Maximizing Lifetime per Unit Cost in Wireless Sensor Networks

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***Abstract*— This paper proposes a new performance metric, called the lifetime per unit cost, to measure the utilization efficiency of the sensors in a wireless sensor network. The lifetime per unit cost is defined as the average network lifetime divided by the number of sensors enabled in the network. We analyze the average lifetime per unit cost of an event-driven linear wireless sensor network with different transmission schemes and different sensor placement schemes. We find that forwarding packets to the nearest neighbor toward the gateway node is the optimal transmission scheme for the network employing the greedy sensor placement. Unlike the network lifetime, which increases monotonically as the number of sensors increases, the lifetime per unit cost of the network decreases when the number of sensors is large. Numerical results show that the optimal number of sensors, which maximizes the lifetime per unit cost of the network, increases as the event arrival rate increases or the sensing energy consumption decreases.**

***Key Words:* Lifetime per unit cost, sensor placement, wireless sensor network.**

I. Introduction

Wireless sensor networks have captured considerable attention recently due to their enormous potential for both consumer and military applications. A wireless sensor network consists of large number of sensors, which are low-cost, low-power, energy-constrained nodes with limited computation and communication capability. Sensors are dedicated to monitoring certain phenomenon within their sensing regions and reporting to the powerful gateway nodes where the end-user can access the data. There are two kinds of reporting methods: event-driven and demand-driven [2]. In the event-driven reporting like traffic accident monitoring, sensors act when triggered by the event of interest. In the demand-driven reporting like continuous temperature monitoring, sensors remain silent until they receive the request from the gateway nodes. The sensors in the network can be deployed either randomly or deterministically. The random sensor deployment is suitable for battlefield or disaster areas while the deterministic deployment can be used in friendly or accessible environment [1]. Generally, less sensors are required in the deterministic deployment than the random deployment to perform the same task.

A. Related Work

One of the promising research challenges in the area of wireless sensor networks is sensor placement. There have been extensive research efforts on the design of sensor placement schemes for different purposes. Dhillon and Chakrabarty [3] propose two algorithms to optimize the number of sensors and their placement for effective coverage and surveillance purposes under the constraint of probabilistic sensor detections and terrain properties. Ganesan *et. al.* [4] jointly optimize the sensor placement and the transmission structure in a one-dimensional data-gathering sensor network. Their approach is aimed at minimizing the total power consumption under distortion constraints. Kar and Banerjee [5] address the optimal sensor placement to ensure connected coverage in sensor networks.

The network lifetime is a critical concern in the design of sensor networks. There is a growing body of literature on the study of sensor network lifetime (see [6], [7] and references therein). Sensor placement schemes that maximize the lifetime has been addressed for different sensor networks. Dasgupta *et. al.* [8] consider a sensor network consisting of two types of nodes, the sensor nodes and the relay nodes. They propose an algorithm to find a placement and role assignment to maximize the lifetime of such network. Hou *et. al.* [10] address the energy provisioning and relay node placement in a two-tiered wireless sensor network. In [9], the placement of the gateway node is studied to maximize the lifetime of a two-tiered wireless sensor network. Cheng *et. al.* [11] propose an energy-aware sensor placement in order to maximize the lifetime or minimize the application-specific cost of many-to-one linear and planar sensor networks.

B. Motivation and Our Contribution

Studying the previous work on maximizing the network lifetime, we notice that most of the available results assume that all the sensors in the network are enabled at the same time. However, we find that as the number of enabled sensors N increases, the network lifetime increases less than linearity when N is large. Hence, enabling a large number of sensors at one time is not an efficient way to utilize the sensors. We thus propose a new performance metric, called the lifetime per unit cost, which is defined as the network lifetime divided by the number of enabled sensors in the network. The lifetime per unit cost shows the rate at which the lifetime network increases as the number of sensors increases. It is an indicator of the utilization efficiency of the sensors in a wireless sensor network. The most efficient way to deploy a large sensor network is to divide all the sensors into small groups, each with the optimal number of sensors which achieves the maximum lifetime per unit cost, and enable one group each time when the previous group malfunctions. It is thus important to analyze the lifetime per unit cost of a network.

This paper studies the lifetime per unit cost of an event-driven linear wireless network with different transmission and sensor placement schemes. Our network model is more general than that in [11] since we take into account not only the transmission energy consumption of each sensor but also other energy consumption, such as sensing energy, circuitry energy and battery draining. We also study the effect of the event arrival rate on the maximum lifetime per unit cost and the optimal number of sensors of the network.

The rest of the paper is organized as follows. Section II describes our network model and defines the lifetime per unit cost. Section III derives the lifetime per unit cost of an event-driven linear sensor network with different transmission and sensor placement schemes. Section IV provides some numerical examples while Section V concludes this paper.

II. Network Model

Let us consider an event-driven linear sensor network with N homogeneous sensors. Let s_i denote the i-th sensor in the network. All sensors have the same amount of initial energy E_{in} and a maximum sensing range D due to the power limit. The sensors are deployed along a straight line of length L with the gateway node connected directly to s_0 at the left end. Let d_i denote the distance between s_i and the gateway node (s_0) where $0 = d_0 < d_1 < d_2 < \ldots < d_{N-1} < L$. The i-th sensor s_i is dedicated to monitoring and reporting the event in the range between s_i itself and its nearest right neighbor s_{i+1}. Specially, the edge sensor s_{N-1} is responsible for the remaining area from d_{N-1} to L. To ensure the continuous coverage of the network, the distance between adjacent sensors should be less than the maximum sensing distance, i.e., $d_i - d_{i-1} < D$ for $i = 1, 2, \ldots, N-1$ and $L - d_{N-1} < D$.

When the event of interest occurs in the monitoring region of a sensor, the sensor generates an equal-sized packet and sends it to the gateway node according to a certain transmission scheme. The simplest transmission scheme, denoted by $\mathcal{X}(1)$, is to send the packet to the nearest left neighbor. That is, the packets from s_k are relayed via $s_{k-1}, s_{k-2}, \ldots, s_1$ to s_0. More generally, the packets can be sent to the l-th nearest left neighbor. Let us denote it by $\mathcal{X}(l)$. That is, the packets from s_k are replayed via $s_{k-l}, s_{k-2l}, s_{k-3l}, \ldots$ to s_0.

We assume that the event of interest is a Poisson random process with mean λ per unit time, i.e.,

$$\Pr\{n \text{ events occur during one unit time}\} = e^{-\lambda}\frac{\lambda^n}{n!}, \quad (1)$$

and the events occur randomly in the coverage area of the network. Hence, the average number of events which occur in an area of Δd is given by

$$\overline{n}(\Delta d) = \frac{\lambda}{L}\Delta d. \quad (2)$$

Let e_{tx} denote the energy required to transmit one packet over the distance of 1m. If the path loss exponent is γ, the energy consumed to transmit the packet over a distance of d can be written as

$$E_{tx}(d) = e_{tx}d^{\gamma} \quad (3)$$

where $2 \leq \gamma \leq 4$. Let e_s denote other energy consumption, such as sensing energy, circuitry energy and battery draining, required to keep the sensor alive during a unit time.

The network lifetime is defined as the amount of time until any sensor runs out of energy [11], which is equivalent to the minimum lifetime of the sensors, i.e.,

$$\overline{T} = \min_k \overline{T_k} \quad (4)$$

where $\overline{T_k}$ is the average lifetime of s_k. The average lifetime per unit cost is defined as the average network lifetime $\overline{T}$ divided by the number of enabled sensors N in the network, i.e.,

$$\overline{LC} = \frac{\overline{T}}{N}. \quad (5)$$

The average lifetime per unit cost is a useful performance measure, which shows the rate at which the average network lifetime increases as the number of sensors increases. It also characterizes the utilization efficiency of the sensors in the network.

III. Lifetime per Unit Cost Analysis

The average energy consumption of each sensor depends on the transmission scheme $\mathcal{X}(l)$ used in the network. Consider the simplest case $\mathcal{X}(1)$ in which packets are always forwarded to the nearest left neighbor. Then the sensor s_k has to transmit a packet to s_{k-1} whenever an event occurs on its right side. Using (2) and (3), we can obtain the average energy consumption of s_k $(k = 1, \ldots, N-1)$ per unit time as

$$\overline{E_k} = e_s + e_{tx}(d_k - d_{k-1})^{\gamma}\frac{\lambda}{L}(L - d_k). \quad (6)$$

As expected, the average energy consumption increases as the mean event arrival rate λ or the sensing energy e_s increases.We also notice that $\overline{E_k}$ increases as the distance between s_k and its nearest left neighbor s_{k-1} increases or the distance between s_k and s_0 decreases. Thus, to balance the energy consumption among the sensors, we should place the sensors with smaller indexes much closer than those with larger indexes.

Next, we consider a general transmission scheme $\mathcal{X}(l)$, in which the packets are forwarded to the l-th nearest left neighbor. Notice that s_j forwards packets directly to s_0 for $1 \leq j \leq l$. Then the average energy consumption of s_k $(k = 1, \ldots, N-1)$ per unit time can be obtained as

$$\overline{E_k} = e_s + e_{tx}(d_k - d_{k-l})^{\gamma}\frac{\lambda}{L}\sum_{n=0}^{\lfloor\frac{N-k-1}{l}\rfloor}(d_{k+ln+1} - d_{k+ln}) \quad (7)$$

where $d_k = 0$ for $k \leq 0$ and $\lfloor x \rfloor$ is the largest integer smaller than x.

A. Greedy Sensor Placement

From [11], we know that given the number of sensors N, the maximum network lifetime is achieved by the greedy sensor placement scheme, in which all sensors run out of energy at the same time. That is, the average unused energy of the sensors after the lifetime expires is zero. We can thus show that the average lifetime per unit cost of our network with greedy sensor placement is given by [12],

$$\overline{LC} = \frac{E_{in}}{N\overline{E_k}} \tag{8}$$

where $\overline{E_k}$ is given in (7) for different transmission schemes $\mathcal{X}(l)$.

Proposition: $\mathcal{X}(1)$ is the optimal transmission scheme for the network employing the greedy sensor placement scheme.

The proof of the proposition is followed from (3). When the location of an event is fixed, the total energy required for the sensors to report this event to the gateway node in $\mathcal{X}(l)$ is more than that in $\mathcal{X}(1)$ since $x_1^\gamma + x_2^\gamma \leq (x_1 + x_2)^\gamma$ when $x_1, x_2 \geq 0$ and $\gamma \geq 1$. Since the total amount initial energy in the network is fixed NE_{in}, $\mathcal{X}(1)$ is able to report more events than $\mathcal{X}(l)$ and thus achieves a longer lifetime. Hence, sending packets to the nearest neighbor toward the gateway node is the optimal transmission scheme for the network employing the greedy sensor placement scheme.

To find the optimal number of sensors which maximizes the average lifetime per unit cost and the corresponding sensor placement, we formulate the problem as follows

$$\begin{aligned} &\text{minimize } N\overline{E_k} \text{ which is given by (7)} \\ &\text{subject to: } \overline{E_2} = \ldots = \overline{E_{N-1}}, \\ &\qquad 0 < d_1 < D, \\ &\qquad 0 < d_k - d_{k-1} < D \text{ for } k = 2, \ldots, N-1, \\ &\qquad 0 < L - d_{N-1} < D. \end{aligned} \tag{9}$$

The multi-variate non-linear optimization problem (9) can be readily solved numerically with the aid of MATLAB.

B. Uniform Sensor Placement

Now, let us consider another sensor placement scheme where the sensors are placed with equal distance in between. Since the lifetime of network with uniform placement is limited by the lifetime of the first sensor s_1, we can show that the average lifetime per unit cost of the linear network with the uniform placement and the transmission scheme $\mathcal{X}(1)$ is given by

$$\overline{LC}(N) = \frac{E_{in}}{N\overline{E_1}} = \frac{E_{in}/N}{e_s + \lambda e_{tx}\left(\frac{L}{N}\right)^\gamma \left(1 - \frac{1}{L}\right)}. \tag{10}$$

Differentiating (10) and setting the derivative to zero, we obtain the number of sensors N^* which maximizes the average lifetime per unit cost of the network (10) as

$$N^* = L\left(\frac{(\gamma-1)\lambda e_{tx}(L-1)}{Le_s}\right)^{\frac{1}{\gamma}}. \tag{11}$$

Since the number of sensors is an integer, the optimal number of sensors is

$$N^{**} = \begin{cases} \lfloor N^* \rfloor, & \overline{LC}(\lfloor N^* \rfloor) > \overline{LC}(\lceil N^* \rceil), \\ \lceil N^* \rceil, & \text{otherwise}, \end{cases} \tag{12}$$

where $\lceil x \rceil$ is the smallest integer larger than x. We find that the optimal number of sensors N^{**} increases as the event arrival rate λ increases or the sensing energy consumption e_s decreases. The optimal N^{**} increases more rapidly when e_s is small.

Notice that forwarding the packets to the nearest neighbor toward the gateway node $\mathcal{X}(1)$ is not the optimal transmission scheme for the network employing the uniform sensor placement scheme. Recall that all the senors run out of energy at the same time in the greedy placement. However, the lifetime of the network with uniform placement and $\mathcal{X}(1)$ is limited by the lifetime of the first sensor s_1. That is, the average wasted energy (the unused energy after the lifetime expires) of the sensors is not zero. That is, when s_1 dies, other sensors still have energy left for sensing or reporting. Hence, more events can be reported if sensors can send their packets a little bit farther away so that the traffic load of s_1 is alleviated. For example, if s_2 can send a portion of its packets directly to the gateway node instead of forwarding all the packets to s_1, a longer lifetime can be achieved.

IV. Numerical Examples

This section provides some numerical examples to show the lifetime per unit cost of the network with different transmission schemes $\mathcal{X}(l)$ and different sensor placement schemes. We also study the effect of the event arrival rate λ and the sensing energy consumption e_s on the maximum lifetime per unit cost of the network and the corresponding optimal number of sensors. In all the figures, the the energy required to transmit one packet over the distance of 1m is $e_{tx} = 1$ unit. All the energy quantities are normalized by e_{tx}. The initial energy of each sensor is $E_{in} = 10$ units. The network coverage area is $L = 10$m and the maximum sensing range of each sensor is $D = 2$m. The path loss exponent is assumed to be $\gamma = 2$.

Fig. 1 compares the average lifetime per unit cost $\overline{LC}$ of the network employing $\mathcal{X}(1)$ with different sensor placement schemes, the greedy placement and the uniform placement. As expected, the greedy sensor placement outperforms the uniform placement. The performance gain of the greedy placement diminishes when the number of sensors is large. Unlike the network lifetime, which increases monotonically as the number of sensors N increases, the average lifetime per unit cost $\overline{LC}$ increases first and then decreases. As the event arrival rate λ increases, the average lifetime per unit cost $\overline{LC}$ decreases and so does the rate at which it decreases.

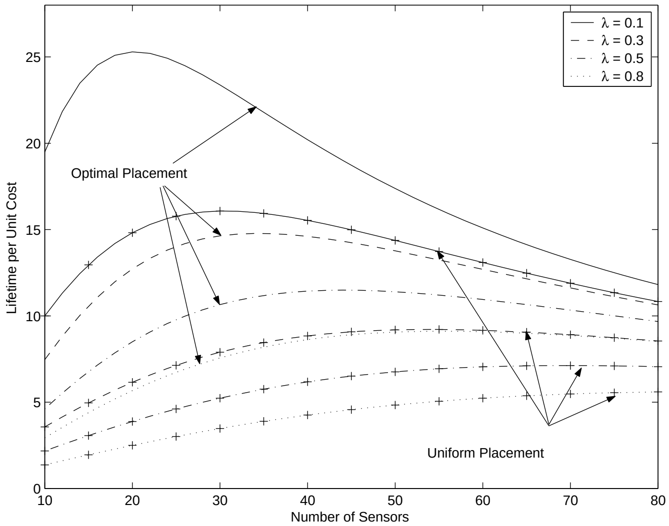

Fig. 1. Average lifetime per unit cost comparison with different sensor placement schemes. $\lambda = \{0.1, 0.3, 0.5, 0.8\}$, $E_s = 0.01$, $\mathcal{X}(1)$.

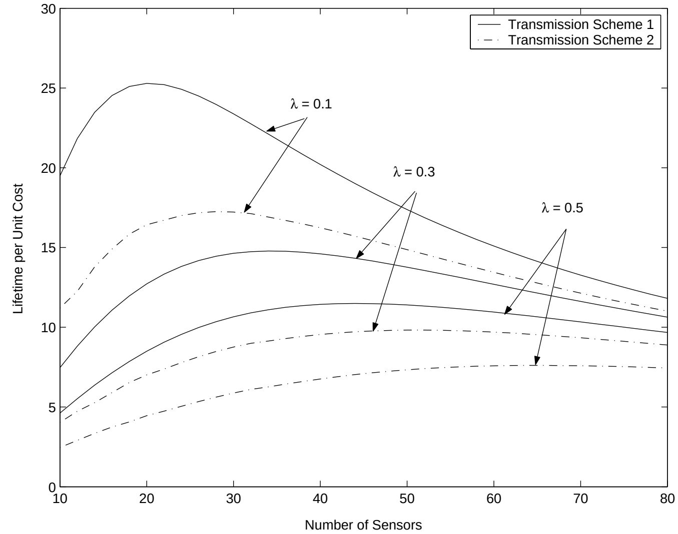

Fig. 3. Average lifetime per unit cost comparison with different transmission schemes. $\lambda = \{0.1, 0.3, 0.5\}$, $E_s = 0.01$.

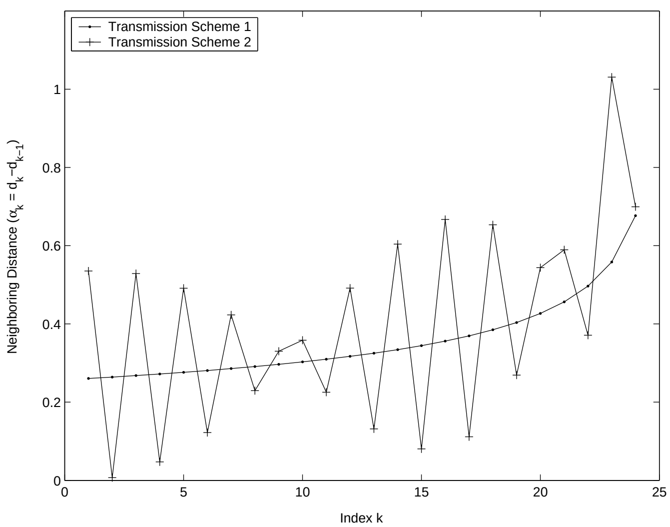

Fig. 2. Optimal greedy sensor placement with different transmission schemes. $N = 25$.

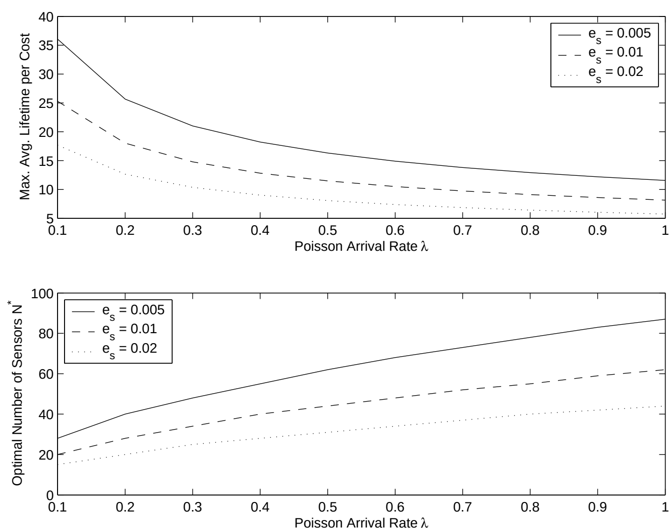

Fig. 4. Maximum average lifetime per unit cost and the optimal number of sensors of the optimal greedy placement. $E_s = \{0.005, 0.01, 0.02\}$, $\mathcal{X}(1)$.

Fig. 2 shows the optimal greedy sensor placement for the network employing different transmission schemes, $\mathcal{X}(1)$ and $\mathcal{X}(2)$. In $\mathcal{X}(1)$, the distance between adjacent sensors $(d_k - d_{k-1})$ increases as the sensor index k increases. It is interesting to notice that the distance between adjacent sensors in $\mathcal{X}(2)$ fluctuates around that in $\mathcal{X}(1)$.

Fig. 3 compares the lifetime per unit cost performance of the network employing the greedy sensor placement with different transmission schemes, $\mathcal{X}(1)$ and $\mathcal{X}(2)$. As expected, we find that $\mathcal{X}(1)$ outperforms $\mathcal{X}(2)$. Let us consider a network with total $N = 80$ sensors employing greedy placement and $\mathcal{X}(1)$. From this figure, we find that when $\lambda = 0.1$, enabling all the 80 sensors at the same time achieves the average lifetime per unit cost $\overline{LC}(80) = 11.8$ and the corresponding average lifetime $\overline{T}(80) = 80 * \overline{LC}(80) = 945$ units. However, if we divide the 80 sensors into four groups each with 20 sensors and enable one group each time, then we can achieve an accumulated average lifetime $\overline{T} = 4 * \overline{T}(20) = 4 * 20 * \overline{LC}(20) = 2023$ units, which is much larger than $\overline{T}(80)$. Hence, when the total number of sensors is large, a longer lifetime can be achieved by dividing the sensors into small groups and enable one groups each time. Fig. 4 can be used to show the optimal number of sensors to enable at each time.

Fig. 4 plots the maximum average lifetime per unit cost achieved by the optimal greedy placement and the corresponding optimal number of sensors N^* enabled in the network. As the event arrival rate λ or the sensing energy e_s increases, the maximum average lifetime per unit cost decreases. As λ increases, the optimal number of sensors N^* increases, and it increases much more rapidly when the sensing energy e_s is smaller. As the sensing energy e_s increases, the optimal number of sensors N^* decreases. Hence, when e_s is large or the event arrival rate λ is low, it is desired to enable a few sensors at each time so that the lifetime per unit cost is maximized.

V. Conclusion

Motivated by the observation that a longer network lifetime can be achieved by dividing sensors into small groups and enabling one group each time, we proposed a new performance metric, the lifetime per unit cost, which is defined as the network lifetime divided by the number of sensors enabled. We analyzed the lifetime per unit cost of an event-driven linear network with different transmission and sensor placement schemes. We find that forwarding the packets to the nearest neighbor toward the gateway node is the optimal transmission scheme for the network employing the greedy sensor placement. However, it is not true for the network employing the uniform sensor placement. We also investigated the effect of the event arrival rate and other energy consumption on the lifetime per unit cost of the network. When the event arrival rate λ is low or other energy consumption e_s is large, it is more efficient to enable a portion of sensors each time in a large sensor network.

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