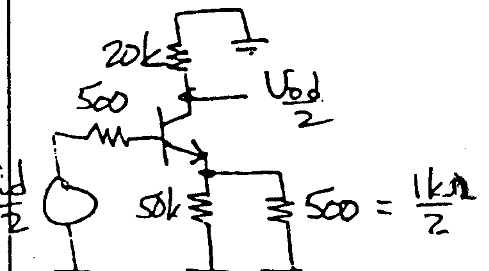
$= \underline{5.2k\Omega}$

$r_{o1} = r_{o2} = \dfrac{V_A}{I_C} = \dfrac{130V}{1mA}$

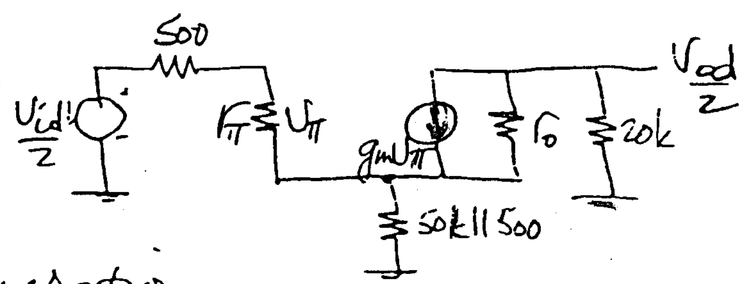
$= \underline{130k\Omega}$

DM $\frac{1}{2}$ ckt analysis

axis of symmetry is small signal virtual gnd.



$\Rightarrow$



this is CE w/ degeneration

$\Rightarrow A_{dm} = A(\text{CE w/deg}) = -\left(\dfrac{R_{id}}{R_{id}+R_s}\right) G_m R_{od} \| R_C$

$\approx -\left(\dfrac{R_{id}}{R_{id}+R_s}\right) G_m R_C$

6 contd)

$$\underline{R_{id}} = 2R_i(\text{CE w/ deg}) = r_\pi + (\beta+1)R_E \quad \leftarrow 50k \parallel 500$$
$$= (5.2k + (201)495)2$$
$$= 209\ \underline{k\Omega}$$

$$\underline{G_m} = \frac{g_m}{1+g_m R_E} = \frac{38.5 m\mho}{1+(38.5m\mho)(495\Omega)} = \underline{1.92 m\mho}$$

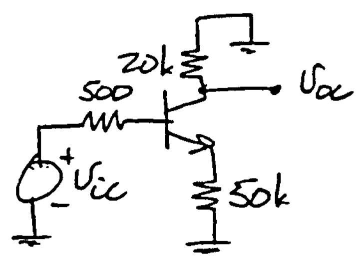
$$A_{dm} = -\left(\frac{209 k\Omega}{209 k\Omega + 500\Omega}\right)(1.92 m\mho)(20k\Omega)$$

$$A_{dm} = -38.4$$

<u>cm ½ ckt analysis</u>

line of symmetry is open ckt for ss analysis. (I = 0)

again this looks like CE w/ degeneration



$$A_{cm} = -\left(\frac{R_{ic}}{R_s + R_{ic}}\right) G_m R_C$$

$$\underline{R_{ic}} = R_i(\text{CE w/ degen}) = r_\pi + (\beta+1)R_E$$
$$= 5.2k + (201)(50k)$$
$$= \underline{10\ M\Omega} \leftarrow \frac{R_{ic}}{R_s + R_{ic}} \approx 1$$

$$\underline{G_m} = \frac{g_m}{1+g_m R_E} = \frac{38.5 m\mho}{1+(38.5 m\mho)(50k)}$$
$$= \underline{19.99 \times 10^{-6}\ \mho}$$

$$A_{cm} = -(19.99\times10^{-6})(20k) \Rightarrow A_{cm} = -0.4$$

$$v_o(t) = -\frac{(-38.4)(-2mV \sin 10t)}{2} + (-.4)(5mV \sin 10t)$$

$$\boxed{v_o(t) = -40.4\ mV \sin 10t\ V}$$