Compressive Sensing Network Inference with Multiple-Description Fusion Estimation

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***Abstract*—We have previously introduced Multiple Description Fusion Estimation (MDFE) framework that partitions a large-scale Under-Determined Linear Inverse (UDLI) problem into smaller sub-problems that can be solved independently and in parallel. The resulting estimates, referred to as multiple descriptions, can then be fused together to compute the global estimate [1]. In this paper, we extend MDFE framework to make it compatible with Compressive Sensing (CS) network inference, where the attributes of interests (i.e. unknowns) are fluctuating rapidly over time and/or space. For this purpose, we propose a new clustering based technique to intelligently divide a large-scale compressive sensing problem into smaller sub-problems where observations between sub-spaces contain redundancy. We apply this new framework, referred to as Compressive Sensing MDFE (CS-MDFE), to three classical inference problems in networking: traffic matrix estimation, traffic matrix completion, and loss inference. Using real topologies and traces, we demonstrate how CS-MDFE can improve the estimation accuracy and speed up computation time, and how it enhances robustness against noise and failures. We also show that this framework is compatible with different CS inference techniques.**

I. Introduction

Designing, monitoring, and managing of today's complex networks depends on providing critical information about aspects of the networks that must be measured or inferred. Direct measurement of some attributes of interest can be challenging or infeasible due to the complexity of current Internet, limited monitoring resources, and exploding traffic volume. In fact, considering measurement resource constraints (e.g. limited number of TCAM entries at switches for flow size measurement or limited bandwidth available in active loss/delay measurement), it is not only impossible to directly measure all attribute of interests but also it is unmanageable, inefficient and expensive to store and process all measurements due to limited memory and processing power. Network inference/tomography methods are powerful tools that can help estimate the internal attributes of interests based on a limited set of measurements and, accordingly, mitigate the limitations and constraints of direct network measurement techniques [2]. Two forms of Network Inference (NI) have been studied rigorously: (a) origin-destination (path-level) traffic volume estimation based on link-level traffic measurements, such as Traffic Matrix (TM) estimation [3] or TM completion [4], and (b) link-level parameter's estimation based on end-to-end measurements [5], [6]. Traffic Matrix (TM) estimation is formulated as a constrained UDLI problem where the main goal is to estimate the origin-destination traffic intensity that can be defined at different levels: between routers, IP-prefixes, or even AS domains. It provides essential information for network design, traffic engineering, and anomaly detection. To uniquely identify the solution of this UDLI problem, side information from different sources/perspectives (e.g. underlined statistical models or NetFlow records) are provided [7],[8]. On the other hand, in matrix completion, the spatial-temporal structure among TMs are used to interpolate missing TM values [4]. Also, in network performance tomography the internal link properties are inferred from end-to-end measurements [9]. For example, in [6], link loss rate inference problem is modeled as an UDLI problem where statistical characteristics of congested links are used as side information to uniquely estimate link loss rates. In these applications, the attribute of interests (or the unknown quantities) are flow size, delay, link loss or bottleneck bandwidth which can be highly fluctuated.

To cope with highly dynamic traffic/network conditions, NI methods must be able to provide accurate estimates at faster time scales with more robust performance against noise and failure in the network or monitoring infrastructures. Since many network inference problems are formulated as Under-Determined Linear Inverse (UDLI) problems (which are naturally ill-posed problems in the sense that the number of measurements are not sufficient to uniquely and accurately determine the solution), side information from different sources must be incorporated into the problem formulation to improve the accuracy of the estimation [4], [5], [6], [7], [8]. On the other hand, due to the dynamic nature of the network, the attributes of interests are fluctuating rapidly over time and/or space. It has also been shown that the distribution of the size and inter-arrival of flows in networks follow heavy-tailed distributions [10], [11]. Therefore, NI techniques must be able to estimate these fluctuated traffic attributes with acceptable precision, depending on the application. This is of particular importance because many recent applications in network monitoring, management and security require timely estimates of both large and small unknown quantities with high precision [12], [13].

Compressive Sensing (CS) inference methods are effective techniques for solving NI problems with the ability of the reconstruction of sparse unknown quantities from a set of limited measurements [14]. However, their performance is limited by different factors: 1) in NI problems, the observation matrix is a low-density sparse matrix with high coherency which is not completely under our control; hence, measurements are not in a well-defined compressed form and their reconstruction using compressive sensing techniques are difficult. 2) Since unknown quantities vary rapidly over time and/or space and compressive inference methods use numerical optimization techniques for solving the UDLI problem, the stability of the reconstruction algorithm is limited for such an ill-posed inference problem, and 3) the number of unknowns are much larger than the number of measurements, which limits not only the performance of the algorithm for recovering large quantities but also makes it computationally untractable.

To address these issues, instead of introducing a particular technique, we propose an efficient, flexible and robust framework which is complementary to the compressive inference techniques used for solving specific NI problem. For this purpose, we extend our MDFE framework previously presented in [1]. Under MDFE, a large scale inference problem can be effectively partitioned into smaller sub-problems and solved independently and in parallel. Then, local estimates/descriptions from subspaces are fused together to reconstruct the global estimate. The MDFE framework is based on least square estimation methods and, accordingly, is effective for inputs without unusual fluctuations. Since CS inference techniques are able to provide more accurate estimates for sparse inputs and to make MDFE framework compatible with CS network inference methods, we propose a new clustering based algorithm to intelligently divide a large-scale CS problem into smaller sub-problems by clustering the sub-set of correlated unknowns and their related observations into sub-spaces. Accordingly, observations among sub-spaces are redundant; thus, set-partitioning problem in [1] is changed to set-covering problem, here. In this way, the measurements/observations are re-utilized to generate multiple descriptions of the sub-set of unknowns where descriptions are considered as side/supplementary information for each other, provided from different perspectives. Using CS-MDFE, the computational complexity in each sub-space is reduced. Moreover, it enhances the robustness against noise and failures in network monitoring infrastructures. CS-MDFE is also compatible with the architecture of multi-processor computing infrastructures and can have important implications on monitoring applications in distributed and dynamic environments such as distributed data centers or clouds.

The improvement in the estimation accuracy using CS-MDFE framework depends on the structure of the problem, sub-space inference technique, set-covering algorithm, and the fusion process which are discussed in this paper. Due to space limitation, we occasionally refer to [15] for the detailed discussion and additional results. It should be noted that, CS-MDFE is a flexible framework and can be used in other compressed sensing applications. Our main contributions are:

- We extend our MDFE framework in [1] for solving compressive sensing NI problems and demonstrate how to effectively design the CS-MDFE framework and realize it in practice.
- We develop a new algorithm to divide the original large-scale problem into smaller sub-problems under CS-MDFE; we also introduce optimal and heuristic fusion methods to combine the multiple descriptions and reconstruct the global solution.
- We demonstrate the efficacy of CS-MDFE in practice by applying it to three important problems in network monitoring and management: TM Estimation (TME), TM completion and loss inference. Using realistic network topologies and traffic data, we show how CS-MDFE could effectively improve the accuracy of the global estimate and enhance the efficiency and robustness of the computing system.

The rest of this paper is organized as follows. In Section II we develop the theory of CS-MDFE and in Section III we address main steps in the implementation of this framework in practice. In Section IV the performance and efficiency of this framework are evaluated for different applications in networking. Finally, Section V summarizes the paper.

II. Compressive Sensing Inference with Multiple Description Fusion

Compressive Sensing (CS) is a recently emerging field in signal processing which offers an effective framework for simultaneous acquiring and reconstructing a signal by finding solutions to an UDLI problem. In fact, it recovers a sparse signal with only k non-zero quantities (in some basis) from $m = O(k\log(n/k))$ properly designed linear measurements, compactly represented by [14]:

$$Y = HX, \quad H = [h_1, ..., h_n] \in R^{m\times n}, \{h_i \in R^m\}_{i=1}^{n}. \quad (1)$$

where Y is an $(m\times 1)$ observation vector as $Y = [y_1, ..., y_m]^T$, $H = [h_{ij}]_{1\leq i\leq m, 1\leq j\leq n}$ is an $(m \times n)$ observation matrix $(m < n)$ and X is an $(n \times 1)$ vector of unknowns as $X = [x_1, ..., x_n]^T$. The general solution to this problem is of the form $\hat{X} = X + \mathcal{N}_0(H)$ where $\mathcal{N}_0(H)$ represents a solution from the span of the null space of H; therefore, there are many solutions for this problem. A compressive inverse problem is defined as the process of uniquely inferring X as a function of observation Y via the constrained optimization problem [14]:

$$\hat{X} = \underset{X}{min} \left\| X \right\|_0 \qquad \text{s.t. Y=HX}. \quad (2)$$

This is an NP-hard problem and, hence, it is infeasible. If the coherence of H, defined as:

$$\mu = \underset{i\neq j}{max} \frac{\left| h_i^T h_j \right|}{\left\| h_i \right\|_2 \left\| h_j \right\|_2} \quad (3)$$

is sufficiently small, then the following convex optimization program exactly reconstructs the signal:

$$\hat{X} = \underset{X}{min} \left\| X \right\|_1 \qquad \text{s.t. Y=HX}. \quad (4)$$

Interestingly it has been shown that random observation matrices with i.i.d. Gaussian or random ± 1 entries, and sufficient number of rows can achieve small coherence with overwhelmingly high probability [14]. There is a large body of literature which extends these results for various applications. Among those, the most relevant works in the literature of CS to our framework are sparse signal recovery techniques using structured sparsity models where certain sparse support patterns are allowable and the signal is modeled using more concise models. Such additional structures can be captured in terms of restricting the feasible signal support to a small subset of possible selections of non-zero coefficients for a sparse signal [14]. For example, in cases where non-zero coefficients appear in clusters, the structure can be expressed in terms of a sparse union of sub-spaces [16]. Or, in distributed compressed sensing when multiple signals are simultaneously recorded and their supports are correlated [17]. In these works, the optimal observation matrix H is designed or assumed to meet specific conditions (e.g. restricted isometry property) so that it can provide a set of well-formed compressed measurements, and consequently, the CS recovery algorithm can reconstruct particular sparsity structure(/s) [14].

In CS-MDFE framework, the original (global) UDLI inference problem described by Eq.(1) is divided into L local redundant sub-problems shown in Eq.(5), which are independently solved using proper inference techniques such as Eq.(4). Then, sub-space estimates/descriptions $\{\hat{X}_i\}_{i=1}^L$ are fused together to provide a more accurate solution in an efficient way. The fusion process is accomplished by applying appropriate weights $\{\omega_i^F\}_{i=1}^L$ to each local estimate during the fusion phase. Fusion weight $\{\omega_i^F\}_{i=1}^L$, for each sub-space, can be a scalar or a vector which appropriately equalizes each unknown $\{x_j\}_{j=1}^n$ at multiple sub-spaces. Eq.(6) describes this process where operator $\oplus$ denotes the fusion process of the partitioned problem, where weighted linear combination of subset of unknowns observed and estimated by different sub-spaces is computed. The feasibility of the idea of MDFE is shown by the sufficient condition represented in Theorem.1.

$$Y = HX \Leftrightarrow \begin{bmatrix} \underline{Y_1} \\ \underline{\vdots} \\ Y_L \end{bmatrix} = \begin{bmatrix} \underline{H_1 X_1} \\ \underline{\vdots} \\ H_L X_L \end{bmatrix} \tag{5}$$

$$\hat{X_C^F} = \oplus_{i=1}^L \omega_i^F \hat{X}_i \tag{6}$$

Theorem.1: Let X and $\hat{X}^G$ represent the unknown vector and its global estimate. Let for some set $\mathcal{C}$, covering the set of observations, and for some fusion weights $\{\omega_i^F\}_{i=1}^L$, the CS-MDFE estimate $\hat{X}_C^F$ is computed using Eq.(6). Then, we have (Proof is in [15]):

$$\left\|\hat{X}_C^F - X\right\|_2^2 \leq \left\|\hat{X}^G - X\right\|_2^2 \quad \text{if} \quad \left\langle \left(\hat{X}^G - \hat{X}_C^F\right), \left(\hat{X}_C^F - X\right) \right\rangle \geq 0. \tag{7}$$

Therefore, computing $\hat{X}_C^F$ is feasible. Through our direct observations we have realized that, at least, for $X \geq 0$ (or for $X \leq 0$) and for binary observation matrices with low-density ($\frac{\#of non-zero entries}{m \times n} := D$) and high coherency, where classic CS techniques can not provide accurate estimation, the CS-MDFE framework can be applied to improve the performance. This of particular importance, because the observation matrix H in many applications are binary matrices. For example, in network monitoring and sensor network applications, the entries of H indicates the ability($h_{ij} = 1$)/inability($h_{ij} = 0$) of measuring the j^{th} attribute of interest (x_j) by i^{th} monitor/sensor. Due to practical constraints and the nature of the problem, the observations are redundant; also, all unknowns can not be desirably measured by any set of sensors. Therefore, observation matrices are sparse with low densities and high coherencies. Based on our experience, to improve the precision of CS-MDFE for sparse and heavy-tailed distributed inputs, it is necessary to have redundancy between measurements in sub-spaces.

Accordingly, the overall performance of CS-MDFE framework is a joint function of sub-space estimation technique, cover of the set of observations $\mathcal{C}$, and fusion process F. Hence, to successfully apply the CS-MDFE framework in practice, three steps must be accomplished correctly: a) effectively divide the problem into sub-problems so that the set of observations is covered, b) construct multiple descriptions by adopting proper sub-space estimation techniques to solve the sub-problems, and c) fuse the sub-space estimates to provide more precise and robust description. The essence of this joint optimization problem lies in an NP-hard set covering problem that is extremely difficult to solve. Hence, we decouple and address steps a-c, independently. Accordingly, in the following sub-sections, we fix the CS inference technique and discuss how the existing CS sparse signal reconstruction methods can be leveraged to provide sub-space estimates (step b). Then, taking practical constraints into account, we discuss the design of the most effective set-covering and fusion methods.

Our framework is called CS-MDFE for two reasons: 1) the number of measurements in network inference problems are very limited, that is, measurements are compressed and 2) sub-space inference techniques are basic CS reconstruction algorithms, as effective techniques for recovering sparse vector(/s) of unknowns. However, our framework also differs from previous works in two aspects: 1) the observation matrix in network inference problems is fixed and it is not under our control, completely. Therefore, network measurements can not be acquired in a desirable compressible form via a well-designed observation matrix and CS reconstruction techniques can not uniquely and accurately determine the unknown vector; 2) the fusion process in our framework optimally equalizes estimates of unknowns, observed in different sub-spaces. In fact, here it is assumed that the unknown vector X lies in a union of sub-spaces with possibly lower sparsity in each sub-space (i.e. $\{k_i \leq k\}_{i=1}^L$ where k_i denotes sparsity at i^{th} sub-space). Accordingly, the centralized Eq.(1) can be intelligently decentralized into multiple sub-spaces to capture the structure of the signal from different perspectives. The improvement in the accuracy of the estimation is achieved by reutilizing the measurements to generate and fuse multiple descriptions. This framework is flexible and it is also compatible with CS recovery techniques.

Our new CS-MDFE is also different from our previous work in [1] where the inference problem is partitioned into L sub-spaces where observations between sub-spaces are not redundant and Linear Least Square Estimation (LLSE) methods are mainly applied to NI problems with non-sparse inputs (note that LLSE techniques have poor performance in the presence of highly fluctuated inputs). Here, in Section IV-C and, also, in [15], we show that MDFE with set-covering is compatible with LLSE techniques.

III. CS-MDFE IN PRACTICE

To implement the CS-MDFE framework in practice, the following three steps (A-C) must be accomplished.

A. Multiple Description Construction

To construct multiple descriptions, sub-inference problems must be properly defined and the best sub-space CS estimation technique is selected based on the characteristics of the input X, matrix H and problem's side information or constraints.

Let I denotes the set of all indicies of observations ($I = \{1, 2, ..., m\}$) and I_i denotes the i^{th} set of indices of measurements where $I = \bigcup_{i=1}^L I_i$. Then, set $\mathcal{C} = \{I_i\}_{i=1}^L$ covers I. Let J denotes the set of all indices of unknowns ($J = \{1, 2, ..., n\}$) and J_i denotes the i^{th} set of indices of unknowns where $J = \bigcup_{i=1}^L J_i$. Now, lets $Y_i := \{y_k\}_{k \in I_i}$, $H_i := H(I_i, J_i)$ and $X_i := \{x_k\}_{k \in J_i}$. Accordingly, the original problem Eq.(1) is divided into L sub-problems as $Y_i = H_i X_i$ (see Eq.(5)).

The most important sub-space inference techniques used in this paper are represented in Eq.(8-10) where global solution is computed by estimating X from measurement vector Y (where $Y = HX$) and local descriptions are produced by considering appropriate unknown vector X_i and local measurement vectors $Y_i (= H_i X_i)$ in sub-spaces. Eq.(8) is a convex reformulation of Eq.(4) where λ controls the amount of regularization. Here, different variations of this formulation is used as inference technique for estimating the attributes of interests in network monitoring applications such as Traffic Matrix Estimation (TME) where the unknown quantities are highly fluctuated and have heavy-tailed distributions [10] [11]. Among these, Eq.(10) is our new method utilized for TME, here.

$$\hat{X} = \underset{X}{min} \frac{1}{2} \|Y - HX\|_2^2 + \lambda \|X\|_1 \quad \text{s.t. } X \geq 0. \tag{8}$$

$$\hat{X} = \underset{X}{min} \|Y - HX\|_1 + \lambda \|X\|_1 \quad \text{s.t. } X \geq 0 \quad \text{from [8].} \tag{9}$$

$$\hat{X} = \hat{x}_0 + \hat{\mathcal{X}} = \underset{x_0, \mathcal{X}}{min} \|Y - H(x_0 + \mathcal{X})\|_2^2 + \lambda \|\mathcal{X}\|_1 \text{ s.t. } [x_0 \in \mathbb{R}; \mathcal{X} \in \mathbb{R}^n] \geq 0 \tag{10}$$

These optimization problems are solved using Linear Programming techniques with complexity $O(n^3)$ (e.g. interior point methods) which is still highly impractical in many applications [14]. To reduce the complexity, iterative greedy algorithms (e.g. Orthogonal Matching Pursuit algorithm) with complexity $O(kmn)$ is used where the coherency of observation matrix (μ) is at most $\frac{1}{2k}$ [14]. Clearly, no matter which inference technique is used, the complexity is reduced with the CS-MDFE process because the dimension of the problem in each sub-space is reduced. In this paper, we use CVX to solve these optimization problems. Note that the solution of global and local problems could be different because the null space of H and H_i are not necessarily equal (see App.B in [15]).

B. Set-Covering Design

The accuracy of redundant estimates from sub-spaces depends on the design of the cover $\mathcal{C}$ that can be formulated as an integer optimization problem to achieve the best possible performance. Assuming there are m measurements and L sub-spaces, then there are $S_{m,L} = \frac{1}{L!} \sum_{j=0}^{L} (-1)^{L-j} \binom{L}{j} j^m$ partitions ($S_{m,L}$ denotes Stirling number of the second kind) and, accordingly, the number of different ways to cover the set of observations is higher. To simplify this NP-hard problem and improve the performance, a heuristics algorithm is developed.

The main idea in this clustering algorithm (Alg.1) is grouping the most correlated attributes of interest ($\{x_j\}_{j=1}^{n_i}$) and corresponding measurements $\{y_i\}_{i=1}^{m_i}$ in the i^{th} cluster to produce the i^{th} description $\hat{X}_i$, representing the signal from a new perspective. For this purpose, here, we use k-means clustering algorithm with Euclidean and Correlation distance measures. Since k-means converges to a suboptimal local minimum, we run the algorithm with different random initial points and choose the solution with optimal performance. The performance is measured on the training data set. It should be noted that, clusters are redundant in the sense that there are common unknowns and their corresponding measurements among sub-spaces. This allows us to create redundant descriptions, which is necessary for CS-MDFE when input vector X is sparse. Other clustering algorithms can also be used to improve the performance and they are currently under investigation.

Algorithm 1 : Correlation Based Set-Covering

Input: Training Data Set X_{Test}, Observation Matrix H, Number of sub-spaces L.
Output: Sub-space characteristics $\{I_i\}_{i=1}^L$ and $\{J_i\}_{i=1}^L$, and accordingly, $\{X_i\}_{i=1}^L$, $\{Y_i\}_{i=1}^L$ and $\{H_i\}_{i=1}^L$.
Initialization: Compute the cross correlation matrix of H as: $C_H^{ij} = \left\{ \frac{|h_i^T h_j|}{\|h_i\|_2 \|h_j\|_2} \right\}_{1 \leq i,j \leq n}$ and set: 1) the distance used in k-means algorithm and 2) the Performance Threshold ($PThr$).
while $Performance < PThr$ **do**
- Apply k-means to C_H to create L redundant clusters of columns of H (equivalently $\{X_i\}_{i=1}^L$).
- For i^{th} cluster and its X_i, choose corresponding measurement vector Y_i and observation matrix H_i. Repeat this for all L clusters to form redundant measurement sub-sets $\{Y_i\}_{i=1}^L$ and sub-space observation matrices $\{H_i\}_{i=1}^L$.
- Check if $\bigcup_{i=1}^L I_i = I$, then, apply the CS-MDFE framework on the training data set and measure its performance (Table I).
end while

C. Fusion Algorithm

CS-MDFE process is completed by applying fusion process F to the local estimates. Here, we consider two different ways to fuse the local descriptions using optimal and heuristic weighting functions. Having the set $\mathcal{C}$ that covers the set of observations and the appropriate sub-space estimation technique (Eq.(8-10)), the optimal fusion weights can be numerically computed by the following procedure. First, apply CS-MDFE framework on the training data set $X_{Test} := \{X_1^T, ..., X_t^T, ..., X_{T_0}^T\}$ where column vector $X_t^T \in \mathbb{R}^n$ for $t = 1, ..., T_0$ and T_0 is small compare to the size of the data. Then, generate local descriptions $\{\hat{X}_t^T\}_{t=1}^{T_0}$ where each $\hat{X}_t^T$ is an $n \times L$ matrix as: $\hat{X}_t^T = \left[\hat{X}_{t_1}^T, ..., \hat{X}_{t_L}^T\right]$. Next, calculate the optimal weights for each X_t^T ($t = 1, ..., T_0$) using the following optimization problem (Eq.(11)), where ω_t is an $n \times L$ matrix that equalizes local descriptions; also, $\mathcal{RS}$ and $.*$ denote row-sum and componentwise multiplication operators, respectively. By doing this on the whole training data set, finally, the optimal weight ω^{Opt} (an $n \times L$ matrix) is computed using Eq.(12). We then apply these weights on the whole data set to evaluate the performance.

$$\omega(:,:,t) = \underset{\omega_t}{min} \left\| X_t^T - \mathcal{RS}\left(\omega_t .* \hat{X}_t^T\right) \right\|_2 \quad \text{s.t. } \omega_t \geq 0 \text{ and } \{\mathcal{RS}\left(\omega_t(j,:) = 1\right)\}_{j=1}^n. \tag{11}$$

$$\omega_{ji}^{Opt} = \frac{1}{T} \sum_{t=1}^{T} \omega(j,i,t) \quad \text{for } j = 1:n, i = 1:L \tag{12}$$

To facilitate the implementation of the CS-MDFE framework, a heuristic weighting function is also proposed whereby for each unknown x_j, the optimal estimate is computed as the average of $\{\hat{x}_{ji}\}_{i=1}^L$ observed at different sub-spaces (Eq.(13)). The computational overhead of these weighting functions are negligible compare to complexity of sub-space estimations and they can remarkably enhance the performance.

$$\omega_{ji}^{Avg} = \frac{1}{\text{\# of repetition of } x_j \text{ among all sub-spaces}} j = 1:n, i = 1:L \tag{13}$$

D. The Efficiency of CS-MDFE

CS-MDFE is an efficient framework that can improve the performance of system from different perspectives. It can provide more accurate estimates by providing redundant local estimates, that is, observing an unknown from different sub-spaces. This redundancy is used by the fusion process to enhance the accuracy of estimation process. The amount of redundancy depends on the number of sub-sets (L), and the structure of observation matrix H. On the other hand, dividing the problem into sub-problems does not change the input-output relationship; however, by re-utilizing the measurements and clustering more-similar unknowns in a sub-space, more coherent solution can be generated. In addition, the row partitioning of observation matrix H improves the Condition Number (CN) of H_i's (Prop.1) which improves the accuracy and robustness of numerical optimization methods used in sub-spaces for local estimation. Since the complexity of subspace estimation techniques are functions of the dimension of sub-problems, and also because processing time and power are functions of the computational complexity of the problem, then, using multi-processor parallel computing architectures results in more efficient computing system by reducing the processing time. It can also lead to reduction in processing power, if the sum of the processing powers of the sub-spaces is less than the processing power for solving the global inference problem. Accordingly, considering other practical constraints, parameter L can be chosen with a reasonable balance between desirable improvement in the accuracy and reduction in processing time/power. In addition, since CS-MDFE can provide more redundant and better estimates; it can also improve the robustness of the system against noise, failure and information-loss, in the system.

Proposition.1: Let H be a matrix in ($R^{m \times n}$ with rank m) and H_i denotes a matrix constructed from a set of rows of H. Then: $CN(H_i) \leq CN(H)$ (see [15] for proof).

E. CS-MDFE: Performance Evaluation Metrics

The performance of the CS-MDFE is evaluated using various criteria, summarized in Table I. Here, $\hat{X^G}$ denotes the global estimate, $\hat{X_w^F}$ denotes the CS-MDFE estimate where ω denotes fusion function (Eq.(12) or Eq.(13)), and $Gain_{L2}$ and $Gain_{RE}$ quantify the performance improvement using CS-MDFE framework comparing with global case. It should be noted that, relative gain $Gain_{RE}$ capture the mean deviation of unknowns from their true values and it is a harder criterion to improve. Parallel processing gains (Δ_p^1 and Δ_p^2) measure the reduction in computation using CS-MDFE structure on a multi-processor parallel computing infrastructure. Because it is difficult to measure sub-space sparsity $\{k_i\}_{i=1}^L$, and since, the complexity of greedy CS algorithms is $O(kmn)$, for Δ_p^1 the computational complexity of solving sub-space CS inference techniques are considered to be $O(mn)$. Since $\{k_i \leq k\}_{i=1}^L$, the achievable processing gains Δ_p^1 are higher in practice. For Δ_p^2, the computational complexity of solving Eq.(8-10) is considered to be $O(n^3)$. Also, $CNI_m = \frac{\min\{CN(H_i)\}_{i=1}^L}{CN(H)}$ and $CNI_M = \frac{\max\{CN(H_i)\}_{i=1}^L}{CN(H)}$ indicate the CN Improvement.

$GE = \frac{\left\| X - \hat{X}^G \right\|_2}{\|X\|_2}, FE_w = \frac{\left\| X - \hat{X}_w^F \right\|_2}{\|X\|_2}$	$Gain_{L2} = 100 \times \frac{GE - FE_\omega}{GE}$
$RGE = \frac{1}{\|N_v\|} \sum_{j:x_j > v} \left\| \frac{x_j - \hat{x}_j^G}{x_j} \right\|$ $RFE_w = \frac{1}{\|N_v\|} \sum_{j:x_j > v} \left\| \frac{x_j - \hat{x}_{\omega_j}^F}{x_j} \right\|$	$Gain_{RE} = 100 \times \frac{RGE - RFE_\omega}{RGE}$
$\Delta_p^1 = 100 \times \frac{mn - max(\{m_i n_i\}_{i=1}^L)}{mn}$	$\Delta_p^2 = 100 \times \frac{n^3 - max(\{n_i^3\}_{i=1}^L)}{n^3}$

TABLE I: Performance evaluation criteria (where $N_v = \{x_j : x_j > v, \text{ for } j = 1, ..., n\}$ and v is chosen so that the unknowns (x's) under consideration carry approximately 90% of the total traffic, as in [8]).

F. CS-MDFE: Illustrative Example

This illustrative example shows the effectiveness of CS-MDFE for estimating sparse signals. Consider UDLI problem $Y = HX$ where $H = [0,1,1,1,0,1;1,0,0,1,0,1;0,0,1,0,1,1]$ and assume that $H1 = H([1,2],:)$, $H2 = H([1,3],:)$ and sub-space CS inference technique is Eq.(4). If $X = [1,0,1,1,0,1]^T$ it can be shown that $GE = 0.8660$, $FE_{Opt} = 0.50$, $FE_{Avg} = 0.5863$ and gains $GE_{L2}^{Opt} = 42.3\%$ and $GE_{L2}^{Avg} = 32.3\%$ are achieved. Also, if $X = [1,0,5,3,0,2]^T$, as a highly fluctuated signal, then, $GE = 0.6304$, $FE_{Opt} = 0.4082$, $FE_{Avg} = 0.5371$, and accordingly, $GE_{L2}^{Opt} = 36.3\%$ and $GE_{L2}^{Avg} = 16.2\%$ (see [15] for details).

IV. Network Inference using CS-MDFE

In this section the effectiveness of CS-MDFE in different applications, including TM estimation, TM completion and loss inference, and its compatibility with different CS inference techniques are shown. Two real networks Abilene [18] and GEANT [19] are considered. Routing matrix $H_{Abilene}$ is a (30×144) matrix with density $D = 0.0353$ and coherency $\mu_{Abilene} = 0.9127$. H_{Geant} is a (74×529) matrix with $D = 0.036$ and $\mu_{Geant} = 0.8942$. Both routing matrices are binary and full row-rank. Also, real network data (Table III), which are highly fluctuating, are used for our experiments.

A. Traffic Matrix Estimation (TME)

Considering $Y = HX$, TME is an UDLI inference problem where X is the TM (each entry of X represents an Origin-Destination Flow (ODF) in the network) and it is estimated by knowing routing matrix H and observing link load measurement vector Y. Since LLSE methods have poor performance for highly fluctuated TMs [15], here, TMs are inferred using optimization techniques Eq.(8-10), in both global and CS-MDFE framework. Alg.1 is used for set-covering and dividing an UDLI problem into sub-problems and different configurations are considered for CS-MDFE framework. Table II shows the performance of CS-MDFE on these two networks with different configurations and parameters. The improvement in the accuracy of the estimation is considerable. Among these, higher gain on relative errors shows that this framework can enhance the precision in the estimation of larger ODFs. This is an important factor in many network monitoring applications such as traffic engineering and anomaly detection. It is also shown that, applying optimum fusion weights ω^{Opt} can significantly improve the performance, although, more accurate estimates can also be achieved even by applying fusion weights ω^{Avg}

Configuration Parameters	GE	FE_{Avg}	$Gain_{L2}^{Avg}\%$	FE_{Opt}	$Gain_{L2}^{Opt}\%$	RGE	RFE_{Avg}	$Gain_{RE}^{Avg}\%$	RFE_{Opt}	$Gain_{RE}^{Opt}\%$	$\Delta_p^1\%$	$\Delta_p^2\%$	MRUF	CNI_m	CNI_M
1) $L = 5$, Eq.(8)	0.7852	0.7027	10.4995	0.6385	18.6721	1.0044	0.7873	21.6083	0.6015	40.1079	71.7172	77.7388	1.7667	0.42	0.94
2) $L = 4$, Eq.(9)	0.7586	0.7506	1.0558	0.6785	10.5577	0.9861	0.8311	15.7167	0.6442	34.6695	75.3788	77.7388	1.6000	0.44	0.94
3) $L = 5$, Eq.(10)	0.7533	0.6385	15.2406	0.6250	17.0271	0.9606	0.6481	32.5375	0.5421	43.5704	42.1970	43.6936	2.0333	0.48	0.86
4) $L = 5$, Eq.(8)	0.6073	0.5750	5.3171	0.4100	32.4965	0.8374	0.6749	19.4010	0.5373	35.8350	61.2328	56.9733	2.2703	0.52	0.91
5) $L = 5$, Eq.(9)	0.6092	0.5685	6.6742	0.4055	33.4372	0.8379	0.7367	12.0835	0.5661	32.4364	64.7153	61.8454	2.1757	0.41	0.84
6) $L = 3$, Eq.(10)	0.6085	0.5818	4.3902	0.4872	19.9358	0.8344	0.7140	14.4278	0.6346	23.9441	10.1058	6.3810	1.5270	0.47	0.99
7) $L = 4$, Eq.(10)	0.6085	0.5772	5.1486	0.4274	29.7641	0.8344	0.7444	10.7856	0.5851	29.8861	58.3859	59.2955	2.0135	0.60	0.78
8) $L = 5$, Eq.(10)	0.6085	0.5605	7.8933	0.4557	25.1068	0.8344	0.6742	19.2041	0.5732	31.3090	13.4548	8.6323	1.9054	0.31	0.98

TABLE II: Performance of CS-MDFE over Abilene (configurations (1)-(3)) and Geant (configurations (4)-(8)) networks, respectively.

which facilitates the implementation of CS-MDFE framework. Furthermore, our new inference technique Eq.(10) improves the performance of CS-MDFE framework.

Table II also indicates the processing gains obtained using CS-MDFE framework. These remarkable, processing gains are achievable using today's multi-core parallel computing architectures where communication delays for distributing the problem among multi-processors are negligible in comparison with processing times. In this table, Measurement Re-Utilization Factor (MRUF) measures the amount of redundancy between observations among sub-spaces which is necessary to improve the performance of CS-MDFE, and it is defined as: $MRUF := \frac{\sum_{i=1}^{L} m_i}{m}$. Note that, in the design of CD-MDFE framework, the number of subspaces L, the sub-space estimation technique and the fusion algorithm must be selected carefully, considering a reasonable trade-off between accuracy, processing gains and all other practical constraints. Based on the topology of the network, it is also possible to improve sequential processing gains, defined as: $\Delta_s^1 = 100 \times \frac{mn - \sum_{i=1}^{L} m_i n_i}{mn}$, or $\Delta_s^2 = 100 \times \frac{n^3 - \sum_{i=1}^{L} n_i^3}{n^3}$. This is of particular importance, because processing power is also a function of computational complexity. Thus, enhancing Δ_s^1 and Δ_s^2 can reduce the processing power of the system and improve the efficiency of the system. For example, for Abilene network, using configurations (1) and (2) in Table II, considerable sequential processing gains ($\Delta_s^1 = 13.5\%$, $\Delta_s^2 = 41\%$) and ($\Delta_s^1 = 12.6\%$, $\Delta_s^2 = 34\%$) can be achieved, respectively. Note that, although, here, we have used CS-MDFE framework for network monitoring applications, this framework can be applied in many other compressive sensing tomography problems. The improvenemt of the CN of sub-space observation matricies (by Prop.1) are also evident. This improves the stability of numerical algorithms used in sub-spaces for multiple descriptin estimation.

- Robustness of CS-MDFE: CS-MDFE framework improves the robustness of the system against noise in measurements, failures and lossy information (Section III-D). Here, this fact is justified by investigating the robustness of CS-MDFE in TME. According to [8], noise in link load measurements (due to disalignment of polling intervals) can be modeled as a White Gaussian Noise (WGN); therefore, we added WGN

Network	Date	Duration	Resolution	TM Size
Abilene	2004-05-01	1 week	5 min.	144×2016
GEANT	2005-01-08	1 week	15 min.	529×672

TABLE III: Real Datasets under study.

SNR(dB)	6	9	15	20	30
GE	1.0360	0.8712	0.7123	0.6618	0.6328
FE_{Opt}	0.7533	0.6311	0.4884	0.4438	0.4227
FE_{Avg}	0.7739	0.6924	0.6131	0.5928	0.5849
Erased Sub-Space	1	2	3	4	5
GE	0.6457	0.6247	0.6779	0.6302	0.6410
FE_{Opt}	0.4279	0.4491	0.5209	0.4444	0.4594
FE_{Avg}	0.5900	0.5971	0.5988	0.5994	0.6160

TABLE IV: CS-MDFE Robustness against noise and failure.

to link measurement vector Y with different Signal-to-Noise Ratios (SNR) to evaluate the performance. The improvement of CS-MDFE in the presence of noisy link-load measurements, using the 8^{th} configuration in TableII (for Geant network), is shown in TableIV. This table also indicates that CS-MDFE is robust against sub-space erasure (or failure) in the system.

B. Traffic Matrix Completion (TMC)

In [4], a Sparsity Regularized SVD (SRSVD) method is introduced for TM Completion where the columns of traffic matrix Z is formed by the unknown vector X in our TME setup at different times ($t = 1, ..., \mathcal{T}$). Now, assuming Z can be factored as $Z_{n \times \mathcal{T}} = LR^T$; then TMC is formulated as the following optimization problem to find missed entries of Z.

$$\hat{Z} = \min_{L,R} \left\| \mathcal{B} - \mathcal{A}(LR^T) \right\|_F^2 + \lambda \left(\|L\|_F^2 + \|R\|_F^2 \right) \tag{14}$$

Here, $\mathcal{B}$ and $\mathcal{A}$ respectively denote the set of measurements and a linear operator satisfying $\mathcal{A}(Z) = \mathcal{B}$. To apply our MDFE framework, we adopt this method and modified the formulation in Eq.(14). In our Modified SRSVD (MSRSVD) method, $A_t = [diag(M_t); H]$ and $b_t = [X_t. * M_t; Y_t]$ where M_t is a binary column vector (where zeros represent missing entries), H is the routing matrix, $Y_t = HX_t$ denotes t^{th} link load measurement vector; accordingly, $\mathcal{A} = \{blockdiag(\mathcal{A}, A_t)\}_{t=1}^{\mathcal{T}}$, $\mathcal{B} = [b_1; ...; b_{\mathcal{T}}]$ and $\mathcal{M} = [M_1, ..., M_{\mathcal{T}}]$. Figure 1 shows that our new MSRSVD TM completion method significantly improves the performance where TMC is applied onto normalized TMs where X_{base} [4] is assumed to be known. It also compares the TM completion performance between Global-TMC and CS-MDFE-TMC on real Abilene and GEANT networks and data. Here, the 3^{rd} and 8^{th} configurations in Table II are considered and we set MSRSVD as sub-space TMC technique in CS-MDFE framework. Also, $\lambda_{Abilene} = 0.01$ and $\lambda_{GEANT} = 0.1$ and for both networks we fixed $r = 2$ (i.e. rank-2 approximation). The Normalized Mean Absolute Error (NMAE) is computed over

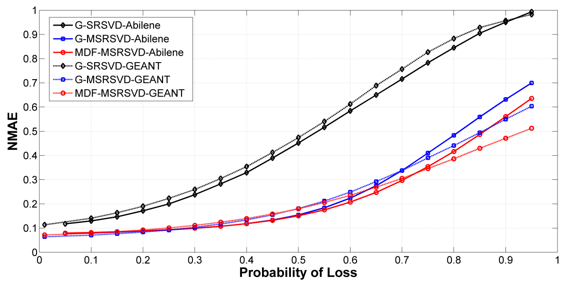

Fig. 1: NMAE v.s probability of loss in TM completion (G: global).

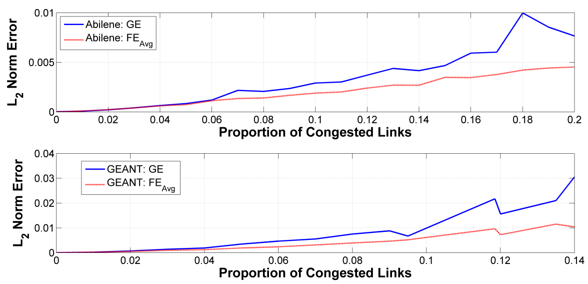

Fig. 2: GE and FE_{Avg} vs. the proportion of congested links for the 3^{rd} and 4^{th} configurations in Table.II where # of beacons S=1000 and # of iterations = 100.

interpolated values as $NMAE = \frac{\sum_{i,j:\mathcal{M}(),|)=0}|Z(i,j)-\hat{Z}(i,j)|}{\sum_{i,j:\mathcal{M}(i,j)=0}|Z(i,j)|}$. It is clear that, CS-MDFE can improve the performance for high loss probabilities where MDFE framework reduces the number of unknowns in each sub-space and helps to improve spatial-temporal correlations. For low loss rates, the performance of both methods are close together. However, CS-MDFE can speed-up the TMC process and improve its robustness.

C. Loss Inference

Considering $Y = HX$, loss inference is an UDLI problem where H is a routing matrix, and X and Y are defined as $X = \{x_j\}_{j=1}^n = \{log\hat{\phi}_{e_j}\}_{j=1}^n$ and $Y = \{y_i\}_{i=1}^m = \{log\hat{\phi}_i\}_{i=1}^m$. Parameter, $\hat{\phi}_i$ represents the fraction of S probes that arrive correctly at the destination and $\hat{\phi}_{e_j}$ is the fraction of probes from all paths passing through link e_j that have not been dropped by that link [6]. The loss inference algorithm is adopted from [6] as the sub-space estimation technique to apply the MDFE framework. Both Abilene and Geant networks are considered and the proportion of the links that are congested is fixed and is varied to evaluate the performance of MDFE framework with set-covering in terms of GE and FE_{Avg}. Here, congested and non-congested links have loss rates uniformly distributed in [0.05, 0.2] and [0,0.002], respectively. Figure 2 shows the improvement achieved by applying MDFE with set-covering for loss inference in two networks and indicates that MDFE is more effective for higher loss rates.

V. Conclusion

In this paper, a novel approach for solving CS inference problems was introduced where a large-scale problem is divided and solved in sub-spaces. By fusing the solution from sub-spaces, we not only showed that the accuracy of the solution is improved, but also the efficiency and robustness of system are enhanced. These are important factors, particularly, in distributed and dynamic environments where accurate, quick and efficient inference are highly demanding. We examined the performance of CS-MDFE in different applications, and we showed that CS-MDFE framework is flexible and compatible with a variety of sub-space estimation techniques, and it is compatible with today's multi-processors architectures.

VI. Acknowledgement

This work is supported in part by HP Labs Innovation Research Award.

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