Metrics for Characterizing the Quality of Service of Multicommodity Flows in Multi-Channel Multi-Radio Multi-Rate Wireless Mesh Networks

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Abstract— **This paper proposes important performance metrics for characterizing the quality of service of multicommodity flows in multi-channel, multi-radio, multi-rate (MC-MR^2) wireless mesh networks. The term "multicommodity" implies different types of applications like audio, video, text etc. being serviced simultaneously by the network. Traditionally, the capacity of the links in a mesh network is assumed to be deterministic. However, this is practically not the case as links are often subject to environmental variations and failures. In such cases, data communication can occur at variable rates and as a result providing proper QoS to applications becomes a challenge. For a particular multicommodity flow demand, we analytically model and evaluate the following three metrics. 1. The expected multicommodity flow achieved by the network 2. The expected multicommodity flow that is achieved by the network under given cost constraints and 3. The quantitative value of the importance of each link in the network with respect to the multicommodity flow demand. In addition, we also state the lower bounds for the first two metrics. A multicommodity flow demand is represented by a rate (capacity) vector of k applications being simultaneously served by the network. Our methodologies for metric evaluation are based on concepts from the theory of probabilistic network flows and run in polynomial time. We corroborate our theory with simulations. Our metrics will be of great significance to network designers and application service providers who could use them to judge the suitability of applications for a given network and/or spend time and effort in maintaining certain high importance links for better QoS support. To the best of our knowledge, this is the first work to address the performance of multicommodity flows in MC-MR^2 wireless meshes.**

I. Introduction

Data communication in multi-hop wireless networks has been an active area of research in the last decade. In the past few years many commercial technologies related to multi-hop wireless networks have emerged. One such technology is wireless mesh networks (WMN's). WMN's consist of nodes that are either mesh routers or mesh clients. Mesh clients could be stationary or minimally mobile and can form a network with themselves and the mesh routers. The nodes of a mesh network are generally not power constrained and therefore not reliant on batteries[10]. One of the main objectives of multi-hop WMN's is to extend the coverage range without sacrificing capacity. Multi-hopping helps in achieving higher throughput without sacrificing effective radio-range via shorter link distances, less interference between the nodes, and more frequency re-use. These networks also serve general purpose user applications and contribute to low-cost, high-bandwidth, seamless multi-hop interconnection service with limited number of Internet entry points. The practical use and economic viability of wireless mesh networks make it a potent research area.

The use of multi-channel, multi-radio (MC-MR) mesh nodes and the study of multi-rate MAC protocols are two new aspects of present WMN research. It has been shown that the network capacity drops off as the number of nodes is increased in single-channel wireless networks. The use of multiple radios is shown to be affordable and as a result the current trend is to equip mesh nodes with multiple radios, each tuned to a distinct orthogonal channel. The use of multiple radios results in the increase in the overall network capacity by employing concurrent transmissions in the network. Multi-rate protocols study the throughput and fairness issues that arise when adaptive modulation schemes modify link data rates dynamically to account for the variation in signal-noise ratio (SNR). Commodity wireless cards, which connect wireless mesh nodes, are equipped with a feature to transmit at multiple transmission rates. Due to the support available from the industry and the research community, MR$^2-$MC WMN's are likely to be extremely popular in the near future. [13] [14]

In this paper we address the problem of QoS evaluation of multicommodity flows in multi-channel, multi-radio, multi-rate wireless mesh networks. Multicommodity flows include multiple applications like audio, video, text etc. being served simultaneously by the network. All these applications require particular data rates(capacities) for proper service. However, due to link capacity constraints and the variation of link capacities due to environmental phenomena like fading and shadowing, the required data rates for various applications are not guaranteed to be served. In such cases, the expected data rate achieved for each application characterizes the QoS for the multicommodity flow.

We propose three metrics for characterizing the QoS of multicommodity flows in MR$^2-$MC wireless mesh networks. For a particular multicommodity flow demand, we propose the following. 1. The expected multicommodity flow achieved by the network 2. The expected multicommodity flow that is achieved by the network under given cost constraints and 3. The quantitative value of the importance of each link in the network with respect to the multicommodity flow demand. A multicommodity flow demand is represented by a rate (capacity) vector of k applications being simultaneously served by the network. Generally, there is a cost involved to transmit an unit of flow from a source to a destination. This cost is borne by the application service provider. Given the budget of the provider, our second metric characterizes the expected flow achieved by the network under the given budget. For any given network, the variation in individual link capacities affects the achievability of the flow demand. A small change in the link capacity of a link may result in an achieved flow vector greater or smaller than that required. In that case, each link in the network is important to the achievability of required QoS of multicommodity flows. The quantitative measure of the link importances will help us rank links in order of importance with respect to a particular multicommodity demand. The values would help network designers spend appropriate time and effort in link maintainence to better QoS support. We use the Birnbaum and Fussell-Vesely techniques [3] [12] [23] to find the numerical values of link importance. To the best of our knowledge, this is the first work in QoS of multicommodity flows in $\mathrm{MR}^2-\mathrm{MC}$ wireless meshes.

A. Related Work

The area of multi-channel multi-radio multi-rate wireless meshes is relatively new and much research has not been done in this area. This section is mainly devoted to multicommodity flow applications in wireless mesh networks. However, since our problem deals with multi-channel multi-radio multi-rate networks, we initially describe other work done in this area.

The authors in [13] and [14] address low latency broadcasting in interference based single-channel single-radio multi-rate and multi-channel multi-radio multi-rate networks respectively. Broadcasting is an important problem that finds significance in video feed applications in community mesh networks and multimedia gaming scenarios. These applications have strict delay requirements that need to be met. In [13] the authors demonstrate that broadcast latency is not always minimized by existing tree-based dissemination topologies where a node performs a single broadcast to its children. They show that efficient broadcasting is achieved by performing multiple multicasts to a subset of children at different rates. They design algorithms that use this feature. Their results are shown to be better than some standard previous approaches in terms of latency and throughput. The work done in [14] extends the single-channel, single-radio scenario to a multi-channel, multi-radio one. This is the first work to address broadcasting in multi-channel, multi-radio multi-rate networks. However scalability is not guaranteed by the authors in [14]. Other works in multi-hop multi-channel, multi-rate networks are addressed in [20] [8] [4] but they address unicast applications rather than multicast applications. The latter shows that the path with the minimum delay is the one that maximizes the throughput between the source and the destination and includes channel contention to account for intra-flow interference. Holland et.al [8] come up with a rate-adaptive MAC protocol for increasing throughput of unicast applications. *None of the above works consider multicommodity flows*.

The authors in [17] [18] address the problem of routing and scheduling of multicommodity flows in single-radio multi-channel and multi-radio multi-channel wireless meshes respectively but do not account for multiple rates. Their works aim at finding the bounds for achieved quality of service of multicommodity flows. They formulate the routing problem as a max-flow linear programming problem and propose a primal-dual algorithm which has a fully polynomial-time approximable solution(FPTAS). The simulation results in [17] show that nearly 67 percent of the optimal solution can be achieved. However, it does not model interference. In [18] the authors model secondary interference and come up with efficient dynamic and static channel assignment algorithms. The question of channel assignment does not arise in [17] as secondary interference is not modeled. Necessary and sufficient conditions are derived to guarantee schedulability.In a strict sense the above works optimize the quality of service i.e maximize the achieved flow, given the demand. The problem of scheduling multicommodity link flows in a spread spectrum system is also considered by the authors in [5]. They show that the problem can be solved in polynomial time. However they also mention the fact that their algorithm is not practical. Post et.al [16] have considered a heuristic procedure for this problem. Both [5] and [16] do not model interference and multiple rates. Our model takes into account multiple rates. Jain et.al [15] have proposed a routing scheme for multicommodity flows in interference based capacitated wireless networks. They model interference via the help of a conflict graph. However, they do not propose a polynomial-time algorithm to solve the routing problem nor do they consider multiple channel rates. Our work aims at evaluating the quality of service as a metric in polynomial time rather than optimizing it. To the best of our knowledge, we are aware of no work which devises QoS metrics for $\mathrm{MR}^2-\mathrm{MC}$ wireless meshes and/or determines link importances in such networks.

The main contributions in this paper are summarized as follows.

1) We develop a network model that characterizes data communication in multi-radio, multi-channel, multi-rate wireless mesh networks.

2) We analytically model the problem of evaluating the expected multicommodity flow demand achieved in a $\mathrm{MR}^2-\mathrm{MC}$ WMN and provide an efficient polynomial time algorithm to compute the same. The difference in the performance allows the network designer or the application service provider to judge the suitability of the network to the applications. Some applications may tolerate a moderate difference/deterioration in performance whereas real-time applications are quite strict on meeting the exact requirements. Our algorithm is scalable and can be extended to all types of networks.

3) We analytically model the problem of evaluating the expected multicommodity flow demand in a $\mathrm{MR}^2-\mathrm{MC}$ WMN under given cost constraints and provide an efficient polynomial time algorithm to compute the same. Nearly every service company creates a budget during the planning phase of its new product/service. From a business perspective, the company would expend according to the budget and would not want to exceed its cost limit. However, the service provided to the user under cost constraints may not be appropriate. In that case, to meet user demands, the company could adjust its budget during the product's next planning phase by taking into account the market cost variations of per unit flows and the difference in the QoS obtained with the present budget. The method proposed is an extension of the polynomial time algorithm mentioned above.

4) We use the **Birnbaum** and **Fussell-Vesely** techniques to characterize link importances with respect to the multicommodity flow demand. The link importances signify the criticality of each link in the achievability of the flow demand. A small to moderate decrease in the capacity of high critical links can affect the achievability in a big way. We find the numerical values of link importance and rank them. The network maintainence group can then focus on high critical links for bettering QoS support. This process is often termed as sensitivity analysis.

5) In addition to quantifying the performance and link importance measures, we perform extensive simulations on random networks to corroborate our theory. Our results highlight the trends of performance improvement/deterioration for changes in various network parameters.

The rest of the paper is organized as follows. Section 2 describes the basic network model and notations used in this paper. The problems to be solved are defined in Section 3 and the rationale behind data communication in our model is discussed in section 4. In Section 5 we solve the problem of determining the QoS of multicommodity flows in $\mathrm{MR}^2-\mathrm{MC}$ wireless mesh networks and state the lower bounds for the same. Section 6 considers the previous problem in cost-constrained $\mathrm{MR}^2-\mathrm{MC}$ wireless mesh networks. The problem of evaluating and ranking the link importances for a particular multicommodity flow demand is addressed in section 7. In Section 8 we discuss and analyze our simulation results. We conclude the paper in Section 9.

II. Notations and System Model

In this section we explain most of the notations used in the paper and describe our network model. Other notations in the paper are explained in the relevant sections.

A. Notations

The various notations used in the paper are as follows.

n The number of nodes in the network
V $\{v_i \mid 1 \le i \le n\}$: the set of nodes in the network
N The number of links in the network
E $\{e_i \mid 1 \le i \le N\}$: the set of links in the network
Pr Probability
Ψ The number of orthogonal channels in the network
K $\{k_i \mid 1 \le i \le \Psi\}$: the set of orthogonal channels
Φ The number of radios on each node
$t(e_i)$ Transmitting node of link e_i; $t(e_i) \,\epsilon\, V$ $\{e_i \mid 1 \le i \le N\}$
$r(e_i)$ Receiving node of link e_i; $r(e_i)$ $\epsilon\, V$ $\{e_i \mid 1 \le i \le N\}$
R_k^i Random variable denoting the available capacities for link i on channel k; $i \,\epsilon\, E$
$|R_k^i|$ Number of capacities(rates) available for transmission on link i channel k
p_k^i The discrete probability distribution of R_k^i on channel k; $k \,\epsilon\, K$
(s,d) (source , destination) node pair for a particular application
S_{sd} The set of all prespecified (source,destination) pairs for different commodities
N_{sd} The number of (source,destination) pairs i.e number of commodities
$F_{s,d}$ The flow from source s to sink d for $(s,d) \,\epsilon\, S_{sd}$
$d_{s,d}$ The required demand for node pair $(s,d) \,\epsilon\, S_{sd}$
P_{sd} The set of all *minimal* paths between $(s,d) \,\epsilon\, S_{sd}$
$|P_{sd}|$ The cardinality of set P_{sd}
TP_{sd} The set of all *minimal* paths between all $(s,d) \,\epsilon\, S_{sd}$
$|TP_{sd}|$ The cardinality of set TP_{sd}
Ω The number of all minimal paths between all $(s,d) \,\epsilon\, S_{sd}$.
C $(C_1, C_2, \ldots\ldots, C_N)$: the maximal system capacity vector. C_i(an integer) denotes the maximal capacity of link e_i for each i = 1, 2,....,N
X $(x_1, x_2, \ldots\ldots, x_N)$: current capacity vector.
$\mathbf{Y} \le \mathbf{X}$ For a vector **Y**, y$_i$ $\le x_i$ for each i = 1,2,.....,N
$\mathbf{Y} < \mathbf{X}$ For a vector **Y**, $\mathbf{Y} \le \mathbf{X}$ and y$_i$ $< x_i$ for atleast one i
DV $(d_{s_1,d_1}, d_{s_2,d_2}, \ldots.., d_{s_{N_{sd}},d_{N_{sd}}})$;the required demand vector
CL Cost limit for a cost-constrained network

The following definition is relevant to the paper.

Definition 1. *A minimal path between two vertices in a graph $G = (V, E)$ is a path whose remainder after removing any arc in it is no longer a path.*

B. System Model

We consider a fixed multi-hop wireless network with n nodes. The network is represented by a directed graph $G = (V, E)$ where V represents the set of nodes and E is the set of data links. The data links are unidirectional. Each node can communicate with a subset of other nodes in the network via these wireless links. We assume that each node is perfectly reliable. If a node u is able to transmit directly to node v in the network, we represent this fact by a directed edge $u \rightarrow v$ from node u to node v. An example of a MR$^2-$MC network with link parameters is shown in Figure 1. The capacity $c(e)$ denoted in the figure is a random variable representing R_k^i for some channel k on link $e = i$. Some specific model characteristics and assumptions are stated next.

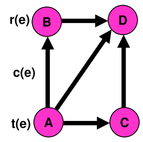

Fig. 1. An example capacitated wireless mesh network

1) Specific Model Characteristics and Assumptions:

1) The nodes are perfectly reliable.
2) The nodes are equipped with multiple radios and are sufficiently powered. All the nodes have equal number of radios and these radios are homogenous.
3) Each radio has access to all the orthogonal channels in the network. A radio can transmit on any channel at a particular time instant. Due to the presence of multiple radios per node, more than one communication can take place per node at a time instant. We assume a **Full Duplex** system i.e a node can transmit and receive in the same time slot.
4) We assume that the system adopts *Dynamic Link Channel Assignment*. Under this channel assignment scheme, the channel on which communication takes place between neighbors is decided at the beginning of each time slot. The decision is transmitted via a common control channel. All the radios in the network are tuned to the control channel. The control channel is taken to be separate from the data channels.
5) The maximal capacity of each channel on a link e_i is C_i
6) The capacity (per unit time) of a link e_i as seen by any radio of $t(e_i)$ at any time instant on channel k is a value taken by the random variable R_k^i. A capacity of 0 indicates unavailability of channel k for data communication. Due to environmental variations, the signal-noise ratio (SNR) varies over time. By adjusting the modulation technique, nodes can transmit at different data rates and account for varying SNR.
7) For a channel k $\epsilon\, K$,we assume that the distribution p_k^i is known through estimation techniques like ***Monte-Carlo simulation*** or ***Gibb's sampling*** on large data sets.
8) The distribution p_k^i for each channel on a link is assumed to be identical. This implies that R_k^i is same for all channels on link i. Our methodology is flexible enough to relax this assumption. According to our model, p_k^i may be different for different links in the MR$^2-$MC wireless mesh network.
9) We assume that interference is present and modelled in the probability distribution of links. i.e when a link capacity is 0 on a channel then the channel cannot be used due to interference or link failure.
10) The capacities of various channels on different data links in the network are statistically independent.
11) The flow in the network satisfies the ***Ford-Fulkerson*** flow-conservation law [7] [1]. According to this law each, each unit of flow is transmitted through one and only one minimal path and no flow is created or destroyed during transmission via such a path.

III. Problem Definition

In this section we define the three problems that are to be solved in this paper.

Problem 1. Given N_{sd} commodities and a required flow demand vector **DV**= $(d_{s_1,d_1}, d_{s_2,d_2}, ..., d_{s_{N_{sd}},d_{N_{sd}}})$, we are required to find the expected flow achieved for each commodity.

Problem 2. Given N_{sd} commodities, a required flow demand vector **DV**= $(d_{s_1,d_1}, d_{s_2,d_2}, ..., d_{s_{N_{sd}},d_{N_{sd}}})$, and the cost limit CL, we are required to find the expected flow achieved for each commodity. Servicing each unit of flow requires a cost. The cost limit represents the total budget of the service provider in transmitting flows from a source to the destination for different commodities.

Problem 3. Given N_{sd} commodities and a required flow demand vector **DV**= $(d_{s_1,d_1}, d_{s_2,d_2}, ..., d_{s_{N_{sd}},d_{N_{sd}}})$, we are required to rank the links of the wireless mesh network in order of importance/criticality with respect to achieving the multicommodity flow demand. The rankings will help the network operators to maintain high critical links and pay more attention to them to better the QoS support.

IV. Rationale Behind Data Communication

As stated in our model assumptions, a node radio in the network has access to all the available channels. At the beginning of every time slot, the channel(if any) through which a node wants to communicate with its neighbour [*Dynamic Channel Assignment*] is decided via some mechanism. This information is conveyed via means of a control channel. If it does not want to be involved in any activity (transmit/receive) during a time slot it remains quiet. For any link i, a node radio can either have access to any channel $k \epsilon K$, at different capacities under consideration, or might have to face link failure. Since, $\forall k \epsilon K$,the $R_k^i{}'s$ are equal, we denote $R_i = R_k^i$ for any k. We assume that each channel on link i can have capacities in the set {0,1,2,.....,$|R_i| - 1$}

Let $p_k^i = [p_k^{i0}, p_k^{i1}, ..., p_k^{i(|R_i|-1)}]$ be the probability distribution vector for channel k on link i at any instant of time. p_k^{im} denotes the probability of link i having capacity m on channel k. m is a value taken by the random variable R_i. By assumption 9, this distribution vector is the same for all the channels on link i. When a node radio wants to transmit information on the link, it can do so via any channel that is free. However, due to variations in link capacity, at any particular time slot, the radio chooses to transmit at rate R_i on some channel.

We denote the number of radios of a node by Φ. At any time slot, either 0 or 1 or ......Φ radios can be in simultaneous communication on a link. Let the total probability space consist of events that either 0 or 1 or....Φ radios communicate concurrently on a link. We assume that the events are uniformly distributed in the space. Therefore the probability that $\alpha (\alpha \epsilon \{0, 1, ..., \Phi\})$ nodes are in simultaneous communication is $\frac{1}{\Phi+1}$.

Let m be any value taken by R_i. Using elementary combinatorics, it is a trivial observation that at any particular time slot, a transmitting radio on a node finds no channel with capacity m in C_0^Ψ ways, it finds 1 channel with capacity m in C_1^Ψ ways. In a likewise manner it finds all channels with capacity m in C_Ψ^Ψ ways.

Let $p^i = [p^{i0}, p^{i1}, ..., p^{i(|R_i|-1)}]$ be the probability distribution vector of a node radio being able to transmit at a particular rate on link i at any instant of time. p^{im} denotes the weighted mean probability of transmitting at rate m. We assume that each element in the vector is obtained from Bernoulli trials in the following manner.

$$p^{im} = \frac{(\sum_{j=0}^{\Psi} C_j^\Psi (p_k^{im})^j (1-p_k^{im})^{\Psi-j})}{(\sum_{j=0}^{\Psi} C_j^\Psi)}$$

Consider the case that only one radio on the transmitting node uses link i. This radio can be chosen in C_1^Φ ways. It finds a channel of capacity m with probability p^{im}. The probability of the event that only one radio transmits on link i is $\frac{1}{\Phi+1}$. In a similar manner two radios can be chosen in C_2^Φ ways and the probability of this event happening is also $\frac{1}{\Phi+1}$. Each radio can independently find a channel of capacity m with probability p^{im}. Following the laws of independence of probabilities [6][19], the probability that any two radios find channels of capacity m for transmission on link i is $\frac{C_2^\Phi (p^{im})^2}{\Phi+1}$. In a similar manner, α radios find channels of capacity m on link i is given by $\frac{C_\alpha^\Phi (p^{im})^\alpha}{\Phi+1}$.

The weighted mean probabilities of transmitting at rate m on link i at any time slot in a multi-channel, multi-radio, multi-rate network can be represented by a vector $p^{i'} = [p^{i0'}, p^{i1'}, ..., p^{i(|R_i|-1)'}]$ where each element is evaluated as

$$p^{im'} = \frac{(\sum_{j=0}^{\Phi} C_j^\Phi (p^{im})^j}{\Phi+1 \cdot (\sum_{i=0}^{\Phi} C_i^\Phi)}$$

The current system capacity vector $\mathbf{X} = (x_1, x_2, ..., x_N)$ is explained as follows. All the links in the vector which form a part of the path from a source to a destination occupies values of capacities at which nodes decide to transmit on some channel. The other links take capacity values of 0 as they are not involved in communication.*The distributions are a small approximation over the non-mean case. We tradeoff complexity with a small approximation without deviating from the actual logic of communication in a MR$^2-$MC network.*

V. Achieving QoS of Multicommodity Flows Without Cost Constraints

In this section we first describe our flow model and intuition behind our solution. Then we propose our algorithm and state its time complexity. Finally we state the lower bounds for the QoS achieved.

A. Flow Model and Intuition

Let $\mathbf{F} = (f_1, f_2, ..., f_{|TP_{s,d}|})$ be a flow assignment vector where f_k denotes integer flow on minimal path mp$_k$ ϵTP_{sd}.

For a particular $(s, d) \epsilon S_{sd}$, according to Ford-Fulkerson flow-conservation law[1] the following statements hold.

1) Any feasible flow pattern from s to d must satisfy
$$F_{s,d} = \sum_{k=1}^{|P_{s,d}|} f_k \geq d_{s,d}$$
2) Except for the source and the sink, the total inflow at each node is equal to its total outflow.
3) Each unit of flow sent from s to d must travel through one of the minimal paths (MP's).

No flow pattern can violate the maximal capacity of any link. This concept is captured in the following inequalities.

$$f_k \leq min\{C_i \mid e_i \epsilon \text{mp}_k\} \text{ for each k = 1,2,...., } |TP_{s,d}|$$

$$\sum_{k=1}^{|TP_{s,d}|} \{f_k \mid e_i \epsilon \text{mp}_k\} \leq C_i \text{ for each i = 1,2,....,} N$$

Given a current system capacity vector $\mathbf{X} = (x_1, ..., x_N)$, the flow pattern **F** is said to be feasible under **X** iff **F** satisfies

$$\sum_{k=1}^{|TP_{s,d}|} \{f_k \mid e_i \epsilon \text{mp}_k\} \leq x_i \text{ for each i = 1,2,....,} N$$

The probability $p_{qos_{\mathbf{DV}}}$, the probability that the wireless mesh network satisfies the QoS given by the demand vector **DV** is

$$p_{qos_{\mathbf{DV}}} = \Pr\{\mathbf{X} \mid \exists \mathbf{F} \epsilon F_{\mathbf{X}} \text{ such that } F_{s,d} \geq d_{s,d} \forall (s,d) \epsilon S_{sd}\}$$

where

$$F_{\mathbf{X}} = \{\mathbf{F} \mid \mathbf{F} \text{ satisfies } \sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \text{mp}_k\} \leq x_i\}$$

There can be many flow vectors satisfying a particular current system capacity vector. Set F_X is the set of flow vectors satisfying feasibility conditions on vector **X**.
Our aim is to find the set of minimal **X**'s (i.e the set of system capacity vectors) under which the demand vector **DV** is satisfied. This can be obtained by filtering out the minimal **X**'s from all the **X**'s that satisfy the system demand.

$$\text{S} = \{ \mathbf{X} \mid \exists \mathbf{F} \,\epsilon\, F_{\mathbf{X}} \text{ such that } F_{s,d} = d_{s,d} \,\forall\, (s,d) \,\epsilon\, S_{sd}\}$$

We define the set of minimal **X**'s as

$$S_{min} = \{\mathbf{X} \mid (\mathbf{X} \,\epsilon\, S) \bigwedge (\mathbf{X} \text{ is minimal in S})\}$$

Such an **X** is called a lower boundary point (LBP) for the vector **DV**. Intuitively, in order to evaluate $p_{qos_{\mathbf{DV}}}$ we need to search for all the LBP's for all demands **DV** . Then $p_{qos_{\mathbf{DV}}}$ can be formulated as

$$p_{qos_{\mathbf{DV}}} = \Pr\{\bigcup_p \{\mathbf{X} \mid \mathbf{X} \geq \mathbf{X}^p \text{ for an LBP } \mathbf{X}^p \text{ for } \mathbf{DV}\}$$

$p_{qos_{\mathbf{DV}}}$ can be evaluated by using the Inclusion-Exclusion principle [9] or the state-space decomposition method [21]. The Inclusion-Exclusion principle for evaluating reliability is suitable generally in case of small and simple networks where the set of lower boundary points is small. The calculations get a bit cumbersome for larger and complicated networks. In such scenarios Aven's state-space decomposition method is useful. The value of $p_{qos_{\mathbf{DV}}}$ obtained is same for all the commodities in the network i.e every commodity has the same proportion of its demand satisfied. This also implies fairness in the satisfaction of the system demand.

Property 1. *For current system capacity vectors* **Y** *and* **X**, $\Pr\{\mathbf{Y} \geq \mathbf{X}\} = \prod_{i=1}^{N} Pr\{y_i \geq x_i\}$ {The probability can be obtained from the distribution vector $p^{i'}$ for each link i}

B. Algorithm and Analysis

The algorithm for finding the achieved QoS given the **DV** is as follows.

Algorithm AcQoS(**DV**)
begin

1. Find all the minimal paths between all (s,d) pairs in S_{sd} using **Al-Ghanim's heuristic** [2].

2. Find all flow patterns **F** satisfying all d_{s_i,d_i}'s in **DV** by solving the following

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \text{mp}_k\} \leq C_i \text{ for each i = 1,2,....,}N$$

$$\sum_{k=1}^{|P_{s,d}|} f_k = d_{s_j,d_j} \text{ for each } j = 1,2,......,N_{sd}$$

3. For each **F**, transform it into $X_{\mathbf{F}} = (x_1, x_2, ..., x_N)$ as follows

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \text{mp}_k\} = x_i \text{ for each i = 1,2,....,}N$$

4. Let S be the family of all $X_{\mathbf{F}}'s$. S = $(X_{\mathbf{F}^1}, X_{\mathbf{F}^2}, ..., X_{\mathbf{F}^T})$. T = |S| . Apply procedure CHECKLBP(**DV**) to generate set S_{min}, the set of minimal $X_{\mathbf{F}}'s$

5. Evaluate $p_{qos_{\mathbf{DV}}}$ using the Inclusion-Exclusion principle or the state-space decomposition method.

6. The **achieved QoS** is given by $[p_{qos_{\mathbf{DV}}} \cdot [\mathbf{DV}]^{\mathbf{T}}]^{\mathbf{T}}$
end

The above algorithm has two dependence algorithms. 1. The Al-Ghanim's heuristic in step 1 and the CHECKLBP procedure in step 2 to check whether a given **X** is a lower boundary points for **DV**.

Advantages of Al-Ghanim's heuristic. This heuristic finds all the minimal paths of a network without determining whether a path is minimal. It is shown to have an approximate linear-time response with the number of nodes. The algorithm is scalable to networks of any size and complexity. It can be applied to non-planar networks as well as networks allowing bilateral flow. The simplicity and multiple advantages of the algorithm motivates us to use it.

Procedure CHECKLBP(**DV**)
begin
1. $S_L = \phi$ (S_L is the stack storing index of non LBP's)
2. For i = 1 to |S|; |S| is the size of family S.
3. For j = i + 1 to |S| and j not in S_L {
4. if $X_{\mathbf{F}^i} > X_{\mathbf{F}^j}$
{$S_L = S_L \bigcup$ i; goto 6. }}
5. $X_{\mathbf{F}^i}$ is an LBP for **DV**
6. Next i.
end

Theorem 1. *The algorithm AcQoS(**DV**) runs time polynomial in max*$\{|P_{s,d}|$ *such that* $(s,d) \,\epsilon\, S_{sd}\}$

Proof. Step 1 of the algorithm executes Al-Ghanim's heuristic which is shown to have linear-time response wit the number of nodes. From step 2 of the algorithm the equation $\sum_{i=1}^{|P_{s,d}|} f_k = d_{s,d}$ for each $(s,d) \,\epsilon\, S_{sd}$ has $(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$ solutions. For all (s,d) pairs the number of solutions is $\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$. Each solution takes $O(|TP_{s,d}|)$ time to test whether it satisfies $\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \text{mp}_k\} \leq C_i$. This check is performed for each link in the network. There are a total of N links. The total time taken in the worst case to run step 2 is $O(N(|TP_{s,d}|)\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))$. Step 3 is just a basic transformation of step 2 and so requires the same amount of time to execute as step 2. The algorithm takes $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))$ time to test the minimality of each solution in step 4. In the worst case it would require $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})).^2)$ time to test all the solutions. Step 5 uses an efficient algorithm depending on the complexity of the network. Step 6 is computed in constant time. Summing up the time complexity at steps 1-6, we get the overall running time of $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))^2)$. We now apply **Stirling's approximation** [22][6] to fine-tune the analysis. According to this approximation technique, $(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$ can be approximated as $\frac{1}{d_{s,d}!}(\frac{(|P_{s,d}|+d_{s,d}-1)}{e})^{d_{s,d}} \cdot e^{d_{s,d}}$. In such a case we have the following result. *For a fixed* **DV** *and* n, algorithm AcQoS *runs in time polynomial in max*$\{|P_{s,d}|$ *such that* $(s,d) \,\epsilon\, S_{sd}\}$. *Q.E.D*

VI. Achieving QoS of Multicommodity Flows With Cost Constraints

A. Flow Model and Intuition

Let **F** $=(f_1, f_2, ..., f_{|TP_{s,d}|})$ be a flow assignment vector where f_k denotes integer flow on minimal path mp$_k$ $\epsilon\, TP_{sd}$.

For a particular $(s,d) \,\epsilon\, S_{sd}$, we assume that the Ford-Fulkerson

flow-conservation law[1] holds as in section 5.
No flow pattern can violate the maximal capacity of any link. This concept is captured in the following inequalities.

$$f_k \leq min\{C_i \mid e_i \,\epsilon\, \mathrm{mp}_k\} \text{ for each k = 1,2,...., } |TP_{s,d}|$$

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq C_i \text{ for each i = 1,2,....,} N$$

Given a current system capacity vector $\mathbf{X} = (x_1, ..., x_N)$, the flow pattern $\mathbf{F}$ is said to be feasible under $\mathbf{X}$ iff $\mathbf{F}$ satisfies

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq x_i \text{ for each i = 1,2,....,} N \quad (1)$$

and

$$\sum_i^n c_i \cdot (\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq x_i) \leq CL \qquad (2)$$

where c_i is the cost to ship each unit of flow along link i.
The probability $p_{cqos_{\mathbf{DV}}}$, the probability that the wireless mesh network satisfies the QoS given by the demand vector **DV** under cost constraint CL is

$$p_{cqos_{\mathbf{DV}}} = \Pr\{\mathbf{X} \mid \exists \mathbf{F} \,\epsilon\, F_{\mathbf{X}} \text{ such that } F_{s,d} \geq d_{s,d} \,\forall\, (s,d) \,\epsilon\, S_{sd}\}$$

where

$$F_{\mathbf{X}} = \{\mathbf{F} \mid \mathbf{F} \text{ satisfies (1) and (2)}\}$$

Our aim is to find the set of minimal **X**'s (i.e the set of system capacity vectors) under which the demand vector **DV** and cost limit CL is satisfied. This can be obtained by filtering out the minimal **X**'s from all the **X**'s that satisfy the system demand.

$$\text{S} = \{\ \mathbf{X} \mid \exists \mathbf{F} \,\epsilon\, F_{\mathbf{X}} \text{ such that } F_{s,d} = d_{s,d} \,\forall\, (s,d) \,\epsilon\, S_{sd}\}$$

We define the set of minimal **X**'s as

$$S_{min} = \{\mathbf{X} \mid (\mathbf{X} \,\epsilon\, S) \bigwedge (\mathbf{X} \text{ is minimal in S})\}$$

Such an **X** is called a lower boundary point (LBP) for the vector **DV** and CL. Intuitively, in order to evaluate $p_{cqos_{\mathbf{DV}}}$ we need to search for all the LBP's for the pair (**DV**,CL) . Then $p_{cqos_{\mathbf{DV}}}$ can be formulated as

$$p_{cqos_{\mathbf{DV}}} = \Pr\{\bigcup_p \{\mathbf{X} \mid \mathbf{X} \geq \mathbf{X}^p \text{ for an LBP } \mathbf{X}^p \text{ for } (\mathbf{DV}, CL)\}$$

$p_{cqos_{\mathbf{DV}}}$ can be evaluated by using the Inclusion-Exclusion principle [9] or the state-space decomposition method [21].

B. Algorithm and Analysis

The algorithm for finding the achieved QoS given **DV** and CL is as follows.

Algorithm AcQoS(**DV**,CL)
begin
1. Find all the minimal paths between all (s,d) pairs in S_{sd} using **Al-Ghanim's heuristic** [2].

2. Find all flow patterns **F** satisfying all d_{s_i,d_i}'s in **DV** and CL by solving the following

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq C_i \text{ for each i = 1,2,....,} N$$

$$\sum_{k=1}^{|P_{s,d}|} f_k = d_{s_j,d_j} \text{ for each } j = 1, 2, ..., N_{sd}$$

$$\sum_i^n c_i \cdot (\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq x_i) \leq CL$$

3. For each **F**, transform it into $X_{\mathbf{F}} = (x_1, x_2, ..., x_N)$ as follows

$$\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} = x_i \text{ for each i = 1,2,....,} N$$

4. Let S be the family of all $X'_{\mathbf{F}}s$. S = $(X_{\mathbf{F}^1}, X_{\mathbf{F}^2}, ..., X_{\mathbf{F}^T})$. T = |S| . Apply procedure CHECKLBP(**DV**, CL)to generate set S_{min}, the set of minimal $X'_{\mathbf{F}}s$

5. Evaluate $p_{cqos_{\mathbf{DV}}}$ using the Inclusion-Exclusion principle or the state-space decomposition method.

6. The **achieved QoS** is given by $[p_{cqos_{\mathbf{DV}}} \cdot [\mathbf{DV}]^{\mathbf{T}}]^{\mathbf{T}}$
end

Procedure CHECKLBP (**DV**,CL)
begin
1. $S_L = \phi$ (S_L is the stack storing index of non LBP's)
2. For i = 1 to |S|; |S| is the size of family S.
3. For j = i + 1 to |S| and j not in S_L{
4. if $X_{\mathbf{F}^i} > X_{\mathbf{F}^j}$
{S_L= $S_L \bigcup$i; goto 6. }}
5. $X_{\mathbf{F}^i}$ is an LBP for **DV** and CL
6. Next i.
end

Theorem 2. *The algorithm AcQoS(DV,CL) runs time polynomial in max*$\{|P_{s,d}|$ *such that* $(s,d) \,\epsilon\, S_{sd}\}$

Proof. Step 1 of the algorithm executes Al-Ghanim's heuristic which is shown to have linear-time response with the number of nodes. From step 2 of the algorithm the equation $\sum_{i=1}^{|P_{s,d}|} f_k = d_{s,d}$ for each $(s,d) \,\epsilon\, S_{sd}$ has $(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$ solutions. For all (s,d) pairs the number of solutions is $\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$. Each solution takes $O(|TP_{s,d}|)$ time to test whether it satisfies $\sum_{k=1}^{|TP_{s,d}|}\{f_k \mid e_i \,\epsilon\, \mathrm{mp}_k\} \leq C_i$. This check is performed for each link in the network. There are a total of N links. The total time taken in the worst case to run step 2 is $O(N(|TP_{s,d}|)\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))$. The inclusion of equation (2) does not alter the running time of step 2. Step 3 is just a basic transformation of step 2 and so requires the same amount of time to execute as step 2. The algorithm takes $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))$ time to test the minimality of each solution in step 4. In the worst case it would require $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})).^2)$ time to test all the solutions. Step 5 uses an efficient algorithm depending on the complexity of the network. Step 6 is computed in constant time. Summing up the time complexity at steps 1-6, we get the overall running time of $O(N\Sigma_{(s,d)\epsilon S_{sd}}(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1}))^2)$. We now apply **Stirling's approximation** [22][6] to fine-tune the analysis. According to this approximation technique, $(C_{d_{s,d}}^{|P_{s,d}|+d_{s,d}-1})$ can be approximated as $\frac{1}{d_{s,d}!}(\frac{(|P_{s,d}|+d_{s,d}-1)}{e})^{d_{s,d}} \cdot e^{d_{s,d}}$. In such a case we have the following result. *For a fixed* **DV** *and* n, algorithm AcQoS *runs in time polynomial in max*$\{|P_{s,d}|$ *such that* $(s,d) \,\epsilon\, S_{sd}\}$. *Q.E.D*

VII. CHARACTERIZING LINK IMPORTANCES FOR BETTERING QOS SUPPORT

The measures quantifying the criticality of links can be used to identify the weaknesses in the network system and prioritize QoS

Fig. 2. Topologies used for simulation

improvement activities. The Birnbaum and Fussell-Vesely [11] [12] [23] measures help in the assessment of the most important link with respect to the overall achieved QoS, given a demand. The measures can be ranked with respect to the impact they have on the achieved QoS. Both of these measures are aimed at identifying how a link affects QoS support.

A. Birnbaum's Measures

We consider two types of Birnbaum's measure. 1. Average of the sum of absolute deviations (SAD) and 2. Mean Absolute Deviation (MAD). As the names suggest, these measures account for the absolute deviation of each link state from the actual value of the achieved QoS. They provide answer to the question of which link most significantly impacts (positively or negatively) the achievability of a given demand vector. Since these measures account for both positive and negative impacts, they are regarded as *risk-neutral* measures. SAD considers the possible state levels of a particular link whereas MAD considers the possible state levels as well as the probability of being in that state.

1) Average of the Sum of Absolute Deviations: Let $\Gamma(\mathbf{X})$ be the function which takes in $\mathbf{X}$ as an argument and returns an achieved demand vector. The required demand vector is **DV**. For a particular link i, let m be a value taken by R_i. The importance of link i is denoted by $BLI1_i$ and is given by the following formula.

$$BLI1_i = \frac{\Sigma_{j=0}^{R} |Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i = j) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})|}{R}$$

where $R = |R_i| - 1$.

A low value of $BL1_i$ indicates that the achieved QoS is not affected by changes in state of the link i. A high value indicates the opposite.

2) Mean/Expected Absolute Deviation: Like in the previous subsection, let $\Gamma(\mathbf{X})$ be the function which takes in $\mathbf{X}$ as an argument and returns an achieved demand vector. The required demand vector is **DV**. For a particular link i, let m be a value taken by R_i. The importance of link i is denoted by $BLI2_i$ and is given by the following formula.

$$BLI2_i = \mathrm{E}[|Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})|]$$

$$= \Sigma_{j=0}^{R} p_{ij} \cdot |Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i = j) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})|$$

where p_{ij} is the probability of link i having capacity j.
A low value of $BLI2_i$ indicates that the achieved QoS is not affected by changes in state of the link i. A high value indicates the opposite.

B. Fussell-Vesely Measures

We consider two types of measures 1. The general Fussell-Vesely measure (GFVM) and 2. The Mean Fussell-Vesely measure (MFVM). Both these measures are relative. GFVM and MFVM account for the average change in achieved QoS when link states negatively contribute to the achieved QoS. These measures can help the network designer identify the link that provides the largest decrease of the achieved QoS when subject to variations. These measures are therefore *risk-averse*. GFVM considers the possible state levels of a particular link whereas MFVM considers the possible state levels as well as the probability of being in that state.

1) General Fussell-Vesely Measure: The general Fussell-Vesely measure quantifies the maximum decrement in the probability of achieving the desired QoS caused by a particular link. Let $\Gamma(\mathbf{X})$ be the function which takes in $\mathbf{X}$ as an argument and returns an achieved demand vector. The required demand vector is **DV**. For a particular link i, let m be a value taken by R_i.

Let $$\beta_{ij} = \frac{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i = j) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})}{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})}$$

The general Fussell-Vesely measure(GFVM) for link i is denoted as $FVM1_i$ and is given as

$$FVM1_i = \Sigma_{j=0}^{R} max(0, \frac{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i = j)}{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})})$$

Or

$$FVM1_i = \Sigma_{j=0}^{R} max(0, -\beta_{ij})$$

A low value of $FVM1_i$ indicates that variations in the capacities of link i have very little negative impact on the achievability of a given demand vector. High values of $FVM1_i$ signifies a considerable impact of variation of link capacities on the achievability of the required QoS.

2) Mean/Expected Fussell-Vesely Measure: Like in the previous subsection, let $\Gamma(\mathbf{X})$ be the function which takes in $\mathbf{X}$ as an argument and returns an achieved demand vector. The required demand vector is **DV**. For a particular link i, let m be a value taken by R_i. The mean Fussell-Vesely importance measure(MFVM) of link i is denoted by $FVM2_i$ and is given by the following formula.

$$FVM2_i = E[max(0, \frac{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}) - Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV}|x_i = j)}{Pr(\Gamma(\mathbf{X}) \geq \mathbf{DV})})]$$

Or

$$FVM2_i = E[max(0, -\beta_{ij})]$$

Or

$$FVM2_i = \Sigma_{j=0}^{R} p_{ij}[max(0, -\beta_{ij})]$$

where p_{ij} is the probability of link i having capacity j.
A low value of $FVM2_i$ indicates that variations in the capacities of link i have very little negative impact on the achievability of a given demand vector. High values of $FVM2_i$ signifies a considerable impact of variation of link capacities on the achievability of the required QoS.

VIII. Simulation Setup and Performance Evaluation

A. Simulation Setup

We consider the topologies as shown in Figures 2a and 2b. Figure 2a represents a collection of nodes uniformly distributed in a 30 X 50 rectangular area. We assume that there are 20 nodes in the network and nodes within 10 units distance from each other can directly communicate amongst themselves. A link between two nodes is converted to a bidirectional link. Figure 2b. is an instance of a random network with 6 nodes and 9 links. This topology is used to assess link importances of a network satisfying a multicommodity flow demand. The links for this network are numbered in the following manner. **Link 1** - (1,3) **Link 2** - (2,1) **Link 3** - (2,3) **Link 4** - (1,5) **Link 5** - (1,4) **Link 6** - (3,4) **Link 7** - (5,4) **Link 8** - (5,6) and **Link 9** - (4,6). The numbers inside brackets represent vertex pairs. The rates for each channel are in the range [0,10] Mbps in discrete steps. As mentioned in section 3, the rate distribution for each channel on a link is assumed to be the same. For cost-constrained networks, the cost of per unit flow on each link is assumed to be 10 and the cost limit is assumed to be 750. In practice, the rate probabilities of channels on each link can be estimated through machine learning techniques like Monte-Carlo simulation or Gibbs sampling. In our simulations we generate the rates with the help of a random number generator. For the topology shown in Fig. 2a we generate 50 random graphs. Our results are representative of the 50 graphs. Our simulations are aimed at four main objectives.

1) To observe the trends of the probability of achieving the required QoS for SC-SR multi-rate and MC-MR multi-rate networks.
2) To rank the importances of links of the network shown in Figure 2b with respect to the multicommodity flow demand.
3) To analyze the different importance measures and comment on the use of each.

B. Performance Evaluation

1) Achieving Required QoS in Single-Channel, Single-Radio Networks: Figure 3 highlights the performance of multicommodity flows in single-channel, single-radio multi-rate wireless meshes. Our results are based on one,three and five simultaneous data transfers. The demand vectors for these commodity flows are **[2]**, **[1,2,4]** and **[1,2,3,4,5]** respectively. The integers in the vector stand for the rates to be achieved in Mbps. We try all possible combinations of one, three and five commodity flows and average our results. From figure 3a we observe that there is a drop of performance by approximately **29** percent and **40** percent respectively when the number of commodities increases from one to three to five. On the other hand for cost constrained networks, the drop is approximately **29** and **57** percent respectively. The thing to notice here is that for one and three commodities the performance is the same for cost-constrained and non cost-constrained networks whereas for five commodities, non cost-constrained networks seem to perform better. This is because the number of feasible system capacity vectors decreases in the case of five commodities which in turn imply that certain flow vectors exceed the given cost limit.

TABLE I
Link Importance Rank Table for Various Measures

Rank	BLI1	FVM1	BLI2	FVM2
1	9	2	9	9
2	2	9	3	3
3	4	6	4	4
4	6	4	2	2
5	3	8	8	8
6	8	3	6	6
7	5	5	5	5
8	7	7	7	7
9	2	1	1	1

2) Achieving Required QoS in Multi-Channel, Multi-Radio Networks: Figure 4 highlights the performance of multicommodity flows in multi-channel, multi-radio multi-rate wireless meshes. We simulate with three orthogonal channels and assume that there could be one, two and three radios present per node. As in the case for single-radio, single-channel networks, the demand vectors are **[2]**, **[1,2,4]** and **[1,2,3,4,5]** for one, two and three commodities respectively. For a single commodity we observe that the performance for two and three radios is the same as compared to a single radio. For three simultaneous flows, the performance improvement over a single-radio is approximately **40** percent and **70** percent for two radios and three radios respectively. This is because more load can be handled with more radios for more than one simultaneous commodity. In the case of five flows, the improvement decreases to **27** percent and **50** percent respectively. The improvement percentage decreases as we add more commodities in the system. The reason for this behaviour is that there could be atmost three simultaneous communications(transmit/receive) for a node at a time. Larger number of simultaneous flows results in many neighbors forwarding flows to a particular node and so bottlenecks might arise.In the case of cost-constrained networks, the trend and the performance is the same as in the case of non cost-constrained networks except in the case of five simultaneous flows, where the performance is lower because of the fact that the number of feasible system capacity vectors decreases which in turn imply that certain flow vectors exceed the given cost limit.

3) Link Importance Measures: Figures 6 and 7 highlight the link importances of the network shown in Figure 2b. For measuring link importances, we assume a three commodity demand of **[5,5,5]**. Figure 7 deals with the mean values and the variances of general Birnbaum and Fussell-Vesely whereas figure 8 addresses the mean and variance of mean Birnbaum and Fussell-Vesely measures. One thing to notice about both these types of measures is that their variances are consistently very low. This indicates their robustness. The link importance ranks based on various measures are shown in table I. The numbers in columns 2 to 5 stand for link numbers. We see that for general Birnbaum, link number 9 is the one whose variation affects QoS the most In case of general Fussell-Vesely it is link number 2. We also observe that the ranks for the general Birnbaum and Fussell-Vesely differ whereas the ranks for mean Birnbaum and Fussell-Vesely are the same. The reason for this behaviour is that the mean measures take into account the probability of an increase/decrease in achieved QoS when link capacities increase/decrease. The general measures do not do that. In terms of robustness both the general and the mean measures are quite good but we would prefer the mean measures because of two main reasons. 1. They provide consistent rankings. 2. Their variance is the lowest. Amongst the mean measures the Fussell-Vesely measure should be preferred due to its lowest variance when compared to that of Birnbaum.

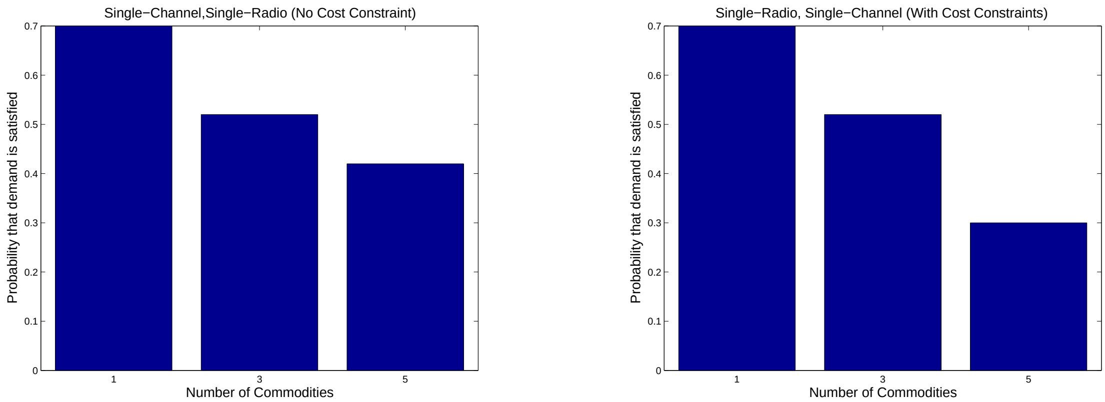

Fig. 3. Performance of a single-radio single-channel multi-rate mesh network with and without cost constraints

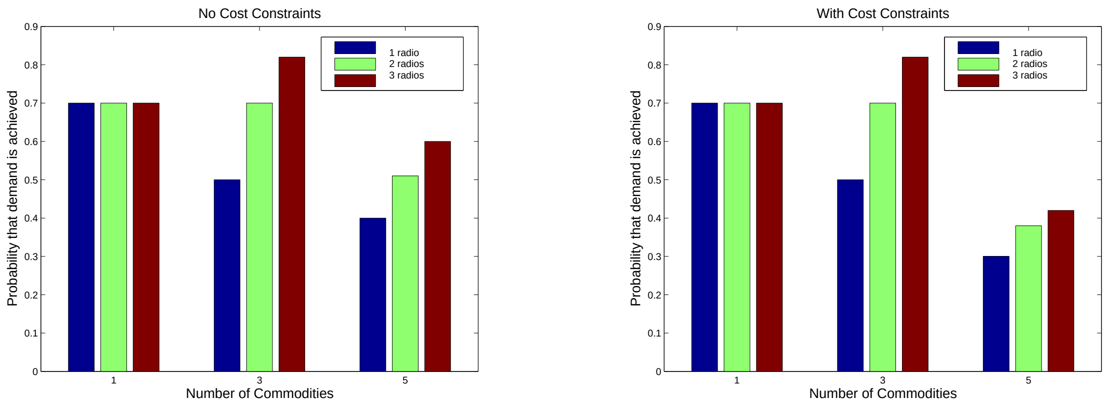

Fig. 4. Performance of a multi-radio multi-channel multi-rate mesh network with and without cost constraints

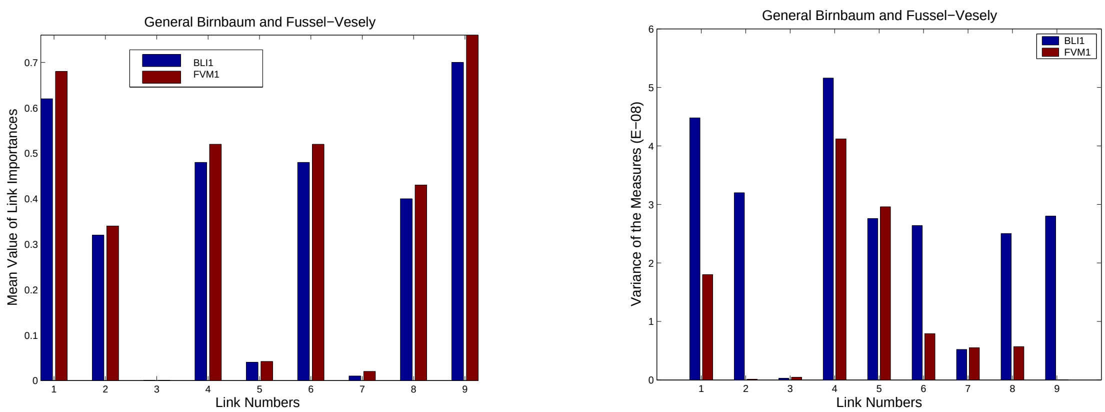

Fig. 5. Characterizing Link Importances via General Birnbaum and Fussell-Vesely (Mean and Variance)

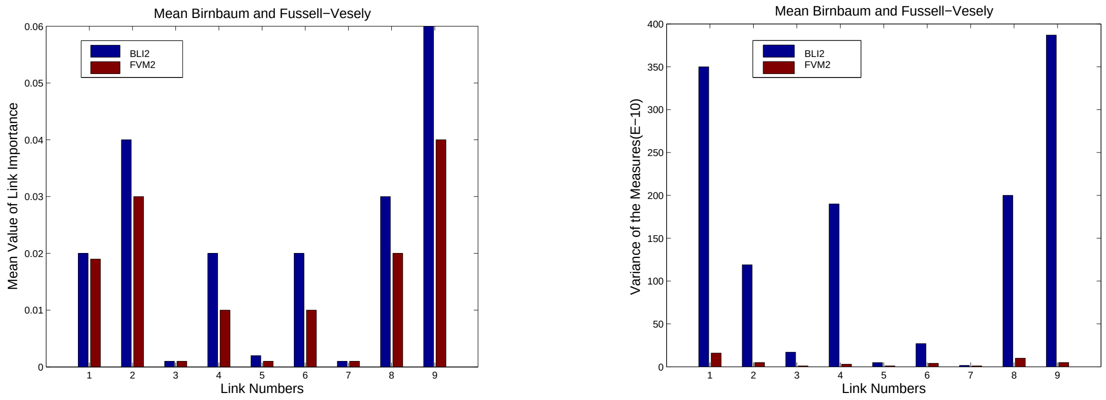

Fig. 6. Characterizing Link Importances via Mean/Expected Birnbaum and Fussell-Vesely (Mean and Variance)

IX. Discussion, Conclusion and Future Work

Supporting multicommodity flows in wireless mesh networks is an important problem studied by the networking community. Various applications including video, audio, text etc. require being simultaneously served at a specified level of service. The dynamically changing nature of the channel conditions pose a challenge to achieving the required quality of service and thus motivates us to characterize QoS metrics for evaluating the level of service achieved, in comparison to the demand. We used probabilistic modeling techniques to formulate our metrics and develop polynomial time algorithms to evaluate the same. Our paper addresses both cost-constrained and non cost-constrained systems. For a given network and a multicommodity flow demand, we also characterize the link importances which can help the network designer/operator to concentrate on maintaining critical links for better QoS support. We use two standard measures 1. The Birnbaum's measure and 2. The Fussell-Vesely measure. Our simulation results show that the Fussell-Vesely measure is the most robust due to its lowest variance. They also reflect the fact that the use of multiple radios significantly improve the performance of applications due to the increase in the overall capacity of the network. There are some issues which we have not addressed in this paper. We have assumed that the links are *statistically independent*. This is hardly the case as poor links affects the flow through the neighboring links also. In that case our computations also need to consider the distributions of all the links affecting a particular link. Our paper also assumes that the channels on each link have equal distributions. In reality, this might not be the situation and we need to model this case separately. This work characterizes QoS metrics in multi-rate mesh networks but does not optimize them. Finally we do not include any interference model in our paper. We have assumed the fact that the probability distributions encapsulate interference information and there is some layer that takes care of it. We plan to address all the above issues as future work. In conclusion, we feel that the methods proposed in this paper will be of immense use to network designers and quality engineers to plan optimal network deployment and tune performance parameters in a timely manner to improve quality of service support.

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