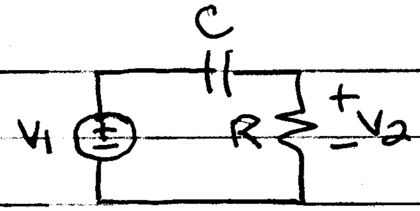
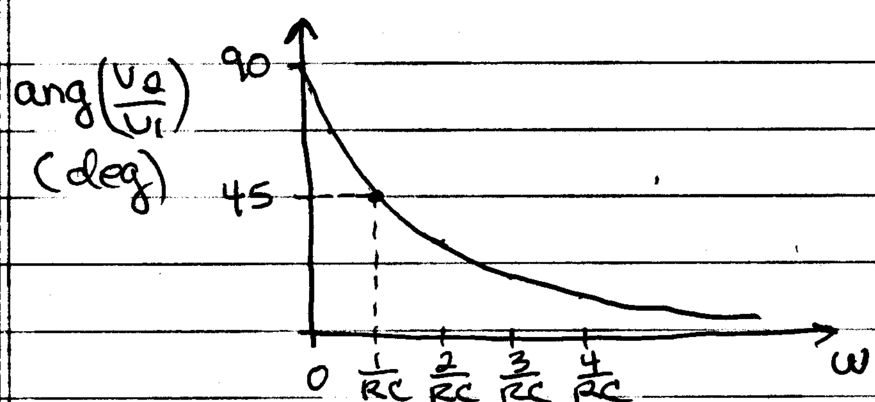
(5.1)



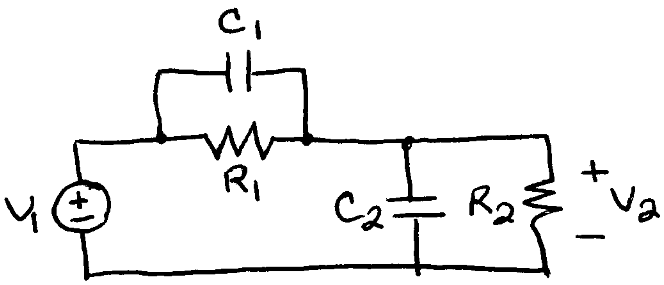
$$\frac{V_2}{V_1} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

$$\text{ang}\left(\frac{V_2}{V_1}\right) = \text{ang}(j\omega RC) - \text{ang}(1 + j\omega CR)$$
$$= 90^\circ - \arctan(\omega CR)$$

ω	$\text{ang}\left(\frac{V_2}{V_1}\right)$
0	90°
$\frac{1}{RC}$	$90^\circ - \arctan(1) = 90^\circ - 45^\circ = 45^\circ$
$\frac{2}{RC}$	$90^\circ - \arctan(2) = 90^\circ - 63^\circ = 27^\circ$
$\frac{3}{RC}$	$90^\circ - \arctan(3) = 90^\circ - 72^\circ = 18^\circ$
$\frac{4}{RC}$	$90^\circ - \arctan(4) = 90^\circ - 76^\circ = 14^\circ$
∞	0°



(5.5)



$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1+Z_2} \quad \text{where } Z_1 = \frac{R_1 \frac{1}{j\omega C_1}}{R_1+\frac{1}{j\omega C_1}} = \frac{R_1}{1+j\omega C_1 R_1}$$

$$Z_2 = \frac{R_2 \frac{1}{j\omega C_2}}{R_2+\frac{1}{j\omega C_2}} = \frac{R_2}{1+j\omega C_2 R_2}$$

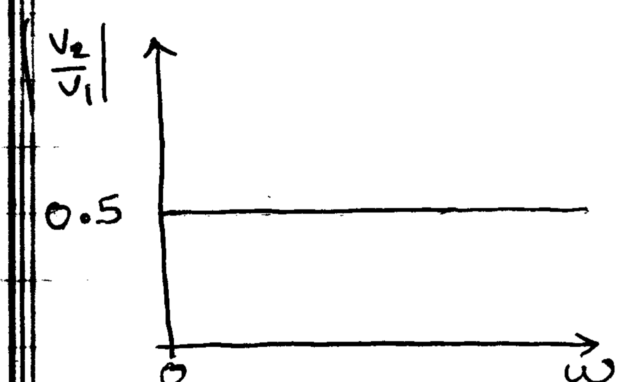
$$\frac{V_2}{V_1} = \frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{R_1}{1+j\omega C_1 R_1} + \frac{R_2}{1+j\omega C_2 R_2}}$$

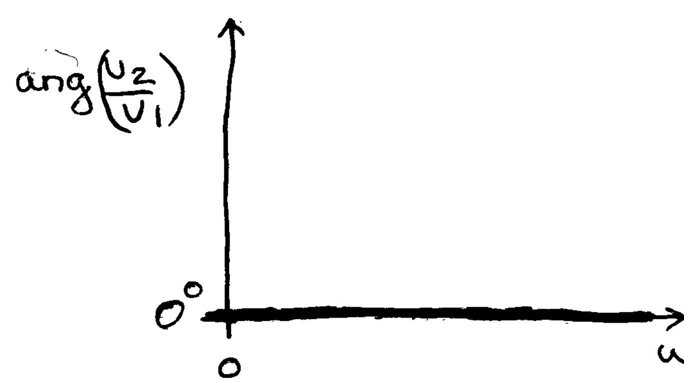
$$\frac{V_2}{V_1} = \frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{R_1(1+j\omega C_2 R_2)+R_2(1+j\omega C_1 R_1)}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_2)}} = \frac{R_2(1+j\omega C_1 R_1)}{R_1+R_2+j\omega R_1 R_2 (C_1+C_2)}$$

$$= \frac{R_2(1+j\omega C_1 R_1)}{(R_1+R_2)\left[1+j\omega \frac{R_1 R_2}{R_1+R_2}(C_1+C_2)\right]}$$

$R_1 = R_2 = R$
$C_1 = C_2 = C$

$$\frac{R}{2R}\frac{(1+j\omega CR)}{\left[1+j\omega \frac{R^2}{2R}(2C)\right]} = \frac{1}{2}\frac{(1+j\omega CR)}{(1+j\omega CR)} = \frac{1}{2}$$





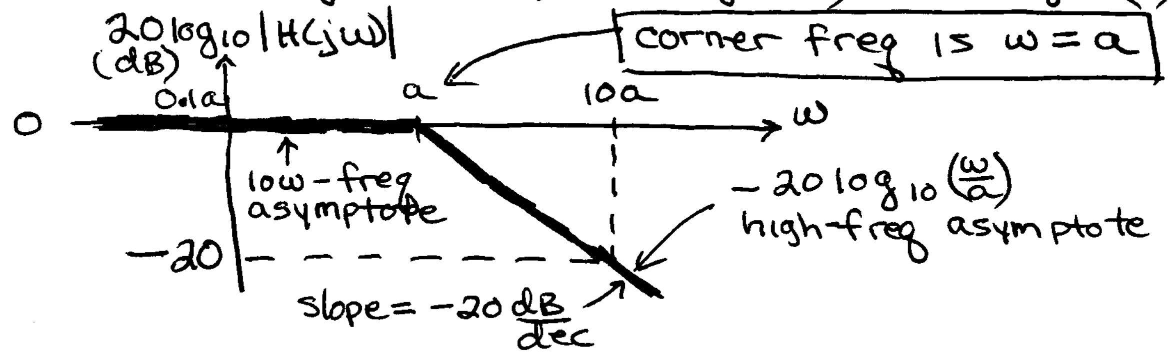
(5.11) $H(j\omega) = \dfrac{a}{a+j\omega} = \dfrac{1}{1+j\frac{\omega}{a}}$ where $a>0$

$|H(j\omega)| = \dfrac{1}{\sqrt{1+\left(\frac{\omega}{a}\right)^2}}$ $\quad$ $\text{ang}[H(j\omega)] = -\tan^{-1}\left(\frac{\omega}{a}\right)$

For small ω, $|H(j\omega)| \simeq 1 \rightarrow 20\log_{10}|H(j\omega)| \simeq 0\,\text{dB}$

For large ω, $|H(j\omega)| \simeq \dfrac{1}{\omega/a} = \dfrac{a}{\omega}$

$\rightarrow 20\log_{10}|H(j\omega)| = 20\log_{10}\left(\frac{a}{\omega}\right) = -20\log_{10}\left(\frac{\omega}{a}\right)$

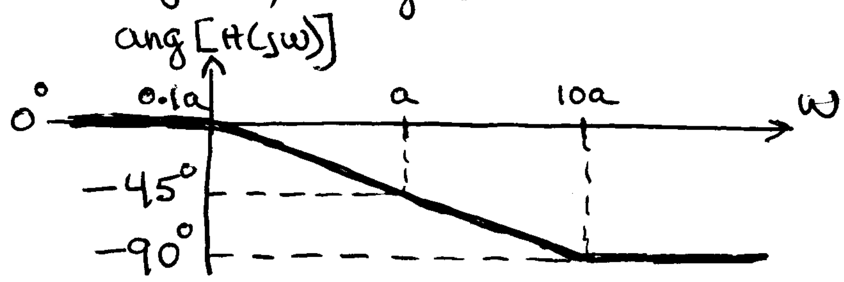


$\text{ang}[H(j\omega)] = -\tan^{-1}\left(\frac{\omega}{a}\right)$

For small ω, $\text{ang}[H(j\omega)] \simeq 0°$

For $\omega = a$, $\text{ang}[H(j\omega)] = -\tan^{-1}(1) = -45°$

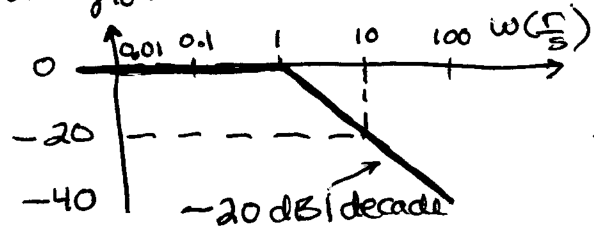
For large ω, $\text{ang}[H(j\omega)] \simeq -90°$

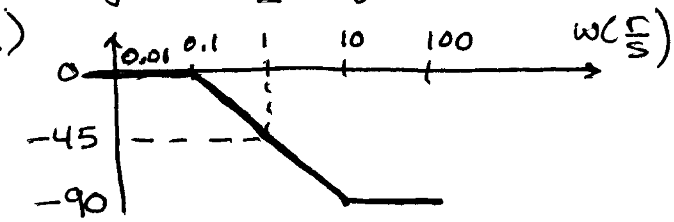


$\boxed{\text{This is a low-pass filter.}}$

(5.13) $H_1(j\omega) = \dfrac{1}{1+j\omega} \rightarrow |H_1(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2}}$ $\quad ang[H_1(j\omega)] = -\tan^{-1}(\omega)$

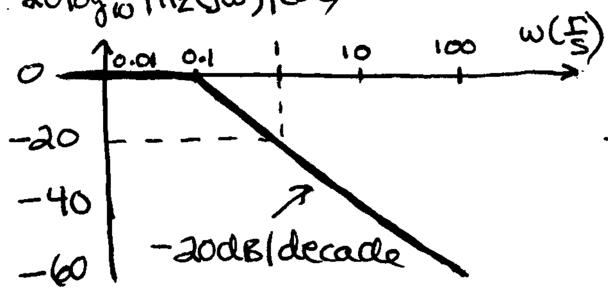
$20\log_{10}|H_1(j\omega)|$ (dB) $\qquad ang[H_1(j\omega)]$ (deg)

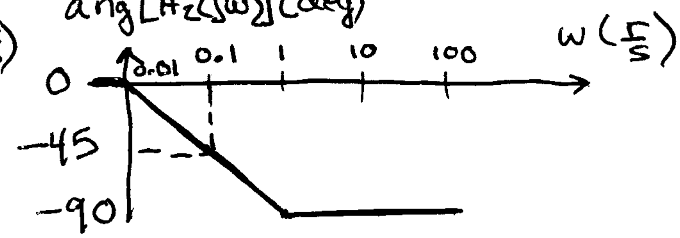




$H_2(j\omega) = \dfrac{1}{1+j\omega(10)} \rightarrow |H_2(j\omega)| = \dfrac{1}{\sqrt{1+100\omega^2}}$ $\quad ang[H_2(j\omega)] = -\tan^{-1}(10\omega)$

$20\log_{10}|H_2(j\omega)|$ (dB) $\qquad ang[H_2(j\omega)]$ (deg)





Adding the plots gives:

$20\log_{10}|H(j\omega)|$ (dB) $\qquad ang[H(j\omega)]$ (deg)