(10 pts) MT 1 Soln E100 F '15

1. a) A filter has a transfer function $H_1(j\omega) = \frac{-j2\omega}{(2+j\omega)}$. What is the phase of this transfer function [or the angle($\mathbf{H}_1$)] at ω = 2 rad/s ?

Phase or angle = ____________________

$$\text{angle } H_1 = \text{angle}(-j\,2\times2) - \text{angle}(2+j2)$$

$$= -90° - \tan^{-1}\frac{2}{2} = -90° - 45°$$

$$= -135°$$

b) At what frequency ω does the amplitude (or magnitude) of $\mathbf{H}_1$ equal –6 dB ?

ω = ____________________

$$-6\text{ dB} = 20\log|H_1| \Rightarrow |H_1| = \frac{1}{2}$$

$$\frac{1}{2} = \left|\frac{-j2\omega}{2+j\omega}\right| =$$

$$= \frac{2\omega}{\sqrt{2^2+\omega^2}}$$

square: $$\left(\frac{1}{2}\right)^2 = \frac{4\omega^2}{4+\omega^2}$$

$$\Rightarrow \frac{1}{4}(4+\omega^2) = 1 + \frac{\omega^2}{4} = 4\omega^2 \Rightarrow \omega = 0.52\,\tfrac{r}{s}$$

1 (con't):

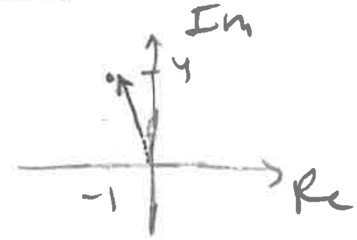
c) What type of filter is $H_1(j\omega) = \dfrac{-j2\omega}{(2+j\omega)}$? Check one box:

- ☐ a low-pass filter.
- ☒ a high-pass filter.
- ☐ a bandpass filter.
- ☐ none of the above.

(6 pts)

2) What are the magnitude and phase of the complex number $-1 + j4$?

a) magnitude or amplitude = ____________________ (***not*** in dB)



$$|-1+j4| = \sqrt{1^2+4^2} = \sqrt{17} = 4.1$$

b) Phase or angle = ____________________

$$\phi = \tan^{-1}\frac{4}{1} = 76^\circ$$

$$\theta = 180^\circ - 76^\circ = 104^\circ$$

3: For the signal v(t) = (0.8 V)cos(1000t) :

$v(t) \rightarrow 0.8\,V_p$

a) What is the peak-to-peak voltage of this waveform? ______________

$$2(0.8V) = 1.6V_{p-p}$$

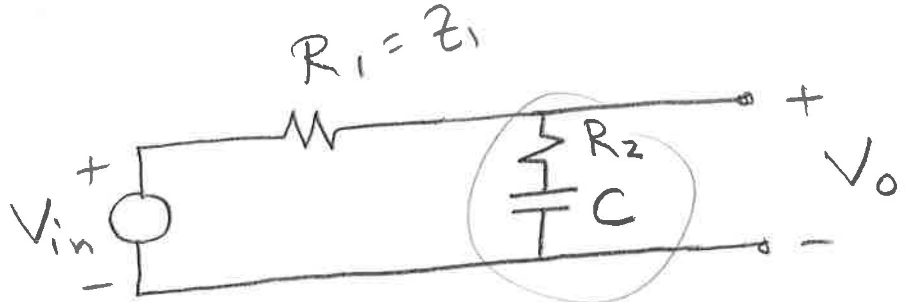
b) What is the rms voltage for this waveform? ______________

$$rms = \frac{0.8V}{\sqrt{2}} = 0.566V$$

6 pts

4. Find the transfer function $\mathbf{H}(j\omega) = \mathbf{V}_o(j\omega)/\mathbf{V}_{in}(j\omega)$ for the circuit below. (The answer should be in the form of a ratio of two complex expressions like 'a + jb', where a and/or b may be a function of ω.)

H(jω) = ______________________________



$$Z_2 = R_2 + \frac{1}{j\omega C}$$

$$\frac{V_o}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} \quad \begin{matrix} \cdot j\omega C \\ \cdot j\omega C \end{matrix}$$

$$= \frac{j\omega R_2 C + 1}{(R_1 + R_2) j\omega C + 1}$$

9 pts

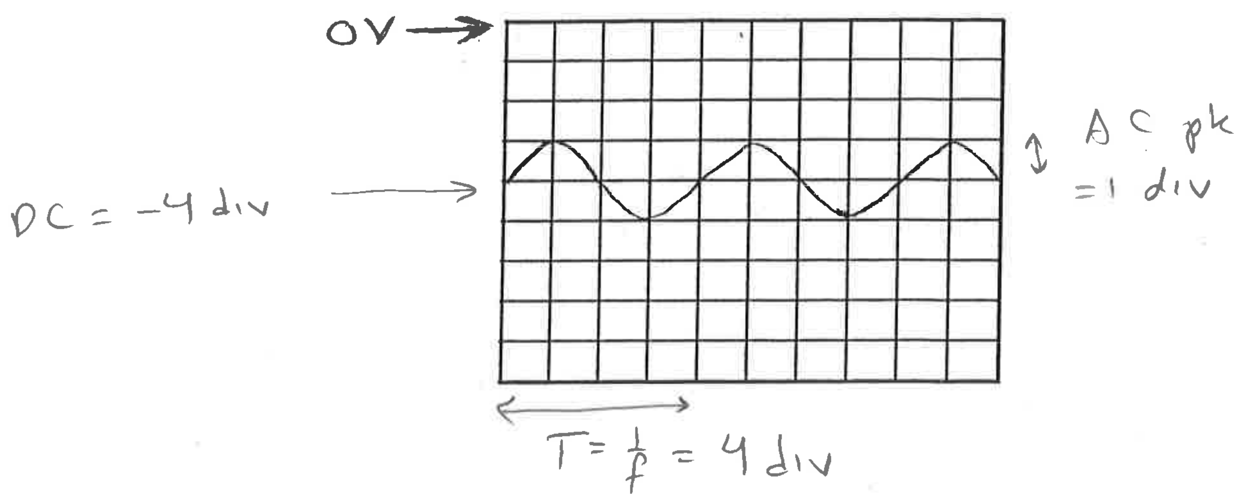
5. A function generator in our lab has generated a voltage signal v(t). That signal is shown below, as it appears on an oscilloscope in our lab. This voltage can be expressed in the form :

   $v(t) = A + B\sin(2\pi ft)$

   What are A, B and f ?

   NOTE: Scope settings are: 1 V / division on the vertical scale and 0.5 ms / division on the horizontal scale. Ground = 0 V is marked on the left.

   A = -4 V   B = 1 V   f = 500Hz



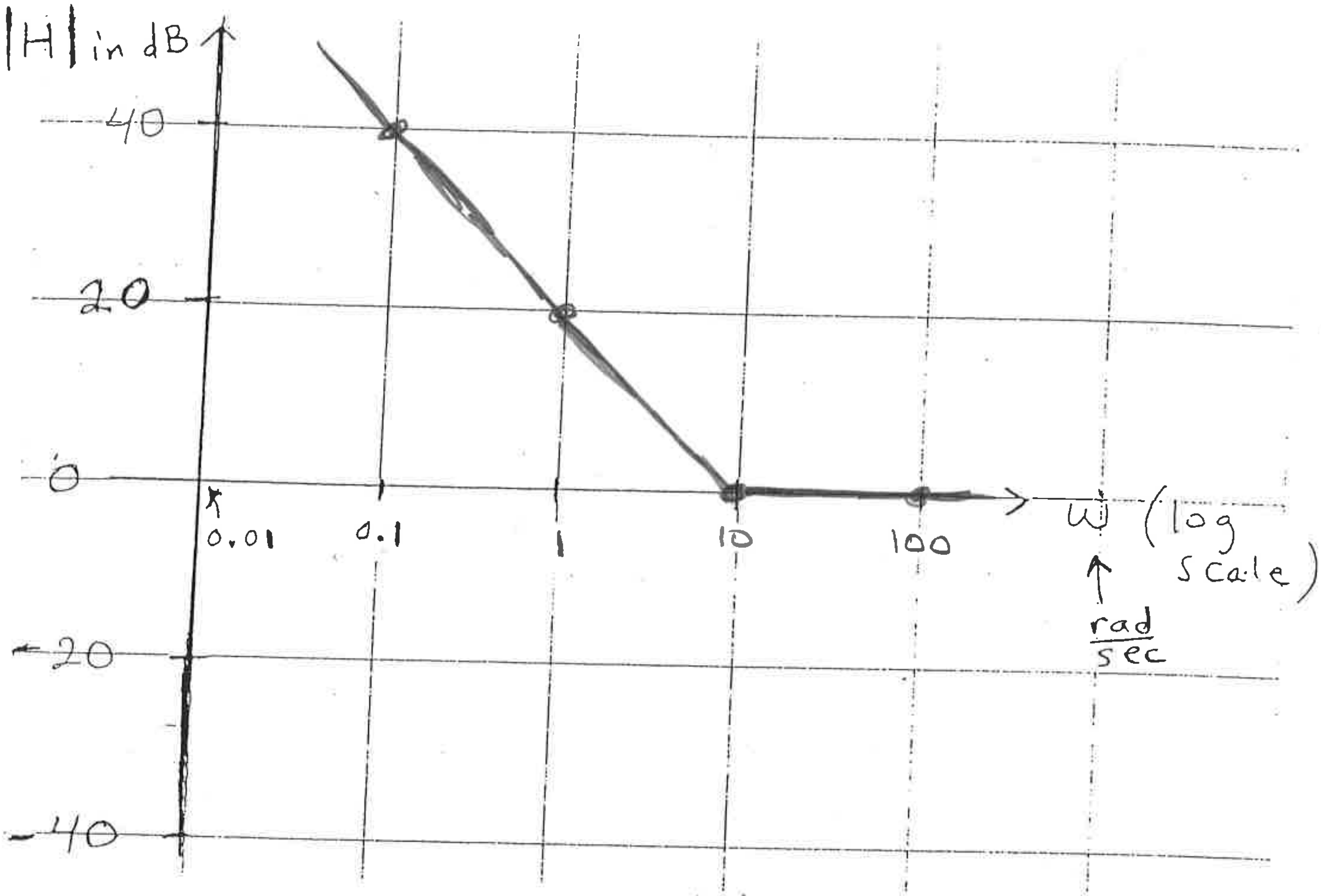
$T = \frac{1}{f} = 4\text{ div} \times 0.5\text{ ms/div} = 2\text{ ms} \Rightarrow f = \frac{1}{T} = 500\text{Hz}$

$A = DC = -4\text{ div} \times 1\text{V/div} = -4\text{V}$

$B = \text{pk AC} = 1\text{ div} \times 1\text{V/div} = 1\text{V}$

(9 pts)

6. a) Draw the approximate Bode magnitude (or amplitude) plot [|H| in dB vs. ω on a log scale] for the transfer function $H(s) = \frac{s+10}{s}$. (Set s = jω.) (A Bode plot using only straight-line segments is acceptable here.)



$$H(j\omega) = \frac{10\left(1+\frac{j\omega}{10}\right)}{j\omega}$$

$$\rightarrow 20\log|H| = \underset{①}{20\log 10} + \underset{②}{20\log\left|1+\frac{j\omega}{10}\right|} - \underset{③}{20\log\omega}$$

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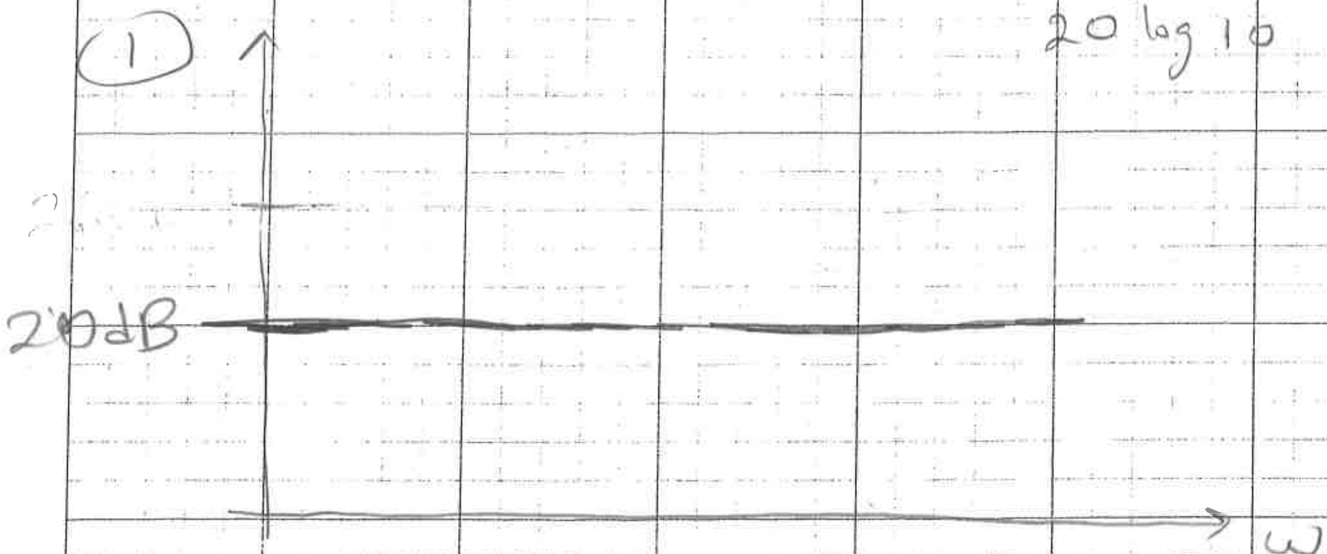
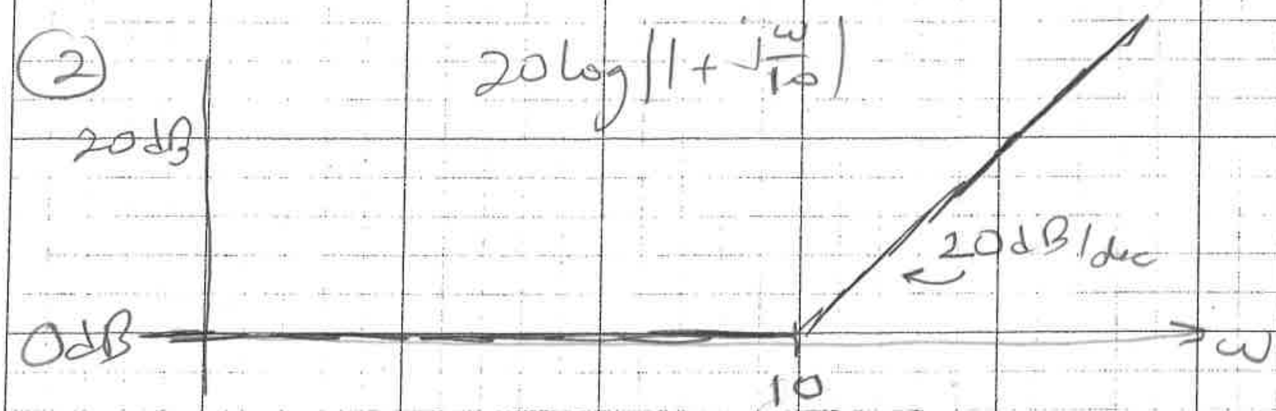
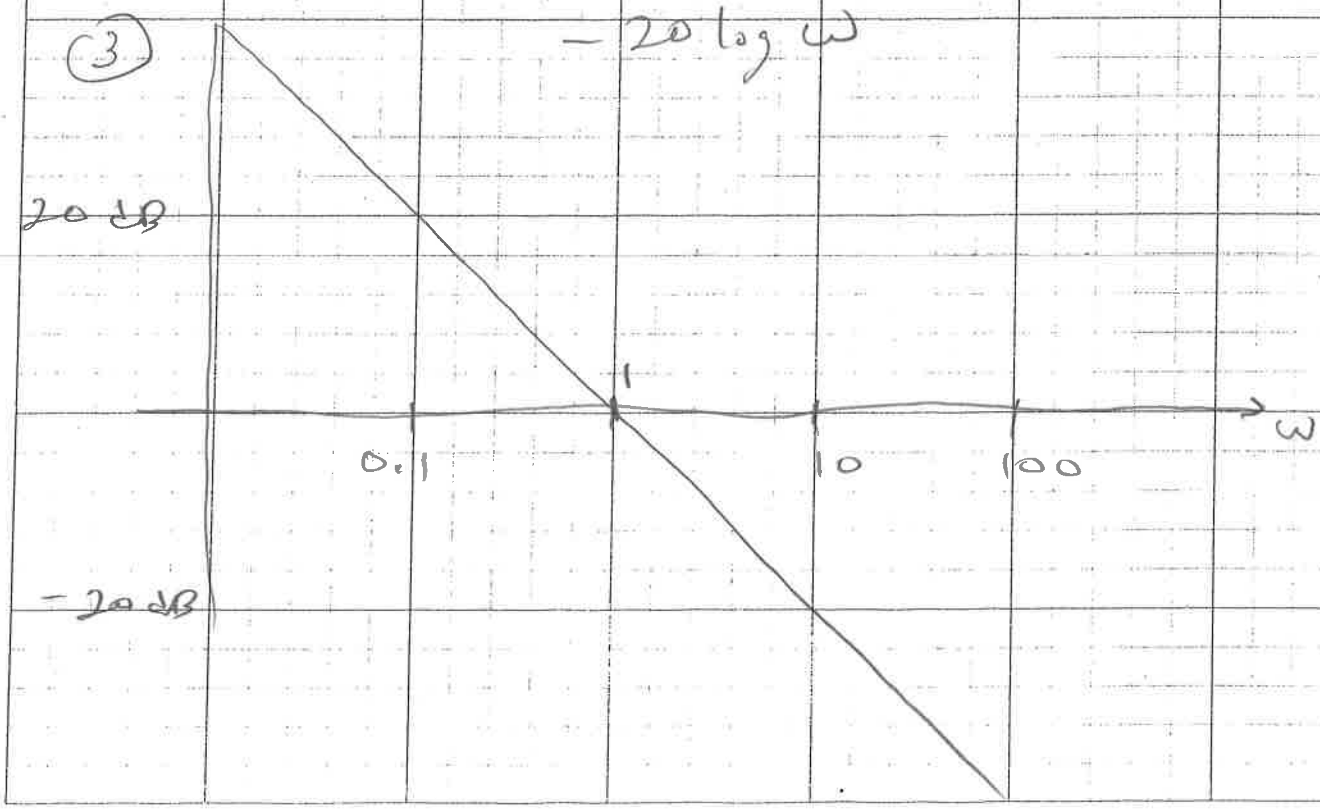


1
20 log 10
20dB
ω



2
20log|1+j ω/10|
20dB
20dB/dec
0dB
ω
10



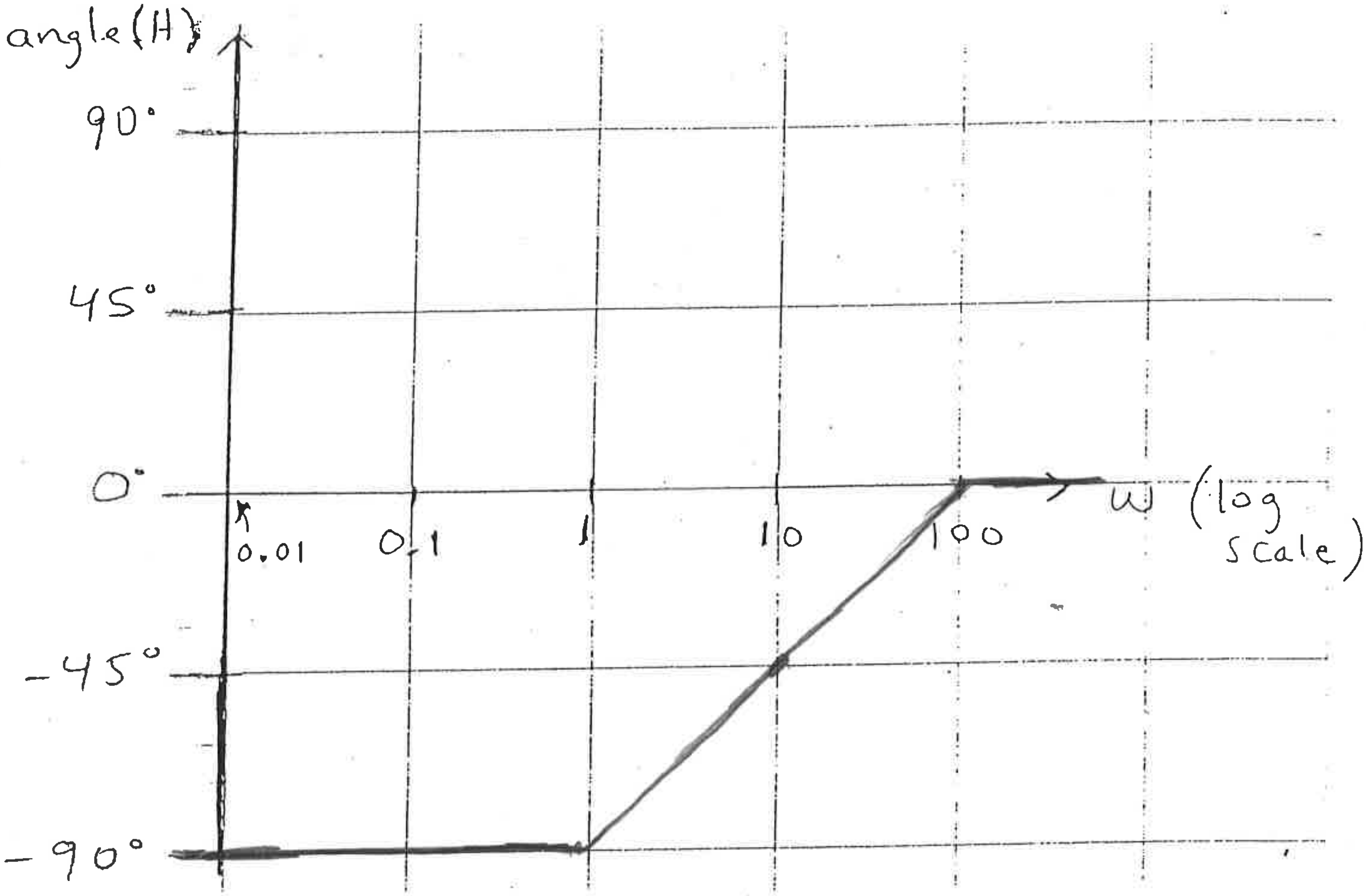
3
−20 log ω
20 dB
1
ω
0.1
10
100
−20 dB

9 pts

6 (continued)

$\frac{10(\frac{s}{10}+1)}{s}$

b) Draw the approximate Bode ~~phase~~ (or angle) plot [angle(H) vs. ω on a log scale] for the transfer function $H(s)=\frac{s+10}{s}$. (Set s = jω.) (A Bode plot using only straight-line segments is acceptable here.)



$$\text{angle } H = \text{angle } 10 + \text{angle}\left(1+j\frac{\omega}{10}\right) - \text{angle}(j\omega)$$

$$= 0° + \tan^{-1}\frac{\omega}{10} - 90°$$

(A) (B)

↑ ignore

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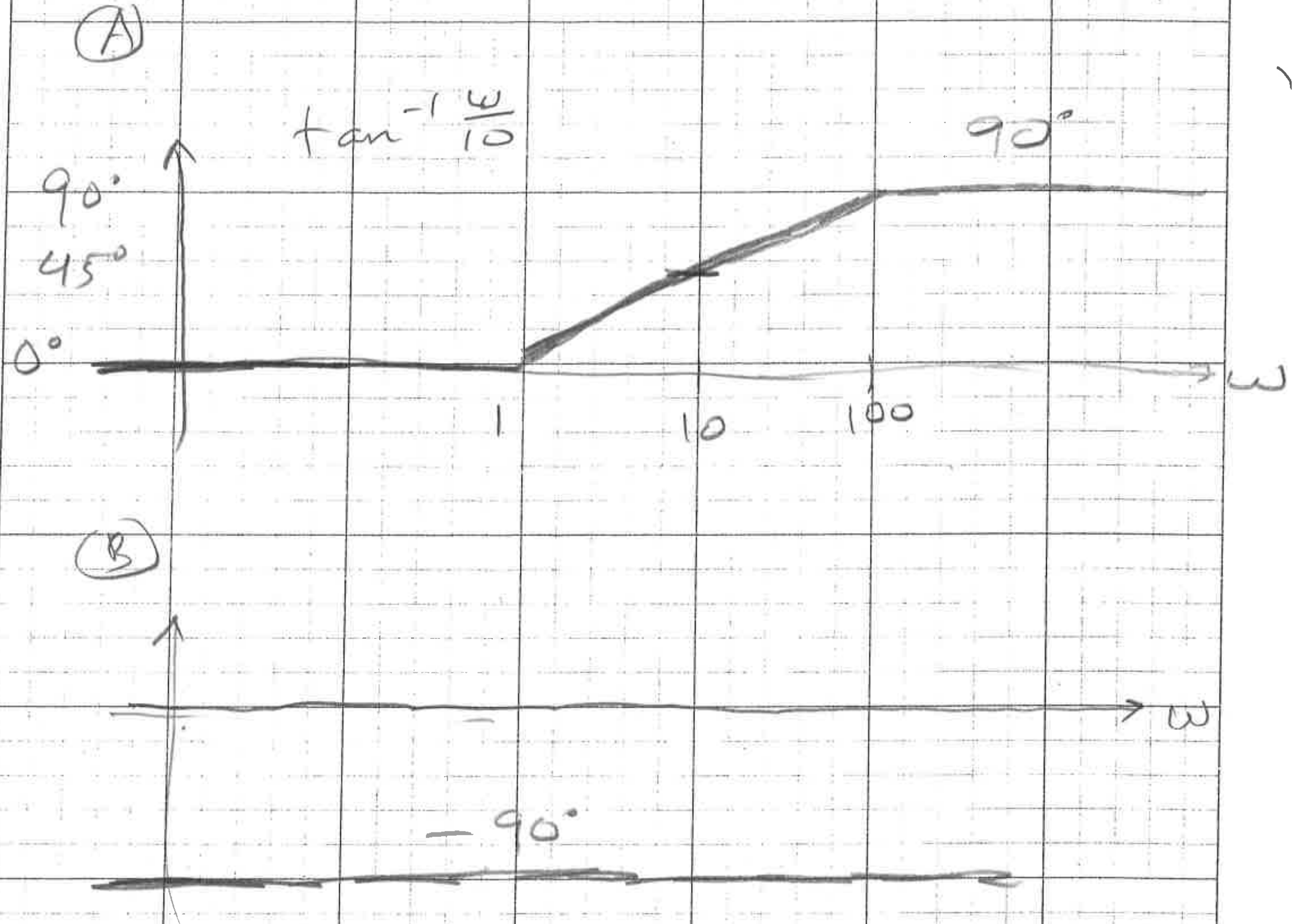
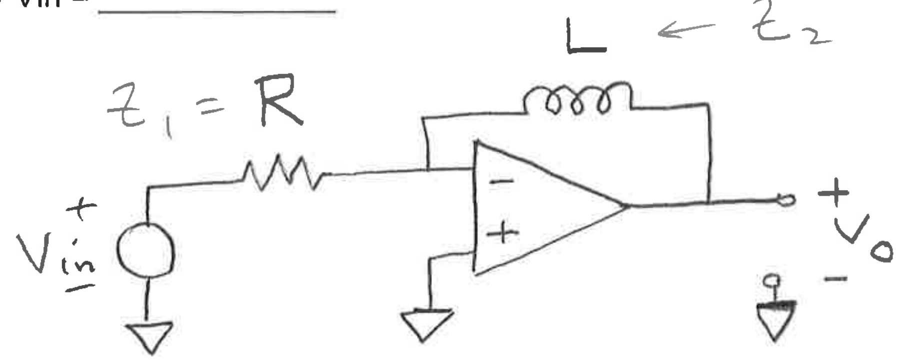


(8 pts)

7. In the circuits below, assume the op amp is ideal.

a) For the circuit below, what is Vo / Vin ? [Write an expression that is a function of L, R and ω.]

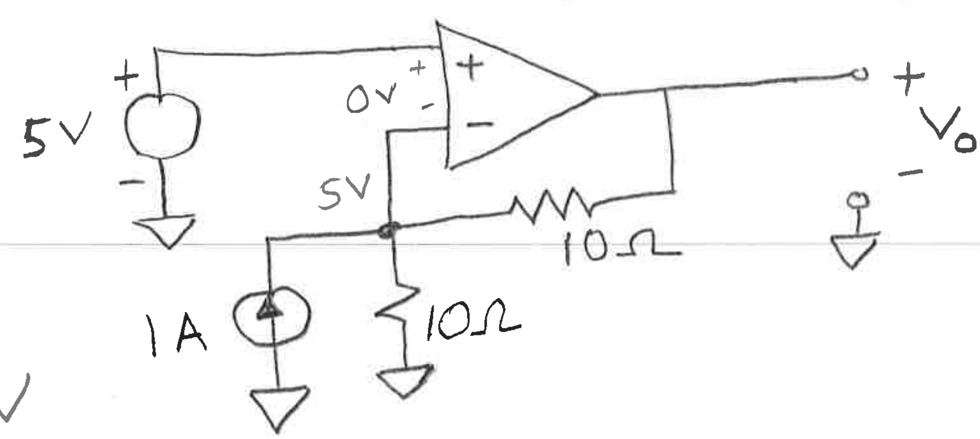
Vo / Vin = ______________



$$\frac{V_o}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{j\omega L}{R}$$

b) For the circuit below, what is the output voltage, Vo ? (The current source and voltage source are DC sources.)

Vo = ______________



$V_e = 0V$

$\Rightarrow V_+ = 5V$

KCL at V_- : $1A = \frac{5V}{10\Omega} + \frac{5V - V_o}{10\Omega}$

$\Rightarrow V_o = 0V$