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Robust Controller Design for a class of MIMO Distributed Parameter Systems

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Abstract: This paper considers the robust stabilization of linear time invariant multi-input-multi-output (MIMO) infinite dimensional systems (with finitely many unstable poles) whose transfer functions are factorized in a special form. Important practical applications include finite dimensional MIMO systems with time delays in the output channels. The controllers are designed from a tangential Nevanlinna-Pick interpolation or solutions of a particular Bézout equation. The controller structure can be seen as an extension of Smith predictors for time delay systems. Finite-dimensional approximations of the controller and its impact on the robust stability of the feedback system are also discussed. A numerical example is given.

Keywords: Linear systems, distributed parameter systems, robust stability, time delay, H-infinity control, interpolation

1. INTRODUCTION

The paper considers linear, time invariant, multi-input multi-output (MIMO) infinite dimensional systems with finitely many unstable poles. It is assumed that the nominal plant model can be factored as MP where M is an inner matrix (stable transfer matrix which is unitary on the imaginary-axis) and P is a finite dimensional transfer matrix (possibly unstable). In this framework, the plant class contains a special case of coupled unstable MIMO systems with time delays in the output channels. Feedback control of such systems has been studied over the last five decades, see e.g., Mirkin and Raskin (2003), Zhong (2006), Deng et al. (2022) and the references therein. For haptic systems with varying time delays Liacu et al. (2013) proposed a Smith-predictor-based controller structure. Moreover, Yegin and Ozbay (2024b) used a "decoupler" from Flesch et al. (2011) for two-input-two-output haptic systems. A necessary and sufficient condition for the existence of a diagonal decoupling controller is that none of the transmission zeros should coincide with the poles of the MIMO plant, Gundes (2021). This condition is generically satisfied. For finite dimensional systems with time delays in the output channels, the decoupler design can be quite complicated unless there are special structures in the MIMO system. Here, we provide an alternative approach (without the need for a decoupler) which can be applied to a large class of distributed parameter systems. The class of plants also capture other distributed parameter systems, as demonstrated by Yegin and Ozbay (2024a), using a flexible beam example. Control of flexible beams has been widely studied in the literature, see e.g. Curtain and Morris (2009), Halevi (2005); Smyshlyaev et al. (2009). We should also mention that there are many works taking into account time delays in systems represented by PDEs, see e.g., recent publications Katz and Fridman (2022); Lhachemi and Prieur (2022) and the references therein. As long as their transfer functions can be computed, these PDE-delay coupled systems can also be put into the framework of the class of plants considered here; see Section 2 for further details.

The present paper considers the MIMO case as an extension of Yegin and Ozbay (2023), where a generalized Smith predictor controller is designed for single-input single-output time delay systems, inspired by the two-parameter control[1] scheme of Sanz et al. (2018) (see their references for historical notes and earlier design methods). The proposed controller structure is given in Section 3; it contains a stable transfer matrix H, which is determined by solving a Bézout equation or by a tangential Nevanlinna-Pick interpolation. Then, a so-called "predictor filter" is constructed: $B = (I - HM)P$, which is stable and infinite dimensional. Next, we approximate B by a stable finite

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[1] Also called two-degree-of-freedom control, see Section 5.6 of Vidyasagar (2011).

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dimensional transfer matrix B_a. In the final step, a C is designed to stabilize P; in this process, uncertainties in the plant model and errors in the approximation of B within the controller are considered and C is designed to robustly stabilize the feedback system. A numerical example is given in Section 4.

Notation. $\mathcal{H}_\infty$ is the Hardy space of bounded analytic functions on $\mathbb{C}_+$, equipped with the usual norm $\|\cdot\|_\infty$, and $\mathcal{H}_\infty^{\mu\times\nu}$ is the set of all transfer function matrices of size $\mu\times\nu$ whose entries are in $\mathcal{H}_\infty$. For finite-dimensional systems, $\mathcal{R}_p$ represents proper rational transfer functions, and stable ones are $\mathcal{S}=\mathcal{R}_p\cap\mathcal{H}_\infty$. Similarly for the MIMO versions $\mathcal{R}_p^{\mu\times\nu}$ and $\mathcal{S}^{\mu\times\nu}$. We say that a square transfer matrix U, whose entries are in $\mathcal{H}_\infty$, is unimodular if U^{-1} has all its entries in $\mathcal{H}_\infty$. The symbol I_μ denotes the $\mu\times\mu$ identity matrix; when the size of the matrix is clear from the context we drop the subscript.

2. THE CLASS OF MIMO UNSTABLE PLANTS

We are interested in the robust stabilization of linear time invariant distributed parameter systems whose transfer function matrices are in the form

$$P_\Delta=(M+M_\delta)P, \quad \text{where} \quad M_\delta = M\Delta \tag{1}$$

$P\in\mathcal{R}_p^{\mu\times\nu}$, $M\in\mathcal{H}_\infty^{\mu\times\mu}$, with $\Delta\in\mathcal{H}_\infty^{\mu\times\mu}$ capturing the modeling uncertainty. Moreover, we assume that there exists a known $W_m\in\mathcal{S}^{\mu\times\mu}$ such that

$$\|\Delta(j\omega)\| < \|W_m(j\omega)\|, \quad \forall\omega\in\mathbb{R}\,; \tag{2}$$

(in many practical applications, by introducing some conservatism, we may assume that W_m is diagonal, or even a scalar times the identity). Thus, the set of all plants is described by P, M and W_m.

Since P is rational, the number of unstable poles of the plant is finite. Accordingly, let $\mathscr{P}_u$ denote the set of poles of P in $\overline{\mathbb{C}}_+$; if $p\in\overline{\mathbb{C}}_+$ is in $\mathscr{P}_u$, then so is its complex conjugate p^*. In Section 3, the general case (unstable poles with multiplicity) is discussed first; then, when interpolation techniques are used, for simplicity we assume that the unstable poles of P are distinct, i.e., $\mathscr{P}_u=\{p_1,\dots,p_n\}$ with $p_i\neq p_j$ for $i\neq j$.

Typically, M is an inner matrix. For example,

$$M(s)=\Lambda(s)=\mathrm{diag}\big[e^{-h_1 s},\dots,e^{-h_\mu s}\big] \tag{3}$$

where $h_j\geq 0$ is the time delay in the j^{th} output channel. There are many practical applications of such plant representations, including control over networks, tele-robotics, haptic systems, see e.g., Fridman (2014), Hokayem and Spong (2006), Liacu et al. (2013).

In general, if we have a distributed parameter system whose transfer function is factored as $P_d=M_iL_oR^{-1}$, where the entries of M_i, L_o, R are in $\mathcal{H}_\infty$, then we can see M_iL_o as the inner-outer factorization of the stable part, and all unstable poles are included in the right-coprime factor, namely R^{-1}. Assume that $R\in\mathcal{S}^{\nu\times\nu}$, and a finite dimensional outer approximation $L\in\mathcal{S}^{\mu\times\nu}$ can be found for the infinite dimensional $L_o\in\mathcal{H}_\infty^{\mu\times\nu}$. Defining $P=LR^{-1}$, the system can be put into the framework of (1) where $\Delta\in\mathcal{H}_\infty^{\mu\times\mu}$ satisfies

$$\Delta(s)\ L(s)=L_o(s)-L(s)\ . \tag{4}$$

The above set-up is demonstrated with a SISO flexible beam example in Yegin and Ozbay (2024a).

In summary, we consider plants of the form (1), where M is inner (infinite dimensional) and P is a proper rational matrix whose $\mathbb{C}_+$ poles are the elements of $\mathscr{P}_u$.

3. CONTROLLER DESIGN

The proposed design uses the two-parameter controller scheme of the Smith-predictor-based structure (the MIMO generalization of Yegin and Ozbay (2023)) as shown in Fig. 1, where the plant is $P_\Delta=(M+M_\delta)P$ as in (1), B_a is a stable rational approximation of the stable infinite dimensional "predictor feedback" defined as

$$B=(I-HM)P\ . \tag{5}$$

Hence, the controller is determined by C and H (both to be designed).

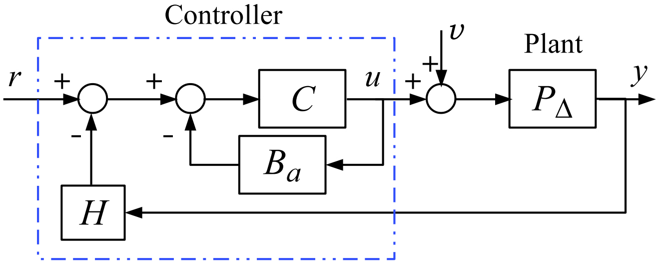

Fig. 1. Two-parameter control scheme

We consider a C such that the feedback system is stable with $M=I$, $M_\delta=0$, and $H=I$ (i.e., C stabilizes the plant P). We should have $H\in\mathcal{S}^{\mu\times\mu}$ such that $B\in\mathcal{H}_\infty^{\mu\times\nu}$. For practical implementations, B is approximated by a finite dimensional B_a. The effect of the approximation error

$$B_\varepsilon := (B_a-B)\in\mathcal{H}_\infty^{\mu\times\nu} \tag{6}$$

is also considered in the robust stability analysis.

3.1 Robust Stability Conditions

Let $M\in\mathcal{H}_\infty^{\mu\times\mu}$ be such that $\mathrm{rank}M(s)=\mu$ for all $s\in\mathscr{P}_u$. There are well-known methods for computing left and right-coprime factorizations of any given finite dimensional P, Vidyasagar (2011). Let $P=D^{-1}N$ be a left-coprime factorization, where $D\in\mathcal{S}^{\mu\times\mu}$, $N\in\mathcal{S}^{\mu\times\nu}$. Furthermore, in this construction, a diagonal D can be found if the transmission zeros of P in $\mathbb{C}_+$ do not coincide with the poles of P, see Gundes (2021), and also Vidyasagar (2011). Hence, we formally assume that $D\in\mathcal{S}^{\mu\times\mu}$ is a matrix satisfying $DP\in\mathcal{S}^{\mu\times\nu}$, and $\mathrm{rank}\big[D \quad DP\big](s)=\mu$ for $s\in\mathscr{P}_u$.

Based on these assumptions on M and D, there exist F, $H\in\mathcal{H}_\infty^{\mu\times\mu}$ that satisfy the following Bézout identity:

$$FD\ +\ HM=I_\mu\ . \tag{7}$$

One way to find the Bézout factors in this equation is to use interpolation: H is such that $F=(I-HM)D^{-1}$ is stable. Then, $B=FN$. In Section 4, we demonstrate this method with an example.

Theorem 1. Robust Stability Conditions. Let $C \in \mathcal{R}_p^{\nu\times\mu}$ be any controller that stabilizes $P \in \mathcal{R}_p^{\mu\times\nu}$. Let F, $H \in \mathcal{H}_\infty^{\mu\times\mu}$ satisfy (7). Define

$$T := PC(I+PC)^{-1},\ S := (I-T) = (I+PC)^{-1}. \tag{8}$$

Then the system shown in Fig. 1 is robustly stable if

$$\|HM_\delta T + B_\varepsilon CS\|_\infty < 1\ . \tag{9}$$

Proof. See Appendix. □

In the special case $B_\varepsilon = 0$, $M_\delta = M\Delta$ with an inner M and Δ satisfying (2), the feedback system is stabilized if

$$\|H\|_\infty < \|W_m T\|_\infty^{-1}\ . \tag{10}$$

Once we find an $H \in \mathcal{H}_\infty^{\mu\times\mu}$ leading to $B = (I - HM)P \in \mathcal{H}_\infty^{\mu\times\nu}$ and satisfying (10), the next step is to make sure that B is approximated by a $B_a \in \mathcal{S}^{\mu\times\nu}$ for which we have

$$\|B_\varepsilon CS\|_\infty < (1 - \|H\ W_m T\|_\infty)\ . \tag{11}$$

where $B_\varepsilon = (B_a - B)$ is the approximation error.

3.2 Construction of H via Tangential Nevanlinna-Pick Interpolation

As before, we begin with a finite dimensional coprime factorization $P = D^{-1}N$. Then,

$$B = (I - HM)P = (I - HM)D^{-1}N. \tag{12}$$

Since (D, N) is a left-coprime pair, B is in $\mathcal{H}_\infty^{\mu\times\nu}$ if and only if $F = (I - HM)D^{-1}$ is in $\mathcal{H}_\infty^{\mu\times\mu}$. Assume that the poles of P are distinct. Then, for each pole p_j we have a vector $y_j \in \mathbb{C}^\mu$ with $\|y_j\| = 1$ satisfying $D(p_j)y_j = 0$. Hence, for all $F \in \mathcal{H}_\infty^{\mu\times\mu}$ we have

$$(I - H(p_j)M(p_j))y_j = F(p_j)D(p_j)y_j = 0.$$

Define $x_j := M(p_j)y_j$. Then, $H(p_j)$ must satisfy

$$H(p_j)x_j = y_j\ ,\ \ j = 1,\ldots,n. \tag{13}$$

Thus, considering the norm condition (10), we want to find an $H \in \mathcal{H}_\infty^{\mu\times\mu}$ with the smallest possible $\mathcal{H}_\infty$-norm satisfying (13). This is a tangential Nevanlinna-Pick interpolation problem, whose solution can be found in Ball et al. (1990). Since there are finitely many interpolation conditions, the central solution H is in $\mathcal{S}^{\mu\times\mu}$.

Once the minimum norm $H \in \mathcal{H}_\infty^{\mu\times\mu}$, satisfying (13) is designed, we need to check (10), which depends on the choice of C. Therefore, in this step we design C to satisfy (if possible)

$$\|W_m T\|_\infty < \|H\|_\infty^{-1}. \tag{14}$$

The left hand side of (14) can be minimized over all C stabilizing P; clearly, the smallest achievable norm depends on how large the uncertainty magnitude $\|W_m(j\omega)\|$ is.

Remark. For the nominal system with $\Delta = 0$ and $B_\varepsilon = 0$ the closed-loop transfer function from r to y is MT, as in the classical Smith predictor. For good "nominal tracking performance" we also consider a sensitivity weight W_1 and design C to minimize the mixed sensitivity level

$$\gamma(C) := \left\| \begin{bmatrix} W_1 S \\ W_2 CS \\ W_3 T \end{bmatrix} \right\|_\infty \tag{15}$$

over all controllers C stabilizing P. The solution is given by the `mixsyn` command of Matlab. With $W_3 = W_m$, if $\gamma(C) < \|H\|_\infty^{-1}$, then C satisfies (10). Furthermore, when $B_\varepsilon \neq 0$, let W_2 be a stable matrix satisfying $\|W_2(j\omega)\| > \|B_\varepsilon(j\omega)\|$, for all ω, then a sufficient condition for (11) is

$$\gamma(C) < 1/(1 + \|H\|_\infty). \tag{16}$$

Note that these are conservative robust stability conditions. For the example given in the next section, we investigate the largest allowable uncertainty magnitude $\|W_m(j\omega)\|$ when C, H and B_a are designed.

4. NUMERICAL EXAMPLE

Let M and $P = D^{-1}N$ be defined as

$$M = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-hs} & e^{-3hs} \\ -1 & e^{-2hs} \end{bmatrix},\quad D = \begin{bmatrix} \dfrac{s^2-2s+2}{s^2+2s+2} & 0 \\ 0 & \dfrac{s-1}{s+1} \end{bmatrix}$$

with $h = 0.15$ sec, and

$$N = \frac{1}{s+3}\begin{bmatrix} \dfrac{2(s+10)}{s+2} & \dfrac{2(s+5)}{s+1} \\ \dfrac{-2(s-5)}{s+2} & \dfrac{2(s+10)}{s+1} \end{bmatrix}.$$

4.1 Design of H using Tangential Nevanlinna-Pick Interpolation

The set of $\mathbb{C}_+$ poles of D^{-1} are $\mathscr{P}_u = \{1-i,\ 1+i,\ 1\}$. The corresponding problem data is

$$y = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^{\mathrm{T}},\quad x = \begin{bmatrix} 0.6018 - i0.0909 & -0.7071 \\ 0.6018 + i0.0909 & -0.7071 \\ 0.4509 & 0.5238 \end{bmatrix}^{\mathrm{T}}$$

x_j is the j^{th} column of x and similarly for y_j, $j = 1, 2, 3$. Using the tangential Nevanlinna-Pick interpolation formula given in Ball et al. (1990), we compute $H \in \mathcal{S}^{2\times 2}$ satisfying $H(p_j)x_j = y_j$ with the minimum $\mathcal{H}_\infty$ norm:

$$H = \begin{bmatrix} \dfrac{(s+8.56)(s-0.08)}{1.4636\ d_H(s)} & \dfrac{(s-39.11)(s-0.096)}{6.6754\ d_H(s)} \\ \dfrac{(s+4.4)(s+0.32)}{0.7566\ d_H(s)} & \dfrac{(s+20.13)(s+0.36)}{3.4503\ d_H(s)} \end{bmatrix}$$

where $d_H(s) = (s + 5.424)(s + 0.3394)$, and the $\mathcal{H}_\infty$-norm is $\|H\|_\infty = 1.5228$.

4.2 Approximation of B

Recall that $B = (I - HM)D^{-1}N$ is infinite dimensional. Let $M_a \in \mathcal{S}^{2\times 2}$ be the matrix obtained by replacing the time delays in M by their 3^{rd} order Padé approximations. Then, an unstable approximation of B, denoted as $B_{a,u}$, can be obtained by using

$$B_{a,u} = (I - HM_a)D^{-1}N.$$

Next, decompose this system into its stable and anti-stable parts using the `stabsep` command of Matlab:

$$B_{a,u}(s) = B_a(s) + B_u(s) \tag{17}$$

where the anti-stable part is $B_u(s) = 10^{-8} \times \widetilde{B}_u$ with

$$\widetilde{B}_u = \begin{bmatrix} \dfrac{1.43(s^2-1.92s+1.95)}{(s-1)(s^2-2s+2)} & \dfrac{6.03(s^2-1.99s+1.99)}{(s-1)(s^2-2s+2)} \\ \dfrac{2.53(s^2-1.96s+1.99)}{(s-1)(s^2-2s+2)} & \dfrac{10.7(s^2-1.99s+2)}{(s-1)(s^2-2s+2)} \end{bmatrix}.$$

By neglecting B_u, a stable finite dimensional approximation of B is determined from (17) as B_a, where each element contains a 13^{th} order stable transfer function. The Bodé plots of B and B_a are given in Fig. 2.

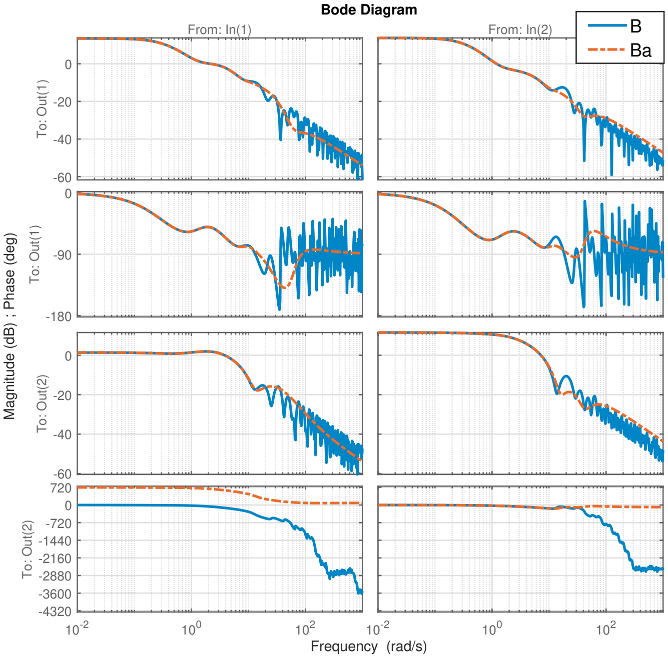

Fig. 2. Bodé plots of B and 13^{th} order B_a.

Magnitude of the error $B_\varepsilon = (B_a - B)$ is shown in Fig. 3. An upper bound $|W_b(j\omega)|$ with

$$W_b(s) = \frac{0.001(2s+1)^2(1+s/400)}{(1+s/20)^2(1+s/10)} \tag{18}$$

is used in Section 4.3 to design C and to estimate the maximum allowable uncertainty.

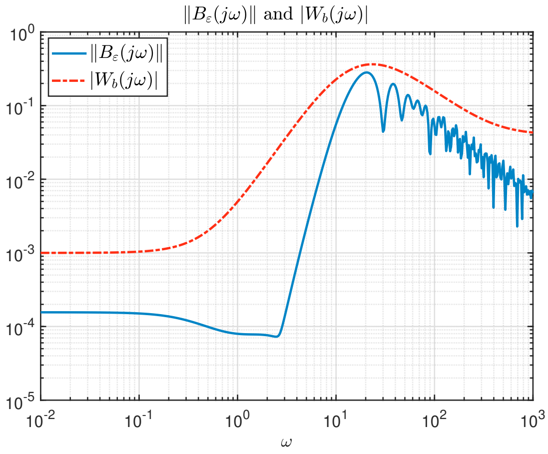

Fig. 3. Maximum singular value of $B_\varepsilon(j\omega)$ and an upper bound $|W_b(j\omega)|$.

4.3 Design of C

As mentioned above, a simple way to design C stabilizing P and achieving robust stability with good reference tracking is to minimize the mixed sensitivity (15). We use the Matlab command `mixsyn(Po,W1,Wb, [])` with W_b given in (18) and

$$W_1(s) = 2 \text{ diag}\left\{\frac{1}{s+\epsilon_1}, \frac{1}{s+\epsilon_2}\right\}, \tag{19}$$

$\epsilon_1 = 10^{-5}$ and $\epsilon_2 = 10^{-4}$. The resulting controller C is 15th order and its Bodé plots are shown in Fig. 4.

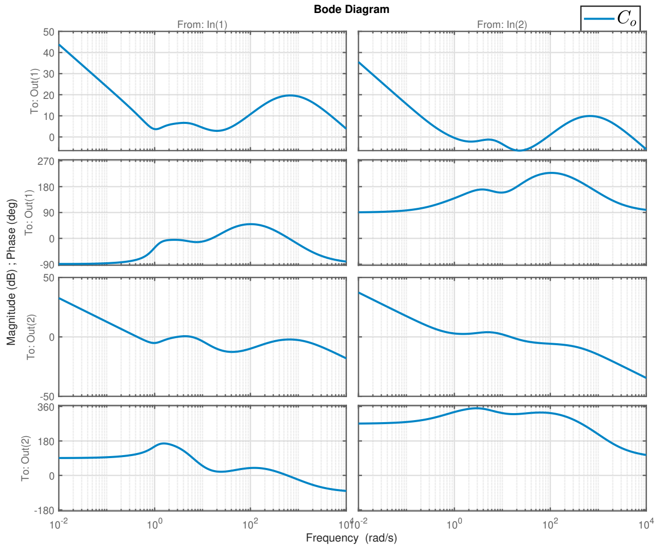

Fig. 4. Bodé plots of $C(j\omega)$.

Recall the inequality (11). The largest allowable uncertainty magnitude $\|W_m(j\omega)\|$ is

$$\|W_m(j\omega)\| < \frac{(1 - \|W_b(j\omega)C(j\omega)S(j\omega)\|)}{\|H(j\omega)T(j\omega)\|}. \tag{20}$$

The impact of the approximation error B_ε (its upper bound W_b) on the allowable W_m is shown in Fig. 5.

An interesting point to note is that in this example $M(0) \neq I$. In this case, we need to pass the reference $r(t)$ through a normalization gain $M(0)^{-1}$ before applying it to the system. This is important in particular for tracking step-like reference signals in $r(t)$: here C is designed to satisfy $T(0) \approx I$. Since the transfer function from r to y in the nominal case ($\Delta = 0$ and $B_\varepsilon = 0$) is $M(s)T(s)$, its value at $s = 0$ is $M(0)$; hence, with the reference scaling $M(0)^{-1}$, we capture the identity at $s = 0$. Obviously, $M(0)$ could be absorbed into $P(s)$ in the system modeling phase:

$$M(s)P(s) = M(s)M(0)^{-1}M(0)P(s) = M_1(s)P_1(s)$$

where $M_1(s) = M(s)M(0)^{-1}$, with $M_1(0) = I$ and $P_1(s) = M(0)P(s)$. Thus, for the class of plants considered in the paper, we may assume $M(0) = I$ without loss of generality.

Time domain responses of the closed-loop feedback system outputs $y(t)$ and $u(t)$, with the above finite dimensional

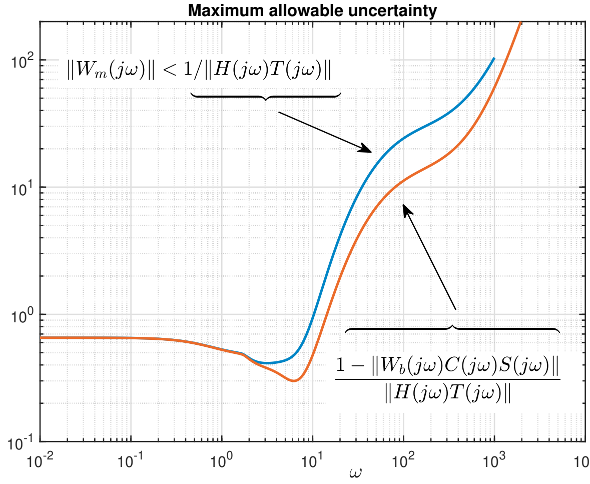

Fig. 5. Upper bound of allowable uncertainty $\|W_m(j\omega)\|$.

controller and the nominal plant, are shown in Fig. 6 and Fig. 7. In these simulations, the external inputs $r(t)$ and $v(t)$ are taken as follows (where $\mathtt{u}_\mathtt{s}(t)$ is the unit step function)

$$r(t) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \mathtt{u}_\mathtt{s}(t),$$

$$v(t) = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \Big(\mathtt{u}_\mathtt{s}(t) - \mathtt{u}_\mathtt{s}(t-2)\Big)$$

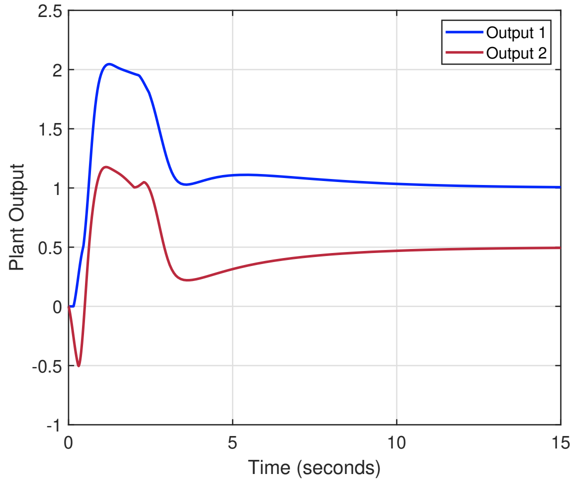

Fig. 6. Plant outputs $y(t) = [y_1(t) \quad y_2(t)]^\mathrm{T}$.

5. CONCLUSIONS

A two-parameter controller is designed for MIMO distributed parameter systems with finitely many unstable poles. The proposed method is an extension of the classical Smith predictor-based design for time delay systems, see Yegin and Ozbay (2023). The nominal plant is assumed to be given by a transfer function matrix MP, where M is inner and P is finite dimensional. We first design a minimum norm stable H as in Fig. 1 such that

$$B = (I - HM)P$$

is stable, then approximate B with a finite dimensional stable B_a . In the next step, we design C stabilizing P using the classical $\mathcal{H}_\infty$ control techniques so that the effects of approximation errors and uncertainties are taken care of. Approximation of B is for practical implementation considerations; if the infinite dimensional B can be implemented precisely, then there is no need for B_a, i.e., $B_\varepsilon = 0$.

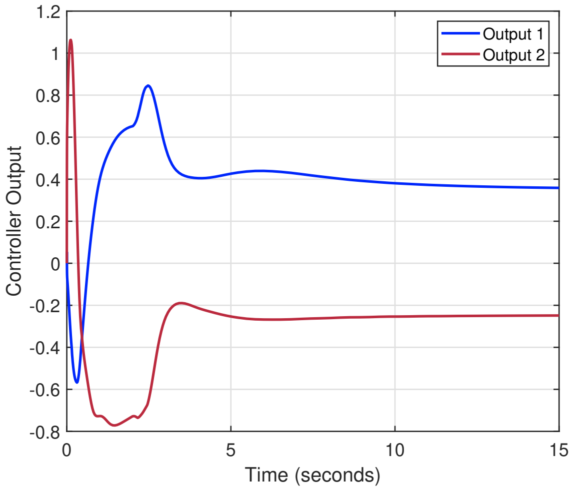

Fig. 7. Controller outputs $u(t) = [u_1(t) \quad u_2(t)]^\mathrm{T}$.

In the numerical example, for implementing the predictor filter $B = (I - HM)P$, a finite dimensional B_a is obtained by using Padé approximations and then separating the stable and anti-stable parts. This is a very simple and naive approach. More rigorous methods, e.g., TF-IRKA, can be applied directly, see Antoulas et al. (2020) and the references therein.

For simplicity of the exposition, the numerical example is taken as a two input two output system; however, the method presented in Section 3 is also applicable to any non-square MIMO system satisfying the structural assumptions of Section 2.

Clearly, the method presented here can also be applied to the class of plants in the form PM, where P is finite dimensional and M is inner, as for example in MIMO systems with delays in the input channels. In that case, dual coprime factorizations and dual Bézout equations are needed. Specifically, the predictor feedback becomes $B = P(I - MH)$ and H is moved from the plant's output to the output of the controller. Then, the design of H and C are essentially the same as in Section 3. The details are left to the expanded version of the paper.

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Appendix A. PROOF OF THEOREM 1.

Let $P = N_r D_r^{-1}$ be any right-coprime factorization of P, and let $C = \tilde{Y}^{-1}\tilde{X}$ be any left-coprime factorization of the controller C that stabilizes P. Then

$$\tilde{Y} D_r + \tilde{X} N_r = U \tag{A.1}$$

is unimodular, i.e., $U^{-1} \in \mathcal{S}^{\nu\times\nu}$. With these factorizations, $T = N_r U^{-1}\tilde{X}$. Write the forward-path controller block as

$$(I + CB_a)^{-1}C = (\tilde{Y} + \tilde{X}(B + B_\varepsilon))^{-1}\tilde{X} \ , \tag{A.2}$$

where B is as in (5). Then we have

$$(\tilde{Y} + \tilde{X}B + \tilde{X}B_\varepsilon)D_r + \tilde{X}HMN_r = U + \tilde{X}B_\varepsilon D_r \ . \tag{A.3}$$

Define $V \in \mathcal{H}_\infty^{\nu\times\nu}$ as

$$V := U + \tilde{X}(HM_\delta N_r + B_\varepsilon D_r) \ . \tag{A.4}$$

If $B_\varepsilon = 0$ and $M_\delta = 0$, then $V = U$. In the presence of uncertainties, the closed-loop maps from the inputs (r, v) to the outputs (y, u) are:

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} (M + M_\delta)N_r \\ D_r \end{bmatrix} V^{-1} \begin{bmatrix} \tilde{X} & (\tilde{Y} + \tilde{X}B_a) \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} - \begin{bmatrix} 0 \\ v \end{bmatrix} \tag{A.5}$$

where $B_a = B + B_\varepsilon$. Since U is unimodular, the closed-loop system is stable if and only if V is unimodular, equivalently, $(I_\nu + \tilde{X}(HM_\delta N_r + B_\varepsilon D_r)U^{-1})$ is unimodular, equivalently,

$$(I + HM_\delta N_r U^{-1}\tilde{X} + B_\varepsilon D_r U^{-1}\tilde{X}) = (I + HM_\delta T + B_\varepsilon CS)$$

is unimodular. A sufficient condition is (9). □