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MIMO Integral-Action Anti-Windup Controller Design and Applications to Temperature Control in RTP Systems

A. N. Mete and A. N. Gündeş

Abstract— **An integral-action controller synthesis is presented, where the controller achieves closed-loop stability and zero steady-state error due to step-input references. Closed-loop stability is maintained even when the integrators in all of the controller channels are limited or completely switched off to protect against integrator windup. The proposed method is applied to temperature control in Rapid Thermal Processing systems, where critical requirements are imposed on tracking of temperature profiles.**

I. INTRODUCTION

We consider controller synthesis for linear, time-invariant (LTI) multi-input multi-output (MIMO) plants subject to input saturation. Our goal is to achieve closed-loop stability and asymptotic tracking of step-input references with zero steady-state error. To achieve robust tracking, the controller is designed to have integral-action.

The performance of integral-action controllers depends on the system operating in a linear range. They suffer serious loss of performance due to a phenomenon called integral windup, which occurs when the actuators in the control-loop saturate (see e.g. [6]). Limitations of actuators put an upper bound on the amplitude of the control signal. Actuators reach their saturation limit when a reference signal requiring a control effort beyond that bound is applied to the system. The integrators in the controller continue to integrate the error although the input is constrained, and once the input comes out of the saturation limit, the initial condition that the integrator has built up to causes a large transient response and serious performance degradation. Effects of integral windup can range from large overshoots in transient response to loss of stability. Therefore, integral-action controllers should be modified by taking actuator limitations into account. Most methods of dealing with the effect of integrator windup are based on the idea of turning off the integrators when the input reaches a limit and resetting the integrators' states. Numerous anti-windup modifications have been proposed, many of which can be considered as observer-based modifications (see e.g., [1]). A synthesis procedure for designing observer-based controller gains in anticipation of windup can be found in [10].

There are several ways to design LTI integral-action controllers (see e.g. [2], [12], [8], [9], [5] for decentralized and centralized integral-action controller synthesis). The simplest controller that achieves integral-action is in the proportional+integral+derivative (PID) form. Closed-loop stability can be achieved using PID-controllers only for certain classes of plants, while many others cannot be stabilized using PID-controllers. Standard observer-based integral-action controller designs apply Linear Quadratic Regulator (LQR) or pole-placement methods to an augmented plant model, which includes the n states of the plant and the n_y states of the integrators [13], [11], [5]. An augmented full-order observer-based design results in an $(n+n_y)$-th order controller. The integrators cannot be completely switched off since the gain matrix corresponding to the integrator states are part of the feedback loop that ensures stability. Instead of turning off the integral-action completely to protect against integrator windup, the integrators are limited so that the input does not reach the saturation limits.

In this paper, we propose an integral-action synthesis procedure based on adding integral-action onto the system, where an initially designed stabilizing controller is already present in the feedback loop. This is achieved in two stages: An initial stabilizing controller is designed for the original plant using any desired method (LQR, H_∞ , etc.) and it does not have integral-action. Then a PID-controller is designed for a stable system associated with the plant. These two blocks are configured so that the final controller achieves closed-loop stability and integral-action. Furthermore, all integral-action controllers can be obtained from this controller by inclusion of a free controller parameter. If a full-order observer-based controller is chosen for the initial stabilizing stage, then only the plant states are estimated and the state feedback gains are only associated with the plant states without augmenting. The transfer-function of the final controller once the PID block is added on is $(n+1)$. To protect against windup due to the integrators in the PID block containing the integral-action, this second block can be completely switched off without affecting closed-loop stability or it can be limited.

The proposed integral-action controller design is applied to temperature control in Rapid Thermal Processing (RTP) systems (see e.g. [14], [3] for modelling and control of RTP). The recent technology of RTP in integrated circuit manufacturing is a fast and efficient multi-chamber single-wafer technology that uses a much smaller chamber than a batch process. Single-wafer processing achieves more uniform film thickness for larger wafers. During an RTP process, it is crucial to maintain uniform temperature on the wafer surface at all times since small temperature variations can lead to large variations in reaction rates [3]. The benefits of RTP cannot be realized without meeting the stringent

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The authors are with Electrical and Computer Engineering, University of California, Davis, CA 95616. anmete@ece.ucdavis.edu & gundes@ece.ucdavis.edu

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temperature uniformity specifications. Fast tracking control laws that achieve near uniform spatial temperature distributions across a semiconductor wafer during both transient and quasi steady-state phases of the process need to be developed for RTP systems, which are inherently nonlinear dynamic processes with actuator saturation.

The paper is organized as follows: The problem description and definitions are in Section II. The main results are in Section III: Section III-A explains the proposed controller synthesis. Theorem 3.1 states that all integral-action controllers can be realized in two stages, where the integral-action is achieved using a separate PID block that can be switched off or limited without affecting closed-loop stability. One method of designing the PID stage is the systematic procedure in Proposition 3.1. Section III-B gives a review of integral-action design using an augmented plant model. In Section IV, the results are applied to a linearized MIMO RTP system model subject to lamp input saturation nonlinearity as described in [4]. The proposed systematic synthesis approach is compared with the standard observer-based controller design with augmented state feedback.

Although we discuss continuous-time systems here, all results also apply to discrete-time systems with appropriate modifications. The following notation is used: Let $\mathbb{C}$, $\mathbb{R}$ denote complex and real numbers. The extended closed right-half complex plane is $\mathcal{U} = \{s \in \mathbb{C} | \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I_n is the $n \times n$ identity matrix. The H_∞-norm of $M(s) \in \mathcal{M}(\mathbf{S})$ is $\|M\| := \sup_{s \in \partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$. We drop (s) in transfer matrices such as $G(s)$ whenever this causes no confusion. We use coprime factorizations over $\mathbf{S}$; i.e., for $G \in \mathbf{R_p}^{n_y \times n_u}$, $G = N_g D_g^{-1}$ denotes a right-coprime-factorization (RCF), where $N_g \in \mathbf{S}^{n_y \times n_u}$, $D_g \in \mathbf{S}^{n_u \times n_u}$, $\det D_g(\infty) \neq 0$; $G = \widetilde{D}_g^{-1} \widetilde{N}_g$ denotes a left-coprime-factorization (LCF), where $\widetilde{N}_g \in \mathbf{S}^{n_y \times n_u}$, $\widetilde{D}_g \in \mathbf{S}^{n_y \times n_y}$, $\det \widetilde{D}_g(\infty) \neq 0$.

II. Problem Description

Consider the LTI, MIMO unity-feedback system $Sys(G, \hat{C})$ in Fig. 1; $G \in \mathbf{R_p}^{n_y \times n_u}$ and $\hat{C} \in \mathbf{R_p}^{n_u \times n_y}$ are the plant's and the controller's transfer-functions, respectively. Assume that the feedback system is well-posed, the plant and controller have no unstable hidden-modes, and the plant $G \in \mathbf{R_p}^{n_y \times n_u}$ is full normal rank. Let $H_{er} = (I_{n_y} + G\hat{C})^{-1} = I_{n_y} - G\hat{C}(I_{n_y} + G\hat{C})^{-1} I_{n_y} - GH_{wr}$ denote the (input-error) transfer-function from r to e.

Definition 2.1: i) The system $Sys(G, \hat{C})$ is said to be stable iff the closed-loop transfer-function from (r, v) to (y, w) is stable. *ii)* The controller $\hat{C}$ is said to stabilize G iff $\hat{C}$ is proper and $Sys(G, \hat{C})$ is stable. *iii)* The stable system $Sys(G, \hat{C})$ is said to have integral-action iff H_{er} has blocking zeros at the origin. ■

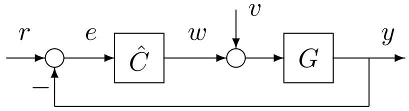
Fig. 1. Unity-Feedback System $Sys(G, \hat{C})$.

Suppose that $Sys(G, \hat{C})$ is stable and that step reference inputs r are applied to the system. The steady-state error due to all step inputs goes to zero if and only if $H_{er}(0) = 0$, i.e., the system has integral-action.

Let $G = N_g D_g^{-1} = \widetilde{D}_g^{-1} \widetilde{N}_g$, $\hat{C} = N_{cr} D_{cr}^{-1} = D_{c\ell}^{-1} N_{c\ell}$ be any RCF and LCF of the plant and the controller. Then $\hat{C}$ stabilizes G if and only if M_L in (1) equivalently, M_R, is unimodular [15], [7]:

$$\widetilde{D}_g D_{cr} + \widetilde{N}_g N_{cr} =: M_L \ , \ D_{c\ell} D_g + N_{c\ell} N_g =: M_R \quad (1)$$

Then H_{er} can be written as in (2), and equivalently, (3):

$$H_{er} = (I_{n_y} + G\hat{C})^{-1} = D_{cr} M_L^{-1} \widetilde{D}_g \ , \quad (2)$$

$$H_{er} = I_{n_y} - G\hat{C}(I_{n_y} + G\hat{C})^{-1} = I_{n_y} - N_g M_R^{-1} N_{c\ell} \, . \quad (3)$$

By Definition 2.1, $Sys(G, \hat{C})$ has integral-action iff $H_{er}(0) = (D_{cr} M_L^{-1} \widetilde{D}_g)(0) = 0$.

Definition 2.2: The controller $\hat{C} = N_{cr} D_{cr}^{-1}$ is said to be an integral-action controller iff $\hat{C}$ stabilizes G and the denominator matrix D_{cr} for any RCF of $\hat{C}$ has blocking zeros at the origin, i.e., $D_{cr}(0) = 0$. ■

By Definition 2.2 and (2), if $\hat{C} = N_{cr} D_{cr}^{-1}$ is an integral-action controller, then $Sys(G, \hat{C})$ has integral-action. Obviously, $D_{cr}(0) = 0$ is sufficient but not necessary for $H_{er}(0) = (D_{cr} M_L^{-1} \widetilde{D}_g)(0) = 0$. If G has poles at $s = 0$, rank$\widetilde{D}_g(0) < n_y$; hence, the system may achieve integral-action even if $D_{cr}(0) \neq 0$. If G has no poles at $s = 0$, then $Sys(G, \hat{C})$ has integral-action if and only if $\hat{C} = N_{cr} D_{cr}^{-1}$ is an integral-action controller, i.e., $D_{cr}(0) = 0$.

Lemma 2.1 gives the necessary conditions imposed on G due to the integral-action requirement:

Lemma 2.1: Let $G \in \mathbf{R_p}^{n_y \times n_u}$. If the system $Sys(G, \hat{C})$ has integral-action, then **i)** (normal) rank$G = n_y \leq n_u$; **ii)** G has no transmission zeros at the origin. ■

Proof: The stability of $Sys(G, \hat{C})$ implies $H_{er}(0) = I_{n_y} - GH_{wr}(0) = 0$, i.e., $GH_{wr}(0) = I_{n_y}$. Therefore, (normal) rank$(GH_{wr}) = n_y \leq \min\{\text{rank}G, \text{rank}H_{wr}\}$ implies $n_y \leq \text{rank}G \leq \min\{n_y, n_u\}$. By (3), $H_{er}(0) = 0$ implies $N_g(0) M_R^{-1}(0) N_{c\ell}(0) = I_{n_y}$; hence, rank$N_g(0) = $ rank$N_{c\ell}(0) = n_y$. ■

III. Main Results

The simplest integral-action controllers are in PID form. We consider a realizable form of proper PID-controllers, where K_p, K_i, $K_d \in \mathbb{R}^{n_u \times n_y}$ are called the proportional, the integral, and the derivative constant, respectively [5]:

$$C_{pid} = K_p + \frac{K_i}{s} + \frac{K_d \ s}{\tau_d s + 1} \ , \quad (4)$$

To implement the derivative term, a pole is typically added to the derivative term (with $\tau_d > 0$) so that C_{pid} in (4) is proper. The integral-action in C_{pid} is present when $K_i \neq 0$. The controller in (4) is in proportional+integral (PI) form

$C_{pi} = K_p + K_i/s$ when $K_d = 0$, integral+derivative (ID) form $C_{id} = K_i/s + K_d s/(\tau_d s+1)$ when $K_p = 0$, pure integral (I) form $C_i = K_i/s$ when $K_p = K_d = 0$.

Although PID-controllers are simple and low order, some (unstable) plants G are not stabilizable with any C_{pid}. Since the plants considered here are not restricted to be stable, existence of stabilizing PID-controllers is not guaranteed. Proposition 3.1 shows that stable systems can be stabilized using PID-controllers, with $K_i \neq 0$, if and only if G has no transmission zeros at the origin, and proposes a method of selecting the constants K_p, K_i, K_d.

Proposition 3.1: Let $N_g \in \mathbf{S}^{n_y \times n_u}$, (normal) rank$N_g = n_y \leq n_u$. *i)* There exist stabilizing PID-controllers with nonzero integral constant $K_i \in \mathbb{R}^{n_u \times n_y}$ if and only if rank$N_g(0) = n_y$. *ii)* Suppose rank$N_g(0) = n_y$. Let $N_g(0)^I \in \mathbb{R}^{n_u \times n_y}$ be any right-inverse of $N_g(0)$. Choose any $\hat{K}_p, \hat{K}_d \in \mathbb{R}^{n_u \times n_y}$, $\tau_d > 0$. With $\hat{C}_{pd}$ defined as in (5) below, let $K_p = \rho \hat{K}_p$, $K_d = \rho \hat{K}_d$, $K_i = \rho N_g(0)^I$, where $\rho \in \mathbb{R}$ is any positive constant satisfying (6):

$$\hat{C}_{pd} := \hat{K}_p + \frac{\hat{K}_d \; s}{\tau_d s + 1} \; ; \tag{5}$$

$$0 < \rho < \| \; N_g(s)\hat{C}_{pd} + \frac{N_g(s)N_g(0)^I - I}{s} \; \|^{-1}. \tag{6}$$

Then N_g is stabilized by the PID-controller in (7):

$$C_{pid} = \rho \hat{K}_p + \frac{\rho N_g(0)^I}{s} + \frac{\rho \hat{K}_d s}{\tau_d s + 1} \; . \quad \blacksquare \tag{7}$$

In (7), $\hat{K}_d = 0$ gives a PI-controller; $\hat{K}_p = 0$ gives an ID-controller; $\hat{K}_d = \hat{K}_p = 0$ gives a pure I-controller.

Proof: For any positive $a \in \mathbb{R}$, define $Z \in \mathcal{M}(\mathbf{S})$ as

$$Z := C_{pid}\frac{s}{s+a} = (\; K_p + \frac{K_d s}{\tau_d s + 1} \;)\frac{s}{s+a} + \frac{K_i}{s+a}. \tag{8}$$

Then $C_{pid} = Z(\frac{s}{s+a}I_{n_y})^{-1}$ is an RCF of C_{pid}. Since C_{pid} stabilizes N_g, the matrix M defined by (9) is unimodular:

$$M := \frac{s}{s+a}I_{n_y} + N_g Z = \left(I_{n_y} + N_g C_{pid}\right)\frac{s}{s+a}I_{n_y} \tag{9}$$

By (9), $a \neq 0$, rank$M(0) = $ rank$(a^{-1}N_g(0)K_i) = n_y \leq min\{\text{rank}N_g(0), \text{rank}K_i\} \leq min\{n_y, n_u\} = n_y$ implies rank$N_g(0) = n_y$. Then there exist a right inverse $N_g(0)^I \in \mathbb{R}^{n_u \times n_y}$, i.e., $N_g(0)N_g(0)^I = I_{n_y}$. Define $\hat{M}$ as

$$\hat{M} = \frac{s+a}{s+\rho}M = \frac{s}{s+\rho}I + N_g C_{pid}\frac{s}{s+\rho} \; . \tag{10}$$

In (10), add and subtract $\frac{\rho}{s+\rho}I_{n_y}$ to obtain

$$\hat{M} = I_{n_y} + \frac{\rho s}{s+\rho}(\; N_g \; \hat{C}_{pd} + \frac{N_g(s)N_g(0)^I - I_{n_y}}{s} \;) \; . \tag{11}$$

Since $\|\frac{\rho s}{s+\rho}\| = \rho$, for any $\rho > 0$ satisfying (6), we have

$$\| \; \frac{\rho s}{s+\rho}(\; N_g \; \hat{C}_{pd} + \frac{N_g(s)N_g(0)^I - I_{n_y}}{s} \;) \; \| < 1 \; . \tag{12}$$

By (12), $\hat{M}$ is unimodular; equivalently, M is unimodular since $a, \rho > 0$; therefore C_{pid} stabilizes N_g. $\blacksquare$

A. Two stage integral-action design

Standard integral-action designs are generally based on augmenting the plant to include the integrator's states in state-feedback. This approach is briefly reviewed in Section III-B. In this section, we propose an approach that does not involve augmenting the plant's states. Theorem 3.1 states that any integral-action controller for $G = N_g D_g^{-1}$ can be expressed as the sum of two blocks: The first is any stabilizing controller $C_g = \widetilde{Y}^{-1}\widetilde{X}$, designed using any method, and does not have integral-action. The second is $\widetilde{Y}^{-1}C_{pid}$, where C_{pid} is any PID-controller that stabilizes N_g; it can be designed using any method including the procedure in Proposition 3.1. This second block provides integral-action and even if it is switched-off, the closed-loop remains stable due to C_g still remaining in the loop.

Theorem 3.1: Let the plant be $G \in \mathbf{R_p}^{n_y \times n_u}$, (normal) rank$G = n_y \leq n_u$; let G have no transmission zeros at $s = 0$. Let $G = \widetilde{D}_g^{-1}\widetilde{N}_g$ be any LCF, $G = N_g D_g^{-1}$ be any RCF. Choose any controller $C_g^o \in \mathbf{R_p}^{n_u \times n_y}$ that stabilizes G. There exists an LCF $C_g^o = \widetilde{D}_c^{-1}\widetilde{N}_c$ of C_g^o that satisfies

$$\widetilde{D}_c D_g + \widetilde{N}_c N_g = I_{n_u}. \tag{13}$$

Let C_{pid} be any PID-controller stabilizing N_g with $K_i \neq 0$. Then $\hat{C}$ is an integral-action controller for G if and only if

$$\hat{C} \; = \; (\widetilde{D}_c - Q\widetilde{N}_g)^{-1} \; (\; \widetilde{N}_c + Q\widetilde{D}_g \; + C_{pid} \;) \; , \tag{14}$$

where $Q \in \mathbf{S}^{n_u \times n_y}$ satisfies $\det(\widetilde{D}_c - Q\widetilde{N}_g)(\infty) \neq 0$. $\blacksquare$

We prove that any $\hat{C}$ in (14) is an integral-action controller for G; a detailed proof that all integral-action controllers are in the form given by (14) can be found in [8]:

Proof: Define $Z \in \mathcal{M}(\mathbf{S})$ as in (8). Since C_{pid} stabilizes N_g, the matrix M in (9) is unimodular. Let $C_g^o = N_c D_c^{-1}$ be an RCF of C_g^o that satisfies

$$\widetilde{D}_g D_c + \widetilde{N}_g N_c = I_{n_y}. \tag{15}$$

Then all stabilizing controllers C_g for G can be expressed as $C_g = \widetilde{Y}^{-1}\widetilde{X} = XY^{-1}$, where

$$\widetilde{Y} := (\widetilde{D}_c - Q\widetilde{N}_g), \; \widetilde{X} := (\widetilde{N}_c + Q\widetilde{D}_g),$$
$$Y := (D_c - N_g Q), \; X = (N_c + D_g Q) \; , \tag{16}$$

and $Q \in \mathbf{S}^{n_u \times n_y}$ satisfies $\det(\widetilde{D}_c - Q\widetilde{N}_g)(\infty) \neq 0$ [15], [7]. Using $\widetilde{Y}^{-1}\widetilde{X} = XY^{-1}$ and (9), $\hat{C}$ in (14) can be written as: $\hat{C} = \widetilde{Y}^{-1}(\widetilde{X} + C_{pid}) = \widetilde{Y}^{-1}\widetilde{X} + \widetilde{Y}^{-1}(\widetilde{Y}D_g + \widetilde{X}N_g)C_{pid} = \widetilde{Y}^{-1}\widetilde{X}(I_{n_y} + N_g C_{pid}) + D_g C_{pid} = XY^{-1}\left(I_{n_y} + N_g C_{pid}\right) + D_g C_{pid} = (X + D_g C_{pid}\frac{s}{s+a}M^{-1}Y) \; Y^{-1}M(\frac{s}{s+a}I_{n_y})^{-1}$. Therefore, an RCF $\hat{C} = N_{cr}D_{cr}^{-1}$ for the controller $\hat{C}$ is given by

$$N_{cr}D_{cr}^{-1} = (X + D_g Z M^{-1}Y)(\frac{s}{s+a}M^{-1}Y)^{-1}, \tag{17}$$

where $N_{cr}, D_{cr} \in \mathcal{M}(\mathbf{S})$ and D_{cr} is biproper. It follows by (9), (15) and $\widetilde{N}_g D_g = \widetilde{D}_g N_g$ that $M_L = \widetilde{D}_g D_{cr} + \widetilde{N}_g N_{cr} = \widetilde{D}_g(\frac{s}{s+a}M^{-1}Y) + \widetilde{N}_g(X + D_g Z M^{-1}Y) =$

$\widetilde{D}_g(\frac{s}{s+a}M^{-1}Y+N_gZM^{-1}Y)+\widetilde{N}_gX=\widetilde{D}_gMM^{-1}Y+\widetilde{N}_gX=\widetilde{D}_gY+\widetilde{N}_gX=I_{n_y}$ is unimodular. The system $Sys(G,\hat{C})$ is stable since M_L in (1) is unimodular. Since $D_{cr}(0)=(\frac{s}{s+a}M^{-1}Y)|_{s=0}=0$, D_{cr} has blocking zeros at $s=0$. By Definition 2.2, any $\hat{C}=N_{cr}D_{cr}^{-1}$ given by (14) is therefore an integral-action controller for G. ∎

The block diagram of $Sys(G,\hat{C})$, with $\hat{C}$ as in (14), is in Fig. 2. For $Q=0$, the integral-action controller $\hat{C}$ becomes

$$\hat{C}^o=C_g^o+\widetilde{D}_c^{-1}C_{pid}\ . \tag{18}$$

The controller $\hat{C}$ in (14) is simplified for stable plants as follows: Let $G\in\mathbf{S}^{n_y\times n_u}$, rank$G(0)=n_y\le n_u$. Let C_{pid}, with $K_i\neq 0$, be a PID-controller that stabilizes G. Then $\hat{C}$ is an integral-action controller for G if and only if $\hat{C}=(I-QG)^{-1}\ (\ Q\ +\ C_{pid}\ )$, where $Q\in\mathbf{S}^{n_u\times n_y}$ satisfies $\det(I-QG)(\infty)\neq 0$.

The parametrization given in (14) can also be obtained using a state-space representation (A,B,C,D) of $G\in\mathbf{R_p}^{n_y\times n_u}$, where $A\in\mathbb{R}^{n\times n}$, (A,B) is stabilizable and (C,A) is detectable. Let $K\in\mathbb{R}^{n_u\times n}$ and $L\in\mathbb{R}^{n\times n_y}$ be such that F_L and F_K defined in (19) are stable:

$$F_L:=(sI-A+LC)^{-1}\ ,\ \ F_K:=(sI-A+BK)^{-1}\ . \tag{19}$$

Then using $G=N_gD_g^{-1}=\widetilde{D}_g^{-1}\widetilde{N}_g$, where

$$\begin{aligned} D_g&=I-KF_KB\ ,\ \ N_g=(C-DK)F_KB+D,\\ \widetilde{D}_g&=I-CF_LL\ ,\ \ \widetilde{N}_g=CF_L(B-LD)+D\ , \end{aligned} \tag{20}$$

a controller $C_g^o=\widetilde{D}_c^{-1}\widetilde{N}_c=N_cD_c^{-1}$ is given by

$$\begin{aligned} \widetilde{D}_c&=I+KF_L(B-LD)\ ,\ \widetilde{N}_c=KF_LL\ ,\\ D_c&=I+(C-DK)F_KL\ ,\ N_c=KF_KL\ ,\\ C_g^o\ &=\ K\ (\ sI-A+BK+L(C-DK)\ )^{-1}\ L\ . \end{aligned} \tag{21}$$

With the nominal full-order observer-based controller C_g^o in (21), the expression for all integral-action controllers in (14) of Theorem 3.1 becomes $\hat{C}=[I+KF_L(B-LD)-Q(CF_L(B-LD)+D)]^{-1}[\,KF_LL+Q(I-CF_LL)+C_{pid}\,]$, where $Q\in\mathbf{S}^{n_u\times n_y}$ is such that $\det(I-Q(\infty)D)\neq 0$. With $Q=0$, the controller $\hat{C}^o$ in (18) is expressed as

$$\hat{C}^o=K(sI-A+BK+L(C-DK))^{-1}L+\widetilde{D}_c^{-1}\ C_{pid}\ . \tag{22}$$

The block diagram of $Sys(G,\hat{C})$, with C_g^o as in (21), is in Fig. 3. The PID block for $N_g=[(C-DK)F_KB+D]$ can be designed using Proposition 3.1: Since G has no transmission zeros at $s=0$, $N_g(0)$ has a right-inverse

$$N_g(0)^I=\left[D-(C-DK)(A-BK)^{-1}B\right]^I\ . \tag{23}$$

The expression (6) is simplified as follows: Since $F_K\in\mathcal{M}(\mathbf{S})$, $F_K(0)=(-A+BK)^{-1}$ exists. By (23), $[N_g(s)-N_g(0)]N_g(0)^I=[(C-DK)F_KB+D-D+(C-DK)(A-BK)^{-1}B]N_g(0)^I=(C-DK)(A-BK)^{-1}[(sI(A-BK)^{-1}-I)^{-1}+I]BN_g(0)^I=(C-DK)(A-BK)^{-1}[(sI(A-BK)^{-1}-I)^{-1}sI(A-BK)^{-1}]BN_g(0)^I=$

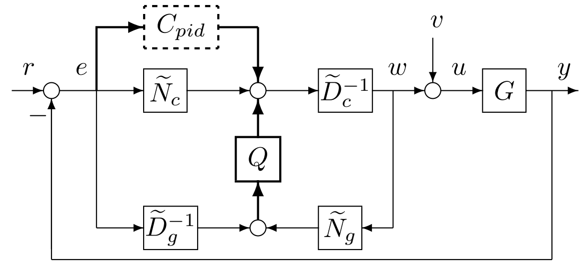

Fig. 2. The system $Sys(G,\hat{C})$ with integral-action controller $\hat{C}$.

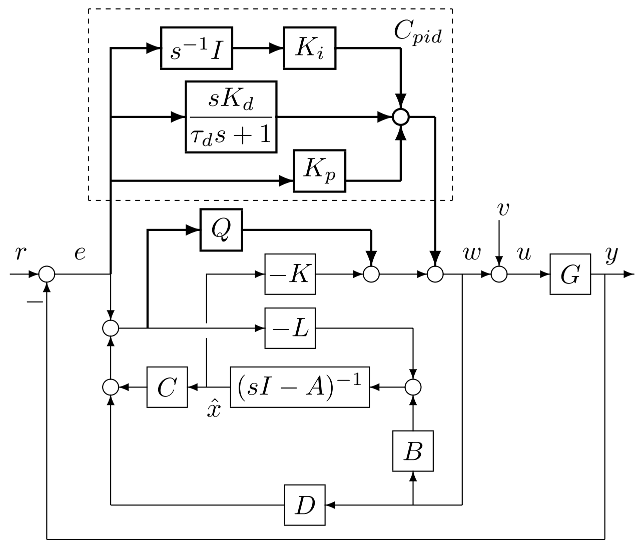

Fig. 3. $Sys(G,\hat{C})$ using full-order observer-based controller C_g^o

$(C-DK)(A-BK)^{-1}(sI-A+BK)^{-1}sBN_g(0)^I$ implies $s^{-1}(N_g(s)N_g(0)^I-I)=(C-DK)(A-BK)^{-1}F_KBN_g(0)^I$. The bound in (6) for $\rho>0$ becomes

$$\rho<\|N_g(s)\hat{C}_{pd}+(C-DK)(A-BK)^{-1}F_KBN_g(0)^I\|^{-1}. \tag{24}$$

Then N_g is stabilized by the PID controller C_{pid} in (7).

B. Integral-action design based on plant augmentation

We briefly review the well-known full-order observer-based integral-action controller synthesis, where the integrator's states are also included in state-feedback [5], [4]. The state-feedback matrix can be designed using pole-placement or LQR. Let (A,B,C,D) be a state-space representation of $G\in\mathbf{R_p}^{n_y\times n_u}$, where $A\in\mathbb{R}^{n\times n}$, (A,B) is stabilizable, (C,A) is detectable. Let $L\in\mathbb{R}^{n\times n_y}$ be such that $F_L\in\mathbf{S}^{n\times n}$. Define the $(n+n_y)$-th order augmented system as

$$A_a:=\begin{bmatrix} A & 0\\ -C & 0\end{bmatrix},B_a:=\begin{bmatrix} B\\ -D\end{bmatrix},C_a:=[\,C\quad 0\,]. \tag{25}$$

The pair (A_a,B_a) is stabilizable if and only if (A,B) is stabilizable and G has no transmission zeros at the origin. A state-feedback $K_a=[\ K_x\quad K_\xi\ ]$ is then determined for the augmented system in (25), and the resulting $(n+n_y)$-th order observer-based controller is called $\hat{C}_a$, given by

$$\hat{C}_a=-K_a\left[sI-A_a+B_aK_a+L_a(C_a-DK_a)\ \right]^{-1}L_a, \tag{26}$$

where $L_a := \begin{bmatrix} -L \\ I_{n_y} \end{bmatrix}$. The block diagram of $Sys(G, \hat{C}_a)$ with the $(n+n_y)$-th order $\hat{C}_a$ is in Fig. 4. The augmented system is stabilized using K_a; if $K_\xi = 0$, the design does not guarantee stability with K_x acting alone. The integrators cannot be taken out of service completely.

IV. Applications to RTP systems

We apply the integral-action controller design procedure of Section III-A to temperature control in RTP systems. We use the linearized model described in [4], with three standard tungsten halogen lamps as actuators, and three temperature sensors. Let $x = [x_1 \ x_2 \ x_3]^T$ denote the temperatures. The linearized MIMO system has a state-space representation (A, B, C, D), where

$$A = \begin{bmatrix} -0.0682 & 0.0149 & 0 \\ 0.0458 & -0.1181 & 0.0218 \\ 0 & 0.04683 & -0.1008 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3787 & 0.1105 & 0.0229 \\ 0 & 0.4490 & 0.0735 \\ 0 & 0.0007 & 0.4177 \end{bmatrix}, \ C = I_3, \ D = 0 \ .$$

We design an integral-action controller following the two-stage procedure of Section III-A; a full-order observer-based stabilizing controller is chosen for the first stage and a PI-controller for the second block. We choose to design the initial controller C_g^o as in (21). The estimator is designed here by pole placement, with the poles of F_L located at $\{-3, \ -4, \ -5\}$. We use LQR to find a state-feedback gain $K \in \mathbb{R}^{3\times 3}$ such that F_K is stable. We choose the state weighting matrix $\hat{Q} = 20 I_3$ and the control weighting matrix $\hat{R} = I_3$. Then $L, K \in \mathbb{R}^{3\times 3}$ are

$$L = \begin{bmatrix} 2.9318 & 0.0149 & 0 \\ 0.0458 & 3.8819 & 0.0218 \\ 0 & 0.0468 & 4.8992 \end{bmatrix}, \tag{27}$$

$$K = \begin{bmatrix} 4.2308 & -0.4739 & -0.0610 \\ 0.6725 & 4.1515 & -0.2611 \\ 0.0966 & 0.4407 & 4.2242 \end{bmatrix}. \tag{28}$$

The controller $C_g^o = K(sI - A + BK + L(C - DK))^{-1} L = (n_{ij})/d$, where $d = (s+4.609)(s+5.921)(s+6.809)$, is a stabilizing controller for the plant G and it does not have integral-action. We follow Proposition 3.1 to design the PID block for N_g in (20) of the RCF $G = N_g D_g^{-1}$. Choosing $\hat{K}_p = 15 I_3$ and $\hat{K}_d = 0$, the inequality (24) becomes $0 < \rho < \|N_g(s)\hat{C}_{pd} + (C - DK)(A - BK)^{-1} F_K B N_g(0)^I\|^{-1} = 0.3292$. Choose $\rho = 0.3$; with $N_g(0)^{-1} = \begin{bmatrix} 4.44 & -0.58 & -0.05 \\ 0.57 & 4.43 & -0.35 \\ 0.09 & 0.32 & 4.46 \end{bmatrix}$, $C_{pi} = \rho(\hat{K}_p + \frac{1}{s} N_g(0)^{-1}) = 4.5 I_3 + \frac{0.3}{s} N_g(0)^{-1}$. Finally, with $Q = 0$, the integral-action controller $\hat{C}^o = C_g^o + \widetilde{D}_c^{-1} C_{pi} = (h_{ij})/\hat{d}$, where $\hat{d} = sd$, is a fourth-order controller due to the third-order observer-based controller C_g^o in the first stage and the first-order PI block in the second stage.

We also design a standard observer-based controller $\hat{C}_a$ as in Section III-B using the augmented plant description (25), with $n = 3$, $n_y = 3$. We use the same state-estimator gain L as in (27) and choose the augmented state weighting $Q_a = \text{diag}[\, 20 I_3 \ , \ I_3 \,]$, the control weighting $R = I_3$. The state feedback $K_a = [\, K_x \quad K_\xi \,]$ is found using LQR. The integral-action controller $\hat{C}_a$ is computed using (26). Fig. 5 shows the closed-loop step responses of the three temperatures individually for $Sys(G, \hat{C}^o)$ and $Sys(G, \hat{C}_a)$, with unit steps applied at each of the three inputs. The step response characteristics are very similar for all three temperatures. The response of $Sys(G, \hat{C}^o)$ (solid line) displayed no overshoot and fast rise time (less than 1 sec.) in the absence of actuator saturation. Temperature uniformity on the wafer surface is also maintained. The responses for $Sys(G, \hat{C}_a)$ are slower and have 22% overshoot. Step responses of $Sys(G, \hat{C}^o)$ and $Sys(G, \hat{C}_a)$ when the integrators are turned off due to actuator saturation are shown in Fig. 6. Saturation nonlinearities included in the control loop saturate at ± 0.3. The step responses of both systems are slowed down because of the actuator saturations, with $Sys(G, \hat{C}^o)$ displaying faster response. Although integral-action is not available due to saturation, the steady-state errors are negligibly small for both systems. Another anti-windup

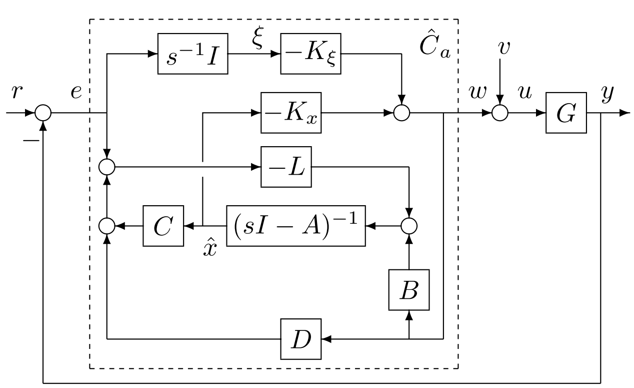

Fig. 4. $Sys(G, \hat{C}_a)$ with augmented observer-based controller.

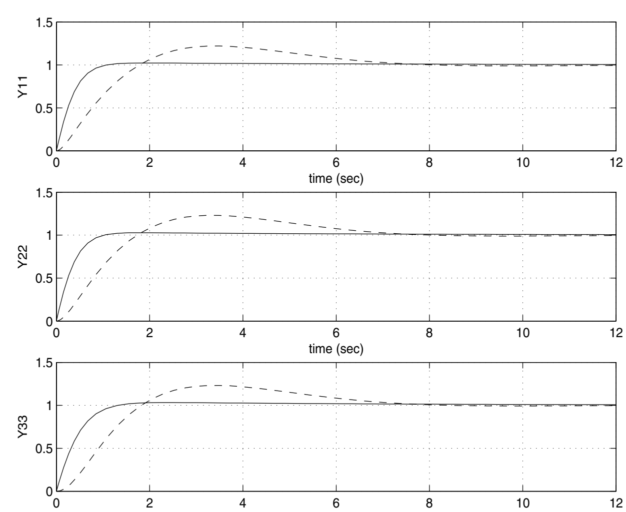

Fig. 5. Step responses of $Sys(G, \hat{C}^o)$ and $Sys(G, \hat{C}_a)$.

technique limits the integral value when saturation occurs. The step responses of the two systems are in Fig. 7; the anti-windup gains (K_{aw}) of integrator limiting feedback loops for $Sys(G, \hat{C}^o)$ and $Sys(G, \hat{C}_a)$ are selected as 0.18 and 1.2, respectively. Slightly faster responses are obtained compared to those in Fig. 6, with zero steady-state errors. In the system $Sys(G, \hat{C}^o)$, where $\hat{C}^o = C_g^o + \widetilde{D}_c^{-1} C_{pi}$, the entire PI block C_{pi} can be turned off when the system has input saturation. Since C_g^o designed to stabilize G is still active when C_{pi} is set to zero, the closed-loop system is still stable. The step response of $Sys(G, C_g^o)$ under actuator saturation is in Fig. 8. The response without the PI block is reasonably fast and the steady-state error is small, although it is not zero due to the absence of integral-action. These simulation results indicate that $Sys(G, \hat{C}_o)$ generally has better step response than $Sys(G, C_a)$ in the absence of actuator saturation. The systems displayed very similar characteristics under actuator saturation: Responses for both systems slowed down and temperature uniformity on wafer surface was not maintained; the third channel shows approximately 2 seconds more delay than the other two.

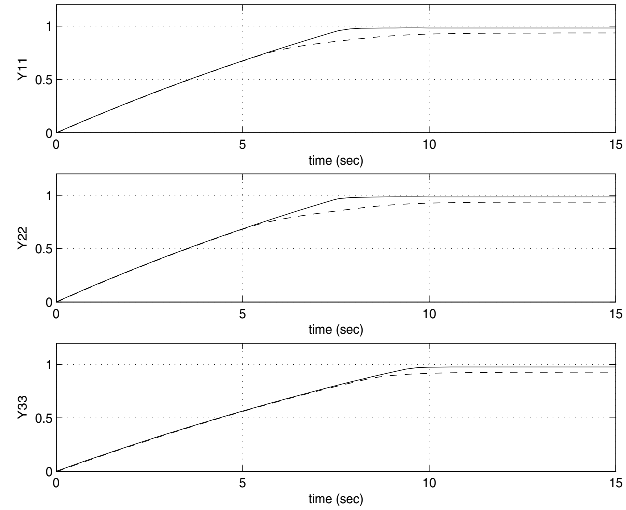

Fig. 6. Step responses of $Sys(G, \hat{C}^o)$, $Sys(G, \hat{C}_a)$ without integrators.

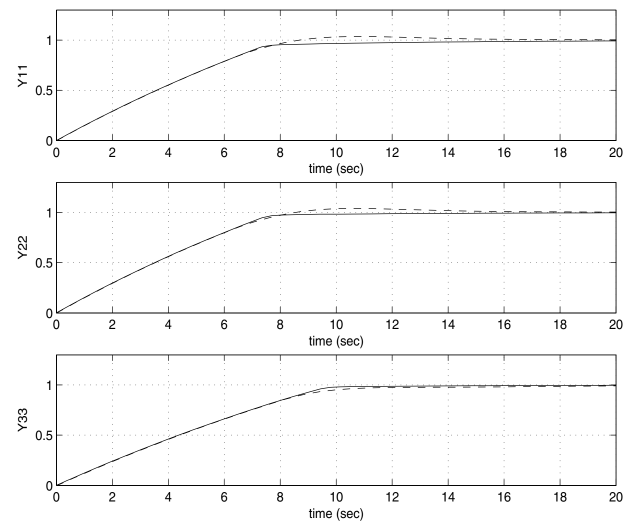

Fig. 7. Step responses of $Sys(G, \hat{C}^o)$, $Sys(G, \hat{C}_a)$ with limited integrators.

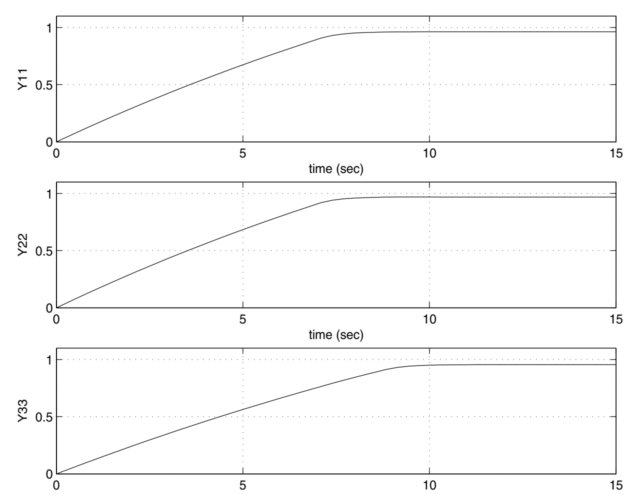

Fig. 8. Step response of $Sys(G, C_g^o)$ with saturated input.

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