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Reliable decentralized integral-action controller design for multi-channel systems

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Abstract

Reliable decentralized stabilizing controller design with integral-action is considered for linear time-invariant, multi-input multi-output systems with stable plants. The proposed design methods achieve closed-loop stability with integral-action in each output channel and guarantee stability with integral-action in the active channels when all controllers are operational and when any of the controllers is set equal to zero.

1 Introduction

Reliable controller design with integral-action (I-A) is considered for linear time-invariant (LTI), multi-input multi-output (MIMO) decentralized systems with stable plants. The goal is to achieve closed-loop stability with I-A at each output so that step-input references applied at each input are tracked asymptotically, and to maintain stability with I-A when any of the controllers fail. Reliable stabilization was studied using full-feedback and decentralized controllers [5, 4, 3, 1, 2]. This paper presents conditions for existence of reliable decentralized I-A controllers and proposes explicit design approaches. The results are explored in detail for two, three and four-channel decentralized systems with MIMO channels; simplifications are given for single-output channels. The design can be extended to more than four channels. The results apply to continuous-time and discrete-time systems. A continuous-time setting was assumed here; discussions involving poles and zeros at $s = 0$ should be interpreted at $z = 1$ in the discrete-time case. *Notation:* The region of instability $\mathcal{U}$ is the extended closed right-half-plane (continuous-time systems) or the complement of the open unit-disk (discrete-time systems). The sets of real numbers, proper rational functions with no $\mathcal{U}$-poles, proper and strictly-proper rational functions with real coefficients, matrices with entries in $\mathcal{R}$ are $\mathbb{R}$, $\mathcal{R}$, $\mathrm{R_p}$, $\mathrm{R_s}$, $\mathcal{M}(\mathcal{R})$; M is stable iff $M \in \mathcal{M}(\mathcal{R})$; $M \in \mathcal{M}(\mathcal{R})$ is unimodular iff $M^{-1} \in \mathcal{M}(\mathcal{R})$. For $M \in \mathcal{M}(\mathcal{R})$, the norm $\|\cdot\|$ is $\|M\| := \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$; $\bar{\sigma}$ is the maximum singular value, $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$. Let $P \in \mathcal{M}(\mathcal{R})$, $\mathrm{rank}P = \rho$; $s_o \in \mathcal{U}$ is called a $\mathcal{U}$-zero of P iff $\mathrm{rank}P(s_o) < \rho$; s_o is called a blocking-zero of P iff $P(s_o) = 0$. Abbreviations: I-A (integral-action), SPD (symmetric, positive-definite), RI (right-inverse).

2 Main Results

Consider the LTI, MIMO, w-channel decentralized feedback system $\mathcal{S}(P, C_D)$ in Figure 1: $\mathcal{S}(P, C_D)$ is well-posed; $P =\in \mathcal{R}^{n_y \times n_u}$, $C_D = \mathrm{diag}[C_1, \ldots, C_w] \in {\mathrm{R_p}}^{n_u \times n_y}$ represent the transfer-functions of the plant and the decentralized controller; P is partitioned so that $P_{ii} \in \mathcal{R}^{n_{yi}\times n_{ui}}$, $P_{ij} \in \mathcal{R}^{n_{yi}\times n_{uj}}$, $C_i \in {\mathrm{R_p}}^{n_{ui}\times n_{yi}}$, $i, j = 1, \ldots, w$, $n_y = \sum_{i=1}^{w} n_{yi}$, $n_u = \sum_{i=1}^{w} n_{ui}$; P and C_D have no hidden modes corresponding to eigenvalues in $\mathcal{U}$. Although $P \in \mathcal{M}(\mathcal{R})$, the decentralized controller C_D is unstable (due to poles at zero for the I-A requirement and other possible $\mathcal{U}$-poles). Let H_{er} denote the (input-error) transfer-function from r to e; $r := [r_1^T \cdots r_w^T]^T$, u, e, y, y_c are defined similarly. A controller that fails is set equal to zero; the failure is recognized and the corresponding controller is taken out of service. When $w = 2$, the failures are due to one controller failure. When $w = 3$, the failures are due to one or two controller failure. When $w = 4$, the failures are due to one, two or three controller failure. For $i = 1, \ldots, w$, $\mathcal{S}(P, C_i)$ is the system with only the i-th controller active. For $j = 2, \ldots, w$, $i = 1, \ldots, i-1$, $\mathcal{S}(P, C_i, C_j)$ is the system with only the i-th and j-th controllers active. For $k = 3, \ldots, w$, $j = 2, \ldots, k-1$, $i = 1, \ldots, j-1$, $\mathcal{S}(P, C_i, C_j, C_k)$ is the system with only i-th, j-th and k-th controllers active. The outputs of the inactive channels (for $\ell = 1, \ldots, w$, $y_{c\ell}$, $\ell \neq i$ of $\mathcal{S}(P, C_i)$, $y_{c\ell}$, $\ell \neq i$, $\ell \neq j$ of $\mathcal{S}(P, C_i, C_j)$, $y_{c\ell}$, $\ell \neq i$, $\ell \neq j$, $\ell \neq k$ of $\mathcal{S}(P, C_i, C_j, C_k)$) are not observed.

2.1 Definitions: a) The system $\mathcal{S}(P, C_D)$ is *stable* iff the transfer-function from (r, u) to (y, y_c) is stable. The stable $\mathcal{S}(P, C_D)$ has *I-A* iff $H_{er}(0) = 0$. For $i = 1, \ldots, w$, $\mathcal{S}(P, C_i)$ is stable iff the transfer-function from (r_i, u) to (y, y_{ci}) is stable. The stable $\mathcal{S}(P, C_i)$ has I-A iff the transfer-function from r_i to e_i has blocking-zeros at zero. For $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, $\mathcal{S}(P, C_i, C_j)$ is stable iff the transfer-function from (r_i, r_j, u) to (y, y_{ci}, y_{cj}) is stable; it has I-A iff the transfer-function from (r_i, r_j) to (e_i, e_j) has blocking-zeros at zero. For $k = 3, \ldots, w$, $j = 2, \ldots, k-1$, $i = 1, \ldots, j-1$, $\mathcal{S}(P, C_i, C_j, C_k)$ is stable iff the transfer-function from (r_i, r_j, r_k, u) to $(y, y_{ci}, y_{cj}, y_{ck})$ is stable; it has I-A iff the transfer-function from (r_i, r_j, r_k) to (e_i, e_j, e_k) has blocking-zeros at zero. **b)** C_D is a *stabilizing controller* for P (or C_D stabilizes P) iff $C_D \in \mathcal{M}(\mathrm{R_p})$ and $\mathcal{S}(P, C_D)$ is stable. **c)** C_D is a *reliable decentralized I-A controller* iff $\mathcal{S}(P, C_D)$ is stable with I-A when all controllers are active and when any subset of the controllers are set to zero; i.e., when $w = 2$, $\mathcal{S}(P, C_D)$, $\mathcal{S}(P, C_i)$, $i = 1, 2$, are stable with I-A, when $w = 3$, $\mathcal{S}(P, C_D)$, $\mathcal{S}(P, C_i)$, $i = 1, 2, 3$, $\mathcal{S}(P, C_i, C_j)$, $j = 2, 3$, $i = 1, \ldots, j-1$, are stable with I-A, when $w = 4$, $\mathcal{S}(P, C_D)$, $\mathcal{S}(P, C_i)$, $i = 1, \ldots, 4$, $\mathcal{S}(P, C_i, C_j)$, $j = 2, \ldots, 4$, $i = 1, \ldots, j-1$, $\mathcal{S}(P, C_i, C_j, C_k)$, $k = 3, 4$, $j = 2, \ldots, k-1$, $i = 1, \ldots, j-1$, are stable with I-A. □

Lemma 2.2 states conditions for existence of w-channel

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reliable decentralized I-A controllers for $w = 2, 3, 4$. Proposition 2.3 gives a reliable decentralized I-A controller design approach. Let $C_i = N_i(\frac{s}{s+\alpha}D_i)^{-1} \in {\mathrm{R_p}}^{n_{ui} \times n_{yi}}$ be a right-coprime-factorization ($N_i, D_i \in \mathcal{M}(\mathcal{R})$, $\det D_i(\infty) \neq 0$, $-\alpha \in \mathbb{R} \setminus \mathcal{U}$), $P_{ii}^I(0) \in \mathbb{R}^{n_{ui} \times n_{yi}}$ be a RI of $P_{ii}(0) \in \mathbb{R}^{n_{yi} \times n_{ui}}$, $N_i(0) := P_{ii}^I(0)$, $i = 1, \ldots, w$; $X_{ij} := P_{jj} - P_{ji}N_iP_{ij}$, $W_{ij} := I + (X_{ij} - P_{jj})(I + s^{-1}k_jP_{jj}^I(0)X_{ij})^{-1}Q_j$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$. If $w \geq 3$, $Y_{\ell m}^k := P_{\ell m} - P_{\ell k}N_kP_{km}$, $k = 1, \ldots, w-2$, $\ell, m = k+1, \ldots, w$, $\ell \neq m$; $Z_{rv}^q := X_{qv} - Y_{vr}^qN_r(I - P_{rq}N_qP_{qr}N_r)^{-1}Y_{rv}^q = X_{qv} - Y_{vr}^qN_r(I + (X_{qr} - P_{rr})N_r)^{-1}Y_{rv}^q$, $W_{rv}^q := I + (Z_{rv}^q - P_{vv})(I + s^{-1}k_vP_{vv}^I(0)Z_{rv}^q)^{-1}Q_v$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$. If $w = 4$, $G := Z_{24}^1 - (Y_{43}^1 - Y_{42}^1N_2(I - P_{21}N_1P_{12}N_2)^{-1}Y_{23}^1)N_3(I + (Z_{23}^1 - P_{33})N_3)^{-1}(Y_{34}^1 - Y_{32}^1N_2(I - P_{21}N_1P_{12}N_2)^{-1}Y_{24}^1)$, $G(0) := Z_{24}^1(0) - (Y_{43}^1 - Y_{42}^1P_{22}^I(X_{12}P_{22}^I)^{-1}Y_{23}^1)P_{33}^I(Z_{23}^1P_{33}^I)^{-1}(Y_{34}^1 - Y_{32}^1P_{22}^I(X_{12}P_{22}^I)^{-1}Y_{24}^1)(0)$, $W_g := I + (G - P_{44})(I + s^{-1}k_4P_{44}^I(0)G)^{-1}Q_4$.

2.2 Lemma: Let $P \in \mathcal{R}^{n_y \times n_u}$. Let $P_{ii}^I(0) \in \mathbb{R}^{n_{ui} \times n_{yi}}$ denote a RI of $P_{ii}(0) \in \mathbb{R}^{n_{yi} \times n_{ui}}$, $i = 1, \ldots, w$. *a) Necessary conditions*: If there exist reliable decentralized I-A controllers, then: *i)* $\mathrm{rank}P(0) = n_y$, $\mathrm{rank}P_{ii}(0) = n_{yi}$, $i = 1, \ldots, w$, *ii)* $\det(X_{ij}(0)P_{jj}^I(0)) \neq 0$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, for some RI $P_{ii}^I(0)$, $P_{jj}^I(0)$ of $P_{ii}(0)$, $P_{jj}(0)$, *iii)* if $w \geq 3$, $\det(Z_{rv}^q(0)P_{vv}^I(0)) \neq 0$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$, for some RI $P_{vv}^I(0)$, $P_{rr}^I(0)$ of $P_{vv}(0)$, $P_{rr}(0)$, *iv)* if $w = 4$, $\det(G(0)P_{44}^I(0)) \neq 0$. *b) Necessary and sufficient conditions*: *i)* There exist reliable decentralized I-A controllers if *1)* the conditions in (a) hold, *2)* for $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, $\det(X_{ij}(0)P_{jj}^I(0)) > 0$ for some RI $P_{ii}^I(0)$, $P_{jj}^I(0)$ of $P_{ii}(0)$, $P_{jj}(0)$, *3)* if $w \geq 3$, $\det(Z_{rv}^q(0)P_{vv}^I(0)) > 0$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$, for some RI $P_{vv}^I(0)$, $P_{rr}^I(0)$ of $P_{vv}(0)$, $P_{rr}(0)$, *4)* if $w = 4$, $\det(G(0)P_{44}^I(0)) > 0$. *ii)* When P_{ij}, or P_{ji}, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, or when any $w-1$ of the w controllers $C_1, \ldots, C_w$ are strictly-proper, or when these have blocking $\mathcal{U}$-zeros, conditions (i2)-(i4) become necessary: there exist reliable decentralized I-A controllers if and only if conditions (i1)-(i4) hold. *c) Sufficient conditions*: There exist reliable decentralized I-A controllers if *1)* the conditions in (a) hold, *2)* $X_{ij}(0)P_{jj}^I(0)$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, is SPD for some RI $P_{ii}^I(0)$, $P_{jj}^I(0)$ of $P_{ii}(0)$, $P_{jj}(0)$, *3)* if $w \geq 3$, $Z_{rv}^q(0)P_{vv}^I(0)$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$, is SPD for some RI $P_{vv}^I(0)$, $P_{rr}^I(0)$ of $P_{vv}(0)$, $P_{rr}(0)$, *4)* if $w = 4$, $G(0)P_{44}^I(0)$ is SPD. □

In some cases, the conditions of Lemma 2.2-(c) and (b) are equivalent (e.g., when $n_{yj} = 1$ for $j = 2, \ldots, w$, or $P_{ij}(0) = 0$ or $P_{ji}(0) = 0$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$).

2.3 Proposition: Let $P \in \mathcal{R}^{n_y \times n_u}$, $\mathrm{rank}P(0) = n_y \leq n_u$, $\mathrm{rank}P_{ii}(0) = n_{yi} \leq n_{ui}$, $i = 1, \ldots, w$, $X_{ij}(0)P_{jj}^I(0)$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, be SPD for some RI $P_{ii}^I(0)$, $P_{jj}^I(0)$ of $P_{ii}(0)$, $P_{jj}(0)$. If $w \geq 3$, let $Z_{rv}^q(0)P_{vv}^I(0)$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$, be SPD for some RI $P_{vv}^I(0)$, $P_{rr}^I(0)$ of $P_{vv}(0)$, $P_{rr}(0)$. If $w = 4$, let $G(0)P_{44}^I(0)$ be SPD. For $i = 1, \ldots, w$, $N_i := (I + s^{-1}k_iP_{ii}^I(0)P_{ii})^{-1}(s^{-1}k_iP_{ii}^I(0) + Q_i) \in \mathcal{R}^{n_{ui} \times n_{yi}}$. Then C_D is a reliable decentralized I-A controller with $C_i = (I - Q_iP_{ii})^{-1}(s^{-1}k_iP_{ii}^I(0) + Q_i)$, $\det(I - Q_iP_{ii})(\infty) \neq 0$; $k_i \in \mathbb{R}$, $k_i > 0$, $Q_i \in \mathcal{R}^{n_{ui} \times n_{yi}}$ are as follows: $k_1 < \|s^{-1}(P_{11}(s)P_{11}^I(0) - I)\|^{-1}$; fix Q_1; $k_2 < \min\{\|s^{-1}(P_{22}(s)P_{22}^I(0) - I)\|^{-1}, \|s^{-1}(X_{12} - X_{12}(0))P_{22}^I(0)\|^{-1}\}$, W_{12} unimodular. If $w \geq 3$, fix Q_2; $k_3 < \min\{\|s^{-1}(P_{33}(s)P_{33}^I(0) - I)\|^{-1}, \|s^{-1}(X_{i3} - X_{i3}(0))P_{33}^I(0)\|^{-1}, \|s^{-1}(Z_{23}^1 - Z_{23}^1(0))P_{33}^I(0)\|^{-1}\}$, $i = 1, 2$, W_{i3}, W_{23}^1 unimodular. If $w = 4$, fix Q_3; $k_4 < \min\{\|s^{-1}(P_{44}(s)P_{44}^I(0) - I)\|^{-1}, \|s^{-1}(X_{i4} - X_{i4}(0))P_{44}^I(0)\|^{-1}, \|s^{-1}(Z_{r4}^q - Z_{r4}^q(0))P_{44}^I(0)\|^{-1}, \|s^{-1}(G - G(0))P_{44}^I(0)\|^{-1}\}$, $i = 1, 2, 3$, $q = 1, 2$, $r = q+1, \ldots, 3$, W_{i4}, W_{r4}^q, W_g unimodular.

2.4 Corollary: Let $P \in \mathcal{R}^{n_y \times n_u}$, $n_{yi} = 1$, $i = 2, \ldots, w$. There exists a reliable decentralized I-A controller $C_D = \left[\frac{K_1}{s}, \ldots, \frac{K_w}{s}\right]$, $K_i \in \mathbb{R}^{n_{ui} \times n_{yi}}$, $i = 1, \ldots, w$, if and only if: 1) $\mathrm{rank}P(0) = n_y$, $\mathrm{rank}P_{ii}(0) = n_{yi}$, 2) $X_{ij}(0)P_{jj}^I(0) > 0$, $j = 2, \ldots, w$, $i = 1, \ldots, j-1$, for some RI $P_{ii}^I(0)$, $P_{jj}^I(0)$ of $P_{ii}(0)$, $P_{jj}(0)$, 3) if $w \geq 3$, $Z_{rv}^q(0)P_{vv}^I(0) > 0$, $v = 3, \ldots, w$, $q = 1, \ldots, v-2$, $r = q+1, \ldots, v-1$, for some RI $P_{vv}^I(0)$, $P_{rr}^I(0)$ of $P_{vv}(0)$, $P_{rr}(0)$, 4) if $w = 4$, $G(0)P_{44}^I(0) > 0$. Furthermore, K_i can be chosen as $k_iP_{ii}^I(0)$ (k_i as in Proposition 2.3). □

In Proposition 2.3, $C_i = s^{-1}k_iP_{ii}^I(0)$ if $Q_i = 0$; the unimodularity conditions hold if $Q_i = 0$; W_{12} is unimodular if $\|Q_2\| < \|(X_{12} - P_{22})(I + s^{-1}k_2P_{22}^I(0)X_{12})^{-1}\|^{-1}$; W_{i3}, W_{23}^1 are unimodular if $\|Q_3\| < \min\{\|(X_{i3} - P_{33})(I + s^{-1}k_3P_{33}^I(0)X_{i3})^{-1}\|^{-1}, \|(Z_{23}^1 - P_{33})(I + s^{-1}k_3P_{33}^I(0)Z_{23}^1)^{-1}\|^{-1}\}$; W_{i4}, W_{r4}^q, W_g are unimodular if $\|Q_4\| < \min\{\|(X_{i4} - P_{44})(I + s^{-1}k_4P_{44}^I(0)X_{i4})^{-1}\|^{-1}, \|(Z_{r4}^q - P_{44})(I + s^{-1}k_4P_{44}^I(0)Z_{r4}^q)^{-1}\|^{-1}, \|(G - P_{44})(I + s^{-1}k_4P_{44}^I(0)G)^{-1}\|^{-1}\}$. When $P_{ii} \in \mathcal{M}(\mathrm{R_s})$, $\det(I - Q_iP_{ii})(\infty) \neq 0$ for all $Q_i \in \mathcal{M}(\mathcal{R})$; $C_i \in \mathcal{M}(\mathrm{R_s})$ if and only if $Q_i \in \mathcal{M}(\mathcal{R}) \cap \mathcal{M}(\mathrm{R_s})$.

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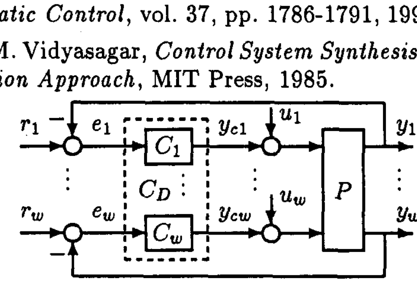

Figure 1: The decentralized system $\mathcal{S}(P, C_D)$.