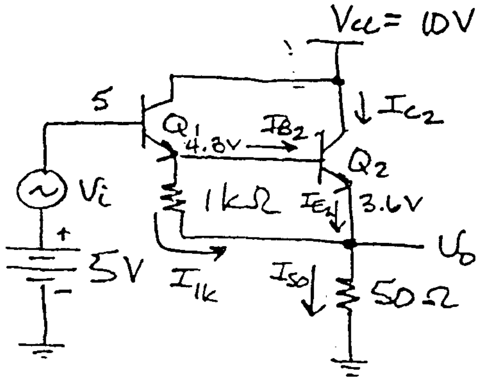
1) Determine the dc collector currents in Q_1 & Q_2 and then R_{in} and A_V for the Darlington emitter follower shown below. Neglect r_μ & r_o. assume $V_{BE(on)} = 0.7V$.



large signal analyses

$V_I = 5V$

$V_{E_1} = V_I - 0.7 = 4.3V$

$V_{E_2} = V_{E_1} - 0.7 = 3.6V$

$I_{50} = \frac{3.6V}{50\Omega} = 72mA$

$I_{1k} = \frac{(4.3-3.6)V}{1k\Omega} = 700\mu A$

$I_{E_2} = I_{50} - I_{1k} = 72mA - 700\mu A = 71.3mA$

$I_{B_2} = \frac{I_{E_2}}{\beta+1} = \frac{71.3mA}{201} = 354.7\mu A$

$\underline{I_{C_2}} = I_{E_2} - I_{B_2} = 71.3mA - 354.7\mu A = \underline{70.95mA}$

$I_{E_1} = I_{1k} + I_{B_2} = 700\mu A + 354.7\mu A = 1.05mA$

$I_{B_1} = \frac{I_{E_1}}{\beta+1} = \frac{1.05mA}{201} = 5.22\mu A$

$\underline{I_{C_1}} = I_{E_1} - I_{B_1} = 1.05mA - 5.22\mu A = \underline{1.04mA}$

you can see that $I_{C_2} \approx I_{E_2}$, $I_{C_1} \approx I_{E_1}$

$\Rightarrow$ could have assumed I_B's $= 0$ for I_C calculations.

however I_{B_2} was significant in calculating I_{E_1}!

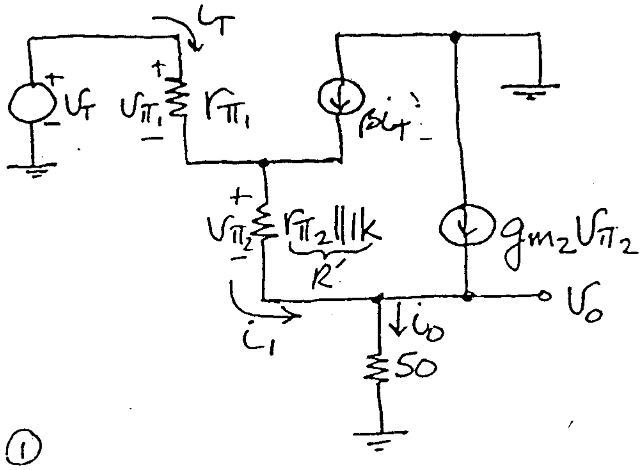
$\underline{g_{m_1}} = \frac{I_{C_1}}{V_T} = \frac{1.04mA}{26mV} = \underline{40m\mho}$ $\quad \underline{g_{m_2}} = \frac{70.95mA}{26m} = \underline{2.73\mho}$

$\underline{r_{\pi_1}} = \frac{\beta}{g_{m_1}} = \frac{200}{40m\mho} = \underline{5k\Omega}$ $\quad \underline{r_{\pi_2}} = \frac{200}{2.73\mho} = \underline{73.3\Omega}$

1 cont'd)

Small Signal analysis

$\boxed{\text{exact analysis}}$:



R_{in}: $v_T = v_{\pi_1} + v_{\pi_2} + v_o$ ①

$v_o = 50 i_o = 50(i_1 + g_{m2} v_{\pi_2})$

$v_{\pi_2} = i_1 R'$

$v_{\pi_1} = i_T r_{\pi_1}$

substitute these into eqn ①

$$v_T = \underbrace{i_T r_{\pi_1}}_{v_{\pi_1}} + \underbrace{i_1 R'}_{v_{\pi_2}} + \underbrace{50 i_1 + 50 g_{m2} i_1 R'}_{v_o}$$

need to get i_1 in terms of i_T

$i_1 = (\beta+1) i_T$

$$\Rightarrow v_T = i_T r_{\pi_1} + i_T(\beta+1)R' + i_T(\beta+1)50 + i_T(\beta+1)50 g_{m2} R'$$

$$= i_T\left[r_{\pi_1} + (\beta+1)R' + (\beta+1)50 + (\beta+1)50 g_{m2} R'\right]$$

$$\boxed{R_{in} = \frac{v_T}{i_T} = r_{\pi_1} + (\beta+1)R' + (\beta+1)50 + (\beta+1)50 g_{m2} R'}$$

from large signal analysis, plug in values

$R_{in} = 5k\Omega + (201)68.3 + (201)50 + (201)(50)(2.73)(68.3)$

$\boxed{R_{in} = 1.9 M\Omega}$

next page

1 cont'd)

A_v: looking at the same ckt:

$$v_T = v_{\pi_1} + v_{\pi_2} + v_o \quad ①$$

$$v_o = (i_1 + g_{m2} v_{\pi_2})\,50 = 50 i_1 + 50 g_{m2} v_{\pi_2}$$

$$i_1 = \frac{v_{\pi 2}}{R'}$$

$$\Rightarrow v_o = 50\frac{v_{\pi 2}}{R'} + 50 g_{m2} v_{\pi 2}$$

$$= 50 v_{\pi 2}\left[\frac{1}{R'} + g_{m2}\right]$$

Solving for v_{π_2}:

$$\Rightarrow v_{\pi 2} = \frac{v_o}{50\left(\frac{1}{R'} + g_{m2}\right)} \quad ②$$

plugging into ①:

$$\Rightarrow v_T = v_{\pi_1} + \frac{v_o}{50\left(\frac{1}{R'} + g_{m2}\right)} + v_o$$

need to get v_{π_1} in terms of v_o

$$i_T + \beta i_T = i_1 = \frac{v_{\pi 2}}{R'} \Rightarrow (\beta+1) i_T = \frac{v_{\pi 2}}{R'}$$

$$\Rightarrow (\beta+1)\frac{v_{\pi_1}}{r_{\pi_1}} = \frac{v_{\pi_2}}{R'}$$

$$v_{\pi_1} = \frac{v_{\pi 2} r_{\pi_1}}{R'(\beta+1)} = \frac{v_o r_{\pi_1}}{R'(\beta+1)50\left(\frac{1}{R'} + g_{m2}\right)}$$

sub eqn ②

plug into eqn ①:

$$v_T = v_o\left[\frac{r_{\pi_1}}{R'(\beta+1)50\left(\frac{1}{R'} + g_{m2}\right)} + \frac{1}{50\left(\frac{1}{R_1} + g_{m2}\right)} + 1\right]$$

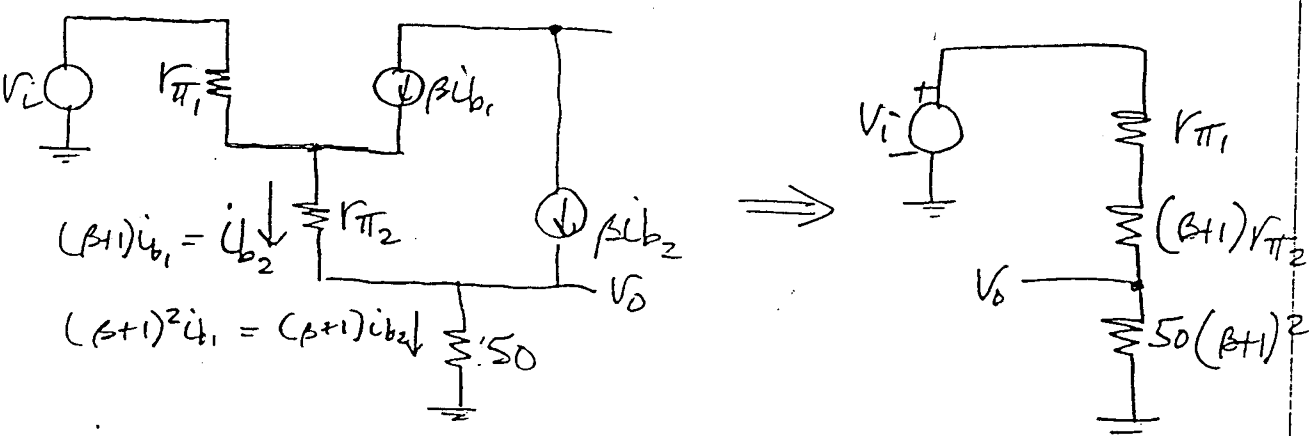
$$A_v = \frac{v_o}{v_T} = \frac{1}{\left[\frac{r_{\pi_1}}{R'(\beta+1)50\left(\frac{1}{R'} + g_{m2}\right)} + \frac{1}{50\left(\frac{1}{R'} + g_{m2}\right)} + 1\right]}$$

$$A_v = \frac{1}{\frac{5k}{(68.3)(201)(50)\left(\frac{1}{68} + 2.73\right)} + \frac{1}{50\left(\frac{1}{68} + 2.73\right)} + 1} \cong \boxed{0.990 = A_v}$$

contd)

approximate analysis:

if we assume that $R' = r_{\pi 2} \| 1k \approx r_{\pi 2}$
then the analysis is much easier.



without the $1k\Omega$ resistor, the current across $r_{\pi 2}$ is i_{b2} and is the only current between the emitter of Q1 and the emitter of Q2. So, Impedance reflection can be used, twice! (as shown above)

R_{in}:
$$R_{in} = r_{\pi_1} + (\beta+1) r_{\pi_2} + 50(\beta+1)^2$$
$$= 5k\Omega + (201)(73.3) + 50(201)^2$$
$$\Rightarrow \boxed{R_{in} = 2.04\ M\Omega}$$

A_v: this is just a voltage divider!

$$V_o = \frac{50(\beta+1)^2}{50(\beta+1)^2 + (\beta+1) r_{\pi_2} + r_{\pi_1}} V_i$$

$$A_v = \frac{50(\beta+1)^2}{50(\beta+1)^2 + (\beta+1) r_{\pi_2} + r_{\pi_1}} = \frac{50(\beta+1)^2}{R_{in}}$$

$$= \frac{50(201)^2}{2.04\ M\Omega} \Rightarrow \boxed{A_v = 0.990}$$

2. Assume All XSTRS identical

$$I_{C_1} = I_{C2} = I_S\left[e^{V_{BE}/V_T}\right]\left(1 + V_{CE}/V_A\right)$$

$$\Rightarrow I_S e^{V_{BE1}/V_T} = \frac{I_{C_1}}{\left(1 + \frac{V_{CE1}}{V_A}\right)}$$

$$\therefore I_{C_3} = I_{C_1}\frac{\left(1+\frac{V_o}{V_A}\right)}{\left(1+\frac{V_{CE_1}}{V_A}\right)} \qquad I_R = I_{C_1} + I_{C_2} = \frac{15 - .7V}{10K\Omega} = 1.4mA$$

$$\therefore I_{C_1} = 0.72mA$$

$$\therefore I_o = 3I_{C_3} = 3I_{C_1}\left(\frac{1 + V_o/V_A}{1 + V_{CE1}/V_A}\right) = 2.15mA\left(\frac{V_A + V_o}{V_A + .7V}\right)$$

$V_A = 130V$

V_o	I_o (mA)
1	2.15
5	2.22
30	2.63

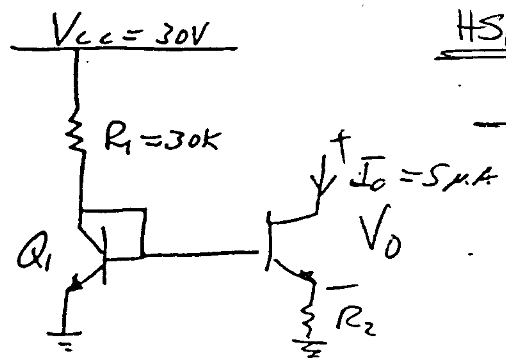
For $R_o \Rightarrow$

$$I_C = I_S e^{V_{be}/V_T}\left(1 + \frac{V_{ce}}{V_A}\right)$$

$$r_o = \left[\frac{\partial I_c}{\partial V_{ce}}\right]^{-1} = \left[I_S e^{V_{be}/V_T}\left(\frac{1}{V_A}\right)\right]^{-1} \qquad \frac{V_A}{I_{co}} = \frac{V_A}{I_C|_{V_{ce}=0}}$$

$$\therefore R_o = r_{o3} \parallel r_{o4} \parallel r_{o5} = \frac{r_{o3}}{3} = \frac{1}{3}\frac{V_A}{I_{co}} = \frac{1}{3}\frac{130V}{.72mA} \boxed{60.6\,K\Omega}$$

3)



HSPICE:

V_o	I_o	R_o
1 Volt	$4.9915\mu A$	$86.96 M\Omega$
5 Volts	$5.0354\mu A$	$95.244 M\Omega$
30 Volts	$5.2668\mu A$	$125 M\Omega$

$$I_R = \frac{V_{CC} - V_{BE}}{R_1} = \frac{30-.7}{30} = .977mA$$

$$V_{BE_1} = V_{BE_2} + I_o R_2 \qquad I_o R_2 = V_T \ln \frac{I_{C1}}{I_o} \simeq V_T \ln \frac{I_R}{I_o}$$

$$R_2 = \frac{V_T}{I_o} \ln \frac{I_R}{I_o} = \frac{26}{5} \ln \frac{.977mA}{5\mu}$$

$$= \underline{\underline{27.4K}}$$

R_o is that of a CEWO with $\frac{1}{g_{m1}} // 30k\Omega$ as R_S:

$$R_o \simeq r_{o2}\left(1 + g_{m2} R_2 \frac{r_{\pi 2}}{r_{\pi 2} + R_{S2} + R_2}\right)$$

$$r_{o2} = \frac{130V}{5\mu A} = 26M\Omega$$

$$r_{\pi 2} = \frac{(200)(26mV)}{5\mu A} = 1.04 M\Omega$$

$$g_{m2} = \frac{5\mu A}{26mV} = 0.192 m\mho$$

$$R_o \approx (26M\Omega)\left(1 + (.192m)(27.4k)\frac{1040k}{1040k + 27.4k}\right)$$

$$\boxed{R_o = 159.5M\Omega}$$

Using formula in book for Widlar Source yields a simiar result of $\underline{\underline{R_o = 159 M\Omega}}$.

4) $$I_{ref} = \frac{15-.7}{20K} = \underline{.72mA}$$

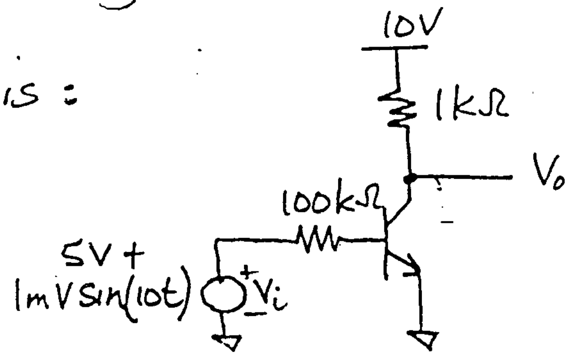
$$I_o R_2 = V_T \ln \frac{I_{C1}}{I_o} \cong V_T \ln \frac{I_{Ref}}{I_o}$$

$$I_o = 26 \times 10^{-3} \ln \frac{.72mA}{I_o}$$

↳ Iterate

$$\boxed{I_o \cong 10.9\mu A}$$

5) Find V_o of the following CE circuit

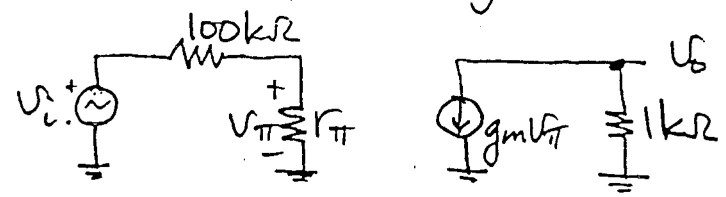


from load line analysis:

$I_B = 44\mu A$
$I_C = 4.6\,mA$
$V_{CE} = 5.25\,V$
see graphs

$$V_o = V_O + v_o = 5.25\,V + v_o$$

<u>small signal analysis:</u>



$$v_o = (-g_m v_\pi)(1k\Omega)$$

$$v_\pi = \left(\frac{r_\pi}{100k\Omega + r_\pi}\right) v_i$$

$$v_o = -g_m(1k\Omega)\left(\frac{r_\pi}{100k\Omega + r_\pi}\right) v_i$$

$$g_m = \frac{I_C}{V_T} = \frac{4.6mA}{26mV} = 176.9\ m℧$$

$$r_\pi = \frac{\beta}{g_m} = \frac{102.5}{176.9m℧} = 579.35\,\Omega$$

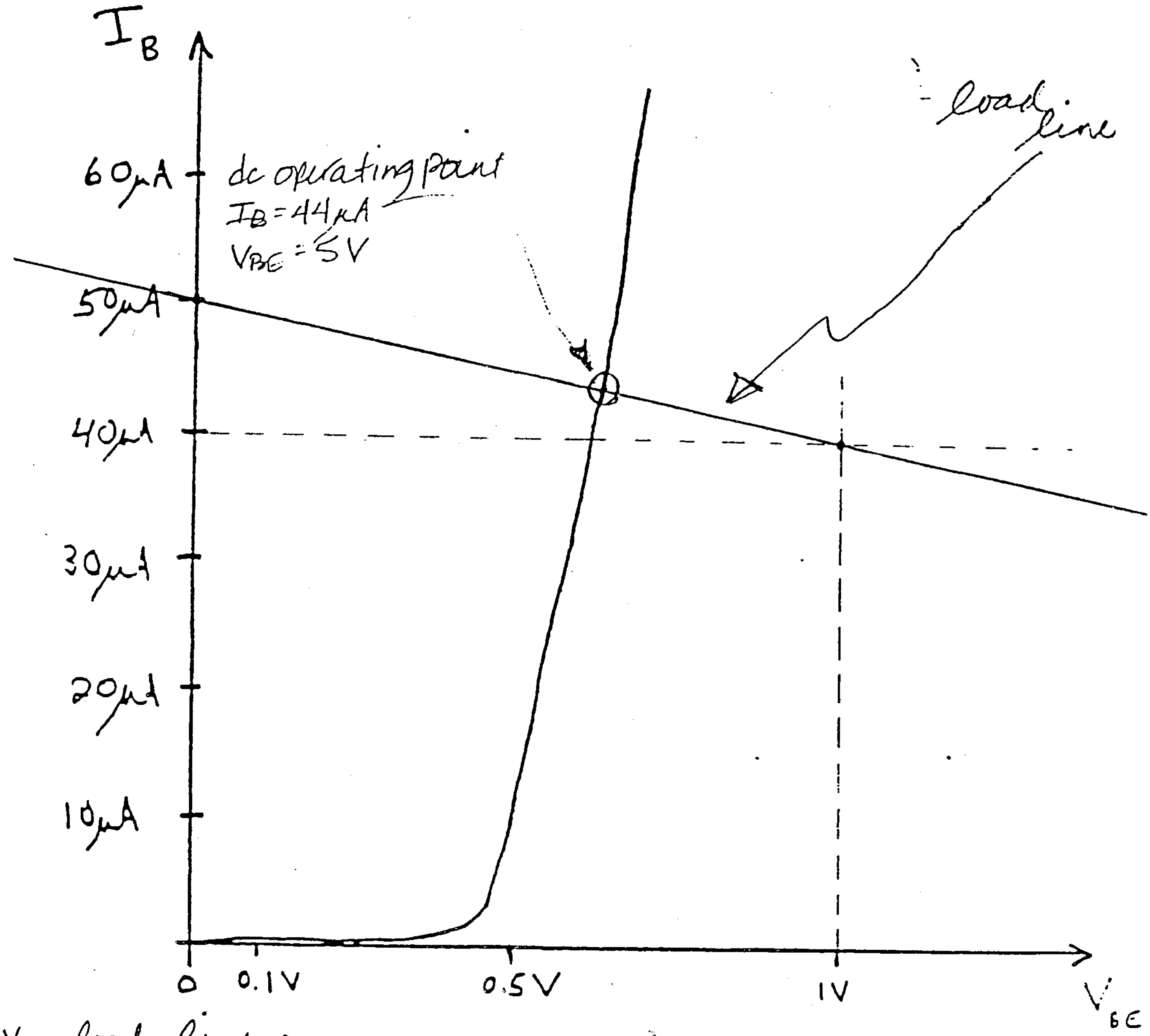
$$v_i = 1mV \sin 10t \leftarrow \text{given}$$

plugging in:

$$v_o = -(176.9\,m℧)(1k\Omega)\left(\frac{579.35\Omega}{100k\Omega + 579.35\Omega}\right)(1mV \sin 10t)$$

$$v_o = -1.02\,mV \sin(10t)$$

$$\boxed{V_o = 5.25V - 1.02mV \sin(10t)}$$



$-V_{BE}$ load line:

$- V_{100k} - V_{BE} = 0$

$-(100k\Omega) I_B - V_{BE} = 0$

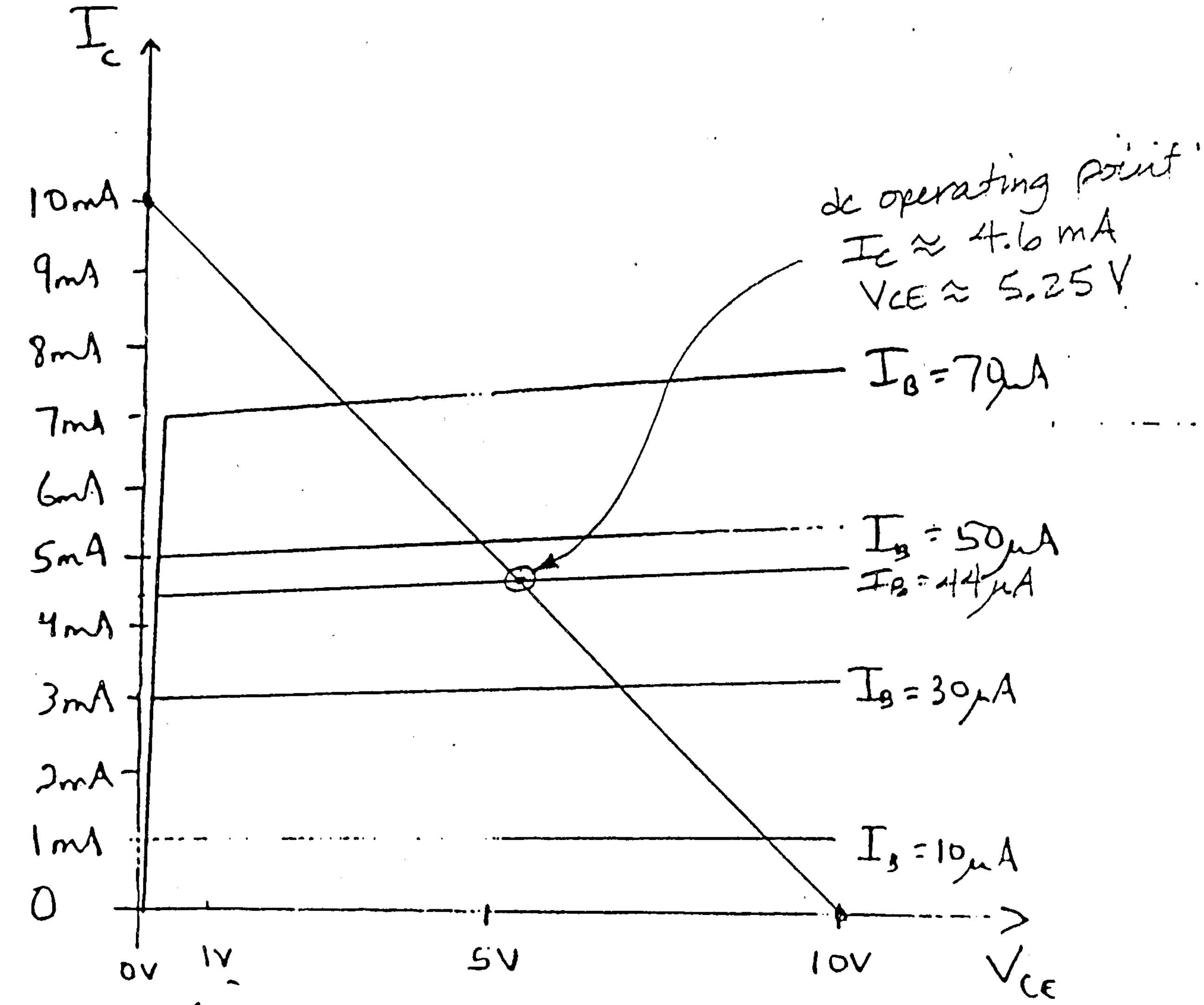
$= \underbrace{\dfrac{5V - V_{BE}}{100k\Omega} = 50\mu A - 10\mu V_{BE}}_{\text{load line}}$

Super Impose on graph above

$V_{open\ ckt} \Rightarrow I_B = 0\ ;\ V_{oc} = 5V$

$I_{short\ ckt} \Rightarrow V_{BE} = 0, I_{SC} = 50\mu A$

draw in $I_B = 44\mu A$ curve on next graph.



$I_c - V_{CE}$ load line :

$$I_c = \frac{V_{cc} - V_o}{R_c} = \frac{10V - V_o}{1k\Omega}$$

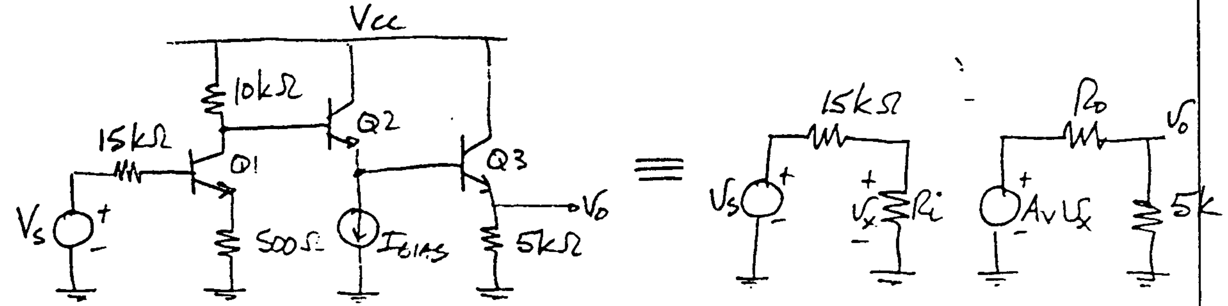
$$V_o = V_{CE} \rightarrow \underbrace{I_c = 10mA - 1m\,V_{CE}}_{\text{load line}}$$

superimpose on graph

$$\beta = \frac{\Delta I_c}{\Delta I_B} \cong \frac{2.05}{20\mu A}$$

$$\underline{\beta_F} \cong \underline{102.5}$$

6) For the circuit below, find the elements R_i, R_o, A_v in the model. Assume DC biasing sets $I_{C1} = I_{C2} = I_{C3}$ and all xtrs are in FAR.



<u>large signal analysis</u>

assume $I_{C1} = I_{C2} = I_{C3} = I_{BIAS} = 1mA$

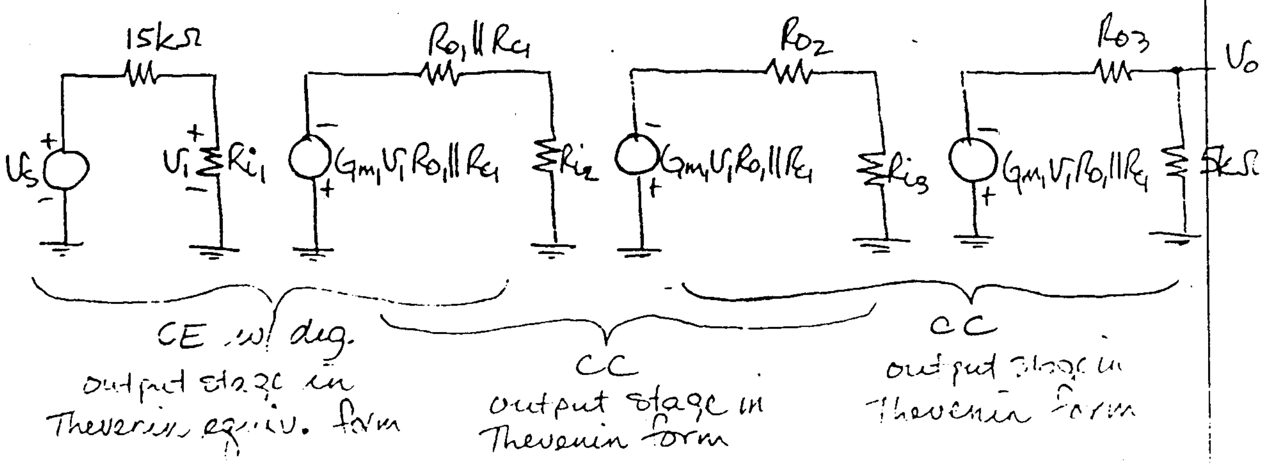
$\underline{g_{m1}} = \underline{g_{m2}} = \underline{g_{m3}} = \frac{I_C}{V_T} = \frac{1mA}{26mV} = \underline{38.5\ m℧}$

$\underline{r_{\pi 1}} = \underline{r_{\pi 2}} = \underline{r_{\pi 3}} = \frac{\beta}{g_m} = \frac{200}{38.5m℧} = \underline{5.2k\Omega}$

$\underline{r_{o1}} = \underline{r_{o2}} = \underline{r_{o3}} = \frac{V_A}{I_C} = \frac{130\ V}{1mA} = \underline{130k\Omega}$

<u>small signal analysis</u>

using the "single transistor" handout, represent the circuit as 3 single transistors in cascade:



6 cont'd)

from small signal model on previous page:

$$R_{in} = R_{i1} = R_{in}(CE \text{ w/ deg})$$
$$= r_{\pi_1}(1 + g_{m_1}R_E) \leftarrow \text{neglecting } r_b$$
$$= (5.2k)(1 + [38.5m\mho][500\Omega])$$

$$\Rightarrow \boxed{R_{in} = 105.3\,k\Omega}$$

$$R_{o3} = R_{out}(CC) = \frac{r_{\pi_3} + R_{s_3}}{\beta + 1}$$

$$R_{s3} = R_{o2} = R_{out}(CC) = \frac{r_{\pi_2} + R_{s2}}{\beta+1}$$

$$R_{o1} = R_{out}(CE \text{ w/ deg}) = r_o\left(1 + g_m R_E \frac{r_{\pi_1}}{r_{\pi_1} + R_{s_1} + R_E}\right)$$
$$= (130k\Omega)\left(1 + (38.5m)(500)\left(\frac{5.2k}{5.2k + 15k + 500}\right)\right)$$
$$= 758.65\,k\Omega \Rightarrow R_{s_2} = R_{o_1} \| R_{c_1} \approx 9.9\,k\Omega$$

$$\Rightarrow R_{s_3} = \frac{r_{\pi_2} + R_{s2}}{\beta+1} = \frac{5.2k + 9.9k}{201}\,\Omega$$
$$= 75\,\Omega$$

$$\Rightarrow R_{o3} = \frac{r_{\pi_3} + R_{s3}}{\beta+1} = \frac{5.2k + 75}{201}\,\Omega$$
$$= 26\,\Omega$$

$$\Rightarrow R_o = R_{o3} = \boxed{26\,\Omega}$$

6 cont'd)

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_1} \cdot \frac{v_1}{v_i}$$

$$v_o = -G_{m_1} v_1 \, R_{o1} \| R_{c1} \left(\frac{5k\Omega}{R_{o3} + 5k\Omega} \right)$$

$$\frac{v_o}{v_1} = -G_{m_1} R_{o1} \| R_{c1} \left(\frac{5k\Omega}{R_{o3} + 5k\Omega} \right)$$

$$G_{m_1} = G_m(\text{CE w/ deg.}) = \frac{g_{m1}}{1 + g_{m1} R_E}$$

$$= \frac{38.5 \, m\mho}{1 + (38.5 \, m\mho)(500\Omega)}$$

$$= 1.9 \, m\mho$$

$$\Rightarrow \left.\frac{v_o}{v_1}\right|_{R_L = \infty} = (-1.9 \, m\mho)(758.65k \| 10k)$$

$$= -18.59$$

$$v_1 = \frac{R_{i1}}{R_{i1} + 15k} v_i \Rightarrow \frac{v_1}{v_i} = \frac{R_{i1}}{R_{i1} + 15k}$$

$$\Rightarrow \frac{v_1}{v_i} = \frac{105.3k\Omega}{105.3k + 15k\Omega} = 0.875$$

$$\Rightarrow A_v = \frac{v_o}{v_1} \cdot \frac{v_1}{v_s} = (-18.59)(0.875)$$

$$\boxed{A_v = -16.3} \quad \left(A_{vs} = \left.\frac{v_o}{v_s}\right|_{R_L = \infty} \right)$$

final model