Network Configuration For Optimal Utilization Efficiency of Wireless Sensor Networks

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Abstract

This paper addresses the problem of configuring wireless sensor networks (WSNs). Specifically, we seek answers to the following questions: how many sensors should be deployed, what is the optimal sensor placement, and which transmission structure should be employed. The design objective is utilization efficiency defined as network lifetime per unit deployment cost. We propose an optimal approach and an approximation approach with reduced complexity to network configuration. Numerical and simulation results demonstrate the near optimal performance of the approximation approach. We also study the impact of sensing range, channel path loss exponent, sensing power consumption, and event arrival rate on the optimal network configuration.

Index Terms

Network lifetime, sensor placement, utilization efficiency, network configuration, wireless sensor network.

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I. INTRODUCTION

Wireless sensor networks (WSNs) have captured considerable attention recently due to their enormous potential for both commercial and military applications. A WSN consists of a large number of low-cost, low-power, energy-constrained sensors with limited computation and com-PSfrag replacements munication capability. Sensors are responsible for monitoring certain phenomenon within their sensing ranges and reporting to gateway nodes where the end-user can access the data.

> In WSNs, sensors can be deployed either randomly or deterministically. A random sensor placement [1] may be suitable for battlefields or hazardous areas while a deterministic sensor placement is feasible in friendly and accessible environments. In general, fewer sensors are required to perform the same task with a deterministic placement. A typical configuration for WSNs with deterministic sensor placement may include the following three aspects: the network size, the sensor placement which determines the location of each sensor, and the transmission structure which specifies how data are relayed to the gateway node. As illustrated in Fig. 1, total N sensors are deployed in a linear WSN. The sensor placement and the transmission structure are specified by $\mathbf{d} \stackrel{\Delta}{=} [d_1, \ldots, d_N]$ and $\mathbb{P} \stackrel{\Delta}{=} \{P_{i,j}\}_{i,j=1}^N$, respectively, where d_i denotes the distance between adjacent sensors and $P_{i,j}$ the probability that sensor i transmits its data to sensor j.



Fig. 1. Network configuration for a linear WSN: network size N, sensor placement $\mathbf{d} = [d_1, \dots, d_N]$, transmission structure $\mathbb{P} = \{P_{i,j}\}_{i,j=1}^N$.

A. Contributions

In this paper, we address all three aspects of network configuration. The contribution of this paper is twofold. First, we introduce a performance measure of utilization efficiency defined as network lifetime per unit deployment cost. In general, both network lifetime and deployment

cost increase with the network size. While deployment cost increases almost linearly with the number of sensors, the increasing rate of network lifetime diminishes when the network is large. Utilization efficiency captures the rate at which network lifetime increases with the network size. It can thus effectively address the tradeoff between network lifetime and deployment cost, providing balanced design guidelines for network configuration. Under the performance metric of utilization efficiency, we study the effect of sensing energy consumption and event arrival rate on the optimal network size. We find that a dense network is desirable when the event arrival rate is large. On the other hand, when the sensing energy consumption is relatively large, a sparse network is preferred.

Second, we formulate network configuration for optimal utilization efficiency as a multivariate non-linear optimization problem by jointly optimizing sensor placement, transmission structure, and network size. The impact of sensing range and channel path loss exponent on sensor placement is studied. We find that sensors should be placed more compact and closer toward the gateway node when their sensing range is large and more uniformly when the channel path loss exponent is large. We also extend our results to a two dimensional WSN with grid structures.

B. Related Work

Sensor placement problem has been addressed in many network applications [2]–[6]. Different performance measures have been used to develop sensor placement schemes. For example, Dhillon and Chakrabarty [7] propose two algorithms to optimize sensor placement with a minimum number of sensors for effective coverage and surveillance purposes under the constraint of probabilistic sensor detections and terrain properties. Lin and Chiu [8] address the sensor placement problem for complete coverage under the constraint of cost limitation. Ganesan *et. al.* [9] jointly optimize sensor placement and transmission structure in a one-dimensional datagathering WSN. Their approach aims at minimizing the total power consumption under distortion constraints. Kar and Banerjee [10] address the optimal sensor placement to ensure connected coverage in WSNs. Sensor placement schemes that maximize network lifetime have also been addressed for different WSNs. For example, Dasgupta *et. al.* [11] propose an algorithm to find the optimal placement and role assignment to maximize the lifetime of a WSN that consists of sensors and relay nodes. Hou *et. al.* [12] address the energy provisioning and relay node

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placement in a two-tiered WSN. In [13], the placement of gateway nodes is studied to maximize the lifetime of a two-tiered WSN. In [14], a greedy sensor placement scheme is proposed to maximize the lifetime of a linear WSN.

The rest of the paper is organized as follows. In Section II, we present a model of linear WSN and define network lifetime and utilization efficiency. In Section III, the utilization efficiency of the linear WSN is analyzed and its asymptotic behavior is studied. In Section IV, we propose an optimal solution and an approximation solution with reduced complexity to network configuration. Section V extends the results to a two-dimensional WSN. Numerical and simulation results are provided in Section VI. Section VII concludes the paper.

II. NETWORK MODEL

Linear WSNs have applications in border surveillance, highway traffic monitoring, and oil pipeline protection [15]. We consider an event-driven linear WSN with N sensors, each powered by a non-rechargeable battery with initial energy E_0 , and a gateway node with fixed location. Sensors are responsible for monitoring and reporting an event of interest. Due to power limitation and hardware constraint, each sensor has a sensing range of R km. We assume that the event arrival process is Poisson distributed with mean λ . Given that an event has occurred, its location is uniformly distributed in the desired coverage area [0, L] km of the network.

A. Sensor Placement

As illustrated in Fig. 1, sensors are placed along a straight line of length L km with the gateway node at the left end¹. Let s_i denote the *i*-th sensor in the network where s_1 is closest to the gateway node and s_N is farthest. Let d_1 be the distance between s_1 and the gateway node, and d_i ($2 \le i \le N$) the distance between adjacent sensors s_i and s_{i-1} . To ensure the coverage of the network under a sensing range R, adjacent sensors should not be placed farther than 2R.

¹Our approach can be readily extended to cases where the gateway node is not located at the end of linear region.

Hence, a sensor placement $\mathbf{d} \stackrel{\Delta}{=} [d_1, \dots, d_N]$ should satisfy the following constraint:

$$\begin{cases} 0 < d_1 \le R, \\ 0 < d_i \le 2R, \\ 0 < L - \sum_{j=1}^N d_j < R. \end{cases}$$
(1)

B. Transmission Structure

When an event occurs, the sensor closest to the event² initiates the reporting process by generating an equal-sized reporting packet. As a consequence, sensor s_i is responsible for reporting the event that occurs in its Voronoi cell with size A_i given by (see Fig. 1)

$$A_{i} = \begin{cases} d_{1} + \frac{d_{2}}{2}, & i = 1, \\ \frac{d_{i} + d_{i+1}}{2}, & 2 \le i \le N - 1, \\ L - \sum_{j=1}^{N-1} d_{j} - \frac{d_{N}}{2}, & i = N. \end{cases}$$

$$(2)$$

The reporting packet is then forwarded to the gateway node according to the network transmission structure $\mathbb{P} \stackrel{\Delta}{=} \{P_{i,j}\}_{i,j=1}^{N}$ whose element $P_{i,j} \in [0,1]$ denotes the probability that s_i transmits its packets to s_j . For ease of presentation, we define $P_{i,0} = 1 - \sum_{j=1}^{N} P_{i,j}$, where $1 \le i \le N$, as the probability that s_i transmits its packet directly to the gateway node. Note that in any energy-efficient transmission structure, sensors always transmit their packets toward the gateway node. Hence, an energy-efficient transmission structure \mathbb{P} should satisfy the following constraint:

$$\begin{cases} 0 \le P_{i,j} \le 1, & 1 \le i, j \le N, \\ 0 \le P_{i,0} \le 1, & 1 \le i \le N, \\ P_{i,j} = 0, & 1 \le i \le j \le N. \end{cases}$$
(3)

We briefly comment on how to enable the sensor closest to the event to initiate the reporting process in a distributed way. When the location of the event can be detected, the sensor closest to the event can be readily determined by the pre-determined sensor placement d. Otherwise,

²The sensor closest to the event will have the most accurate measurement. Similar analysis can be carried out to study the case where the sensor closest to the gateway node is responsible for reporting.

the opportunistic carrier sensing technique [16], [17] can be applied. Specifically, every sensor that detects the event maps the strength of its sensed signal to a backoff time based on a predetermined strictly decreasing function and then listens to the channel. A sensor will transmit with its chosen backoff delay if and only if no one transmits before its backoff time expires. Since the strength of the sensed signal decreases with the sensing distance, the sensor with the strongest sensed signal and hence closest to the event will initiate the reporting process. In this paper, we focus on the optimal network configuration, assuming perfect carrier sensing.

C. Energy Model

Let P_s denote the sensing power consumption of each sensor and E_{rx} the energy consumed in receiving one packet. Let \tilde{E} denote the energy required to transmit one reporting packet over a distance of 1 km. The energy consumed in transmitting one packet over a distance of d km can be modeled as [18]

$$E_{tx}(d) = E_{tc} + \dot{E}d^{\gamma},\tag{4}$$

where E_{tc} is the energy consumed in the transmitter circuitry and $2 \le \gamma \le 4$ is the path loss exponent. Note that the transmission energy consumption increases super-linearly with the transmission distance d.

D. Network Lifetime and Utilization Efficiency

We define network lifetime \mathcal{L} as the average amount of time until any sensor runs out of energy (the first failure) [14]. In general, network lifetime increases with the number of deployed sensors, but at a decreasing rate when the network size is large. That is, the contribution of each individual sensor to the network lifetime diminishes with the network size. We thus propose a performance metric — utilization efficiency η — to study the optimal network configuration. Assuming that the deployment of one sensor has one unit cost, we define utilization efficiency as network lifetime \mathcal{L} divided by the number of deployed sensors N, *i.e.*,

$$\eta = \frac{\mathcal{L}}{N}.$$
(5)

Utilization efficiency indicates the rate at which network lifetime \mathcal{L} increases with the network size N and captures the tradeoff between network lifetime and deployment cost.

Our design goal³ is to find the optimal network size N^* , sensor placement d^* , and transmission structure \mathbb{P}^* that maximize utilization efficiency, *i.e.*,

$$\{N^*, \mathbf{d}^*, \mathbb{P}^*\} = \arg\max_{N \neq \mathbb{P}} \eta.$$
(6)

III. ANALYSIS OF UTILIZATION EFFICIENCY

In this section, we analyze the utilization efficiency of a linear WSN and investigate the effect of network size and transmission structure on utilization efficiency. We find that deploying either an extremely large or an extremely small number of sensors is inefficient. Transmitting packets via multiple short hops is not the optimal transmission structure in dense WSNs.

A. A Closed-Form Expression for Utilization Efficiency

In [19], a general formula has been derived for network lifetime:

$$\mathcal{L} = \frac{\mathcal{E}_0 - \mathbb{E}[E_w]}{P_c + \lambda \mathbb{E}[E_r]} \tag{7}$$

where \mathcal{E}_0 is the network initial energy (not necessarily evenly distributed among sensors), P_c is the constant continuous power consumption over the whole network, $\mathbb{E}[E_w]$ is the expected wasted energy (the unused energy left in the network when the network dies), and $\mathbb{E}[E_r]$ is the expected reporting energy (the energy consumed over the whole network to report an event to the gateway node) in a randomly chosen reporting process. This lifetime formula (7) holds independently of the underlying network model and the definition of network lifetime. Applying (7) to our network setting, we obtain utilization efficiency as

$$\eta = \frac{E_0 - \frac{1}{N} \mathbb{E}[E_w]}{NP_s + \lambda \mathbb{E}[E_r]}.$$
(8)

The expected reporting energy consumption $\mathbb{E}[E_r]$ can be obtained as

$$\mathbb{E}[E_r] = \sum_{i=1}^N p_i \varepsilon_i,\tag{9}$$

where p_i is the probability that the event of interest occurs in the Voronoi cell of sensor s_i :

$$p_i = \frac{A_i}{L},\tag{10}$$

³In the general case where the gateway nodes is not located at one end of the linear region, the design objective is to find the optimal sensor placement **d**, transmission structure \mathbb{P} , and numbers N_l and N_r of sensors on the left and the right sides of the gateway node, *i.e.*, $\{N_l^*, N_r^*, \mathbf{d}^*, \mathbb{P}^*\} = \arg \max_{\{N_l, N_r, \mathbf{d}, \mathbb{P}\}} \eta$.

and ε_i is the expected reporting energy consumption over the whole network⁴ given that s_i initiates the reporting process. Let $E_{i,j}$ and $E_{i,0}$ be the energy consumed by s_i in transmitting one reporting packet to s_j and the gateway node, respectively. Applying (4), we obtained that

$$E_{i,j} = E_{tc} + \tilde{E} \left(\sum_{k=j+1}^{i} d_k \right)^{\gamma}, \qquad 0 \le j < i \le N.$$
(11)

Taking into account the energy E_{rx} consumed in receiving one packet, we can obtain ε_i as

$$\varepsilon_{i} = P_{i,0}E_{i,0} + \sum_{j=1}^{i-1} P_{i,j} \left[E_{i,j} + E_{rx} + \varepsilon_{j} \right]$$

$$= \begin{cases} E_{tc} + \tilde{E}d_{1}^{\gamma}, & i = 1, \\ E_{tc} + \tilde{E}\sum_{j=0}^{i-1} P_{i,j} \left(\sum_{k=j+1}^{i} d_{k} \right)^{\gamma} + E_{rx}\sum_{j=1}^{i-1} P_{i,j} + \sum_{j=1}^{i-1} P_{i,j}\varepsilon_{j}, & 2 \le i \le N. \end{cases}$$
(12)

Clearly, utilization efficiency η given in (8) depends on network size N, sensor placement d, and transmission structure \mathbb{P} .

B. The Effect of Network Size

Since the expected wasted energy is non-negative, *i.e.*, $\mathbb{E}[E_w] \ge 0$, we obtain an upper bound on utilization efficiency:

$$\eta \le \frac{E_0}{NP_s + \lambda \mathbb{E}[E_r]}.$$
(13)

This upper bound is tight when the wasted energy $\mathbb{E}[E_w]$ in the network is relatively small compared to the network initial energy NE_0 . From (13), we find that as the number N of deployed sensors goes to infinity, utilization efficiency approaches to 0:

$$\lim_{N \to \infty} \eta = 0. \tag{14}$$

That is, deploying an extremely large number N of sensors in the network is inefficient. On the other hand, deploying an extremely small number N of sensors reduces the total sensing power consumption NP_s at the expense of increasing the distance d_i between adjacent sensors, which causes more reporting energy consumption $\mathbb{E}[E_r]$ and lower utilization efficiency. Hence, the network size should be carefully chosen for optimal utilization efficiency.

⁴Note that the energy consumption over the whole network does not include the energy consumption of the gateway node.

C. The Effect of Transmission Structure

Note that $\left(\sum_{j=1}^{i} d_{j}\right)^{\gamma} \geq \sum_{j=1}^{i} d_{j}^{\gamma}$. It thus follows that when $E_{tc} = E_{rx} = 0$, the optimal transmission structure is to transmit packets via multiple short hops toward the gateway node, *i.e.*,

$$P_{i,j} = \begin{cases} 1 & j = i - 1, \ 1 \le i \le N; \\ 0 & \text{otherwise.} \end{cases}$$
(15)

In short-distance transmissions, however, neither E_{tc} nor E_{rx} can be ignored, and sending packets via multiple short hops may consume more energy than a single long hop. For example, consider a linear WSN with two sensors and a given placement $\mathbf{d} = [d_1, d_2]$. When the transmission structure \mathbb{P}_1 in which sensors transmit their packets via multiple short hops (*i.e.*, $P_{1,0} = P_{2,1} = 1$) is employed, the energy consumed over the whole network in reporting an event that occurs in the Voronoi cell of s_2 is given by

$$E(\mathbb{P}_1) = E_{tx}(d_2) + E_{rx} + E_{tx}(d_1) = 2E_{tc} + \tilde{E}(d_1^{\gamma} + d_2^{\gamma}) + E_{rx}.$$
(16)

When the transmission structure \mathbb{P}_2 in which sensors transmit their packets via a single long hop (*i.e.*, $P_{1,0} = P_{2,0} = 1$) is employed, the energy consumed over the whole network in reporting an event that occurs in the Voronoi cell of s_2 is given by

$$E(\mathbb{P}_2) = E_{tx}(d_1 + d_2) = E_{tc} + \tilde{E}(d_1 + d_2)^{\gamma}.$$
(17)

When $(d_1 + d_2)^{\gamma} - (d_1^{\gamma} + d_2^{\gamma}) < \frac{E_{tc} + E_{rx}}{\tilde{E}}$, we have $E(\mathbb{P}_1) > E(\mathbb{P}_2)$, *i.e.*, transmitting via multiple short hops consumes more energy than a single long hop. Hence, the optimal transmission structure depends on sensor placement. To achieve the optimal utilization efficiency, we should jointly optimize transmission structure and sensor placement.

IV. NETWORK CONFIGURATION FOR OPTIMAL UTILIZATION EFFICIENCY

In this section, we propose an optimal approach and an approximation approach with reduced complexity to network configuration. Specifically, we address the optimal network size N^* , sensor placement d^* , and transmission structure \mathbb{P}^* under the performance metric of utilization efficiency.

A. The Optimal Approach

The optimal utilization efficiency η^* can be written as

$$\eta^* = \max_{N,\mathbf{d},\mathbb{P}} \frac{\mathcal{L}}{N} = \max_{N} \left(\frac{1}{N} \max_{\mathbf{d},\mathbb{P}} \mathcal{L} \right), \tag{18}$$

which suggests a two-step optimization approach. First, for every given network size N, we jointly optimize sensor placement d and transmission structure \mathbb{P} for maximizing network lifetime. That is, we obtain the optimal network configuration $\{d^*(N), \mathbb{P}^*(N)\}$ for every fixed network size N:

$$\{\mathbf{d}^*(N), \mathbb{P}^*(N)\} = \arg\max_{\mathbf{d}, \mathbb{P}} \mathcal{L},\tag{19}$$

Second, we choose the optimal network size N^* by comparing the maximum utilization efficiencies achieved by each network size, *i.e.*,

$$N^* = \arg\max_N \frac{\mathcal{L}^*(N)}{N},\tag{20}$$

where $\mathcal{L}^*(N) = \max_{\mathbf{d},\mathbb{P}} \mathcal{L}$ denotes the network lifetime achieved by the optimal network configuration $\{\mathbf{d}^*(N), \mathbb{P}^*(N)\}$ when the network size is N.

1) Optimal Sensor Placement and Transmission Structure: To maximize network lifetime \mathcal{L} , we resort to the lifetime formula (7). We find that the optimal sensor placement and transmission structure should minimize both the reporting energy $\mathbb{E}[E_r]$ over the whole network in a randomly chosen reporting process and the wasted energy $\mathbb{E}[E_w]$ of the network. To minimize $\mathbb{E}[E_r]$, we should reduce the energy consumption of each sensor in the reporting process. To minimize $\mathbb{E}[E_w]$, which depends on the residual energy of sensors when the network dies, we should balance the energy consumption among sensors. With the above goals in mind, we propose an optimal strategy which minimizes the reporting energy consumption $\mathbb{E}[E_r]$ over the network under the constraint that the expected energy consumption $\mathbb{E}[E_r^{(i)}]$ of each sensor s_i in a randomly chosen reporting process is the same.

Note that the reporting energy consumption $\mathbb{E}[E_r]$ over the whole network is the sum of the sensor energy consumption $\mathbb{E}[E_r^{(i)}]$, *i.e.*, $\mathbb{E}[E_r] = \sum_{i=1}^N \mathbb{E}[E_r^{(i)}]$. Hence, under the constraint that the expected sensor energy consumption $\mathbb{E}[E_r^{(i)}]$ is the same, minimizing $\mathbb{E}[E_r]$ is equivalent to minimizing $\mathbb{E}[E_r^{(i)}]$. Considering the constraints on sensor placement (1) and transmission

structure (3), we can formulate the optimization problem in (19) as

$$\min_{\mathbf{d},\mathbb{P}} \mathbb{E}[E_{r}^{(i)}]$$
subject to: $\mathbb{E}[E_{r}^{(1)}] = \dots = \mathbb{E}[E_{r}^{(N)}],$
 $0 < d_{1} \leq R,$
 $0 < d_{i} \leq 2R, \quad \text{for } 2 \leq i \leq N-1,$
 $0 < L - \sum_{j=1}^{N} d_{j} < R,$
 $0 \leq P_{i,j} \leq 1, \quad 1 \leq i, j \leq N,$
 $0 \leq P_{i,0} \leq 1, \quad 1 \leq i \leq N,$
 $P_{i,j} = 0, \quad 1 \leq i \leq j \leq N.$
(21)

The key to solving (21) is the average energy consumption $\mathbb{E}[E_r^{(i)}]$ of each sensor s_i in a randomly selected reporting process. Let q_i denote the probability that s_i has to relay the reporting packet generated by other sensors, which can be calculated recursively as

$$q_{i} = \begin{cases} 0, & i = N, \\ \sum_{k=i+1}^{N} P_{k,i}(q_{k} + p_{k}), & 1 \le i \le N - 1. \end{cases}$$
(22)

Recall that p_i is the probability that s_i initiates the reporting process, which is given in (10). We obtain $\mathbb{E}[E_r^{(i)}]$ as

$$\mathbb{E}[E_r^{(i)}] = p_i \sum_{j=0}^{i-1} P_{i,j} E_{i,j} + q_i \left[E_{rx} + \sum_{j=0}^{i-1} P_{i,j} E_{i,j} \right]$$

$$= (p_i + q_i) \left[E_{tc} + \tilde{E} \sum_{j=0}^{i-1} P_{i,j} \left(\sum_{k=j+1}^{i} d_k \right)^{\gamma} \right] + q_i E_{rx},$$
(23)

where $E_{i,j}$ is given in (11). With the closed-form expression (23) for $\mathbb{E}[E_r^{(i)}]$, we can solve the optimization problem in (21) numerically for the optimal sensor placement $\mathbf{d}^*(N)$ and transmission structure $\mathbb{P}^*(N)$ given network size N. We can see that both $\mathbf{d}^*(N)$ and $\mathbb{P}^*(N)$ depend on sensing range R and channel path loss exponent γ , but is independent of event arrival rate λ and sensing power consumption P_s . It, however, should be mentioned that both λ and P_s play important roles in choosing network size N. 2) Optimal Network Size: To obtain the optimal network size N^* given in (20), we need to calculate the optimal lifetime $\mathcal{L}^*(N)$ achieved by the optimal sensor placement $\mathbf{d}^*(N)$ and transmission structure $\mathbb{P}^*(N)$. Unfortunately, the calculation of the average wasted energy $\mathbb{E}[E_w]$ in the lifetime formula (7) is usually intractable. We are thus motivated to find a simple approximation to the optimal network size N^* .

Since $d^*(N)$ and $\mathbb{P}^*(N)$ are designed to balance the energy consumption $\mathbb{E}[E_r^{(i)}]$ of each sensor s_i , the wasted energy $\mathbb{E}[E_w]$ of the network is negligible as compared to the network initial energy NE_0 . Hence, the upper bound on utilization efficiency given in (13) is tight. We can thus approximate the optimal network size N^* as:

$$N^* \approx \arg\max_N \frac{E_0}{NP_s + \lambda \mathbb{E}[E_r]},\tag{24}$$

where $\mathbb{E}[E_r]$ can be obtained by substituting the optimal $\mathbf{d}^*(N)$ and $\mathbb{P}^*(N)$ into (9). Since $\mathbf{d}^*(N)$ and $\mathbb{P}^*(N)$ are obtained to satisfy the constraint $\mathbb{E}[E_r^{(1)}] = \ldots = \mathbb{E}[E_r^{(N)}]$, we have $\mathbb{E}[E_r] = \sum_{i=1}^N \mathbb{E}[E_r^{(i)}] = N\mathbb{E}[E_r^{(N)}]$. Hence, (24) can be simplified as

$$N^* \approx \arg\min_{N} \left\{ NP_s + N\lambda \mathbb{E}[E_r^{(N)}] \right\}$$

= $\arg\min_{N} \left\{ NP_s + \frac{N\lambda A_N^*}{L} \left[E_{tc} + \tilde{E} \sum_{j=0}^{N-1} P_{N,j}^* \left(\sum_{k=j+1}^N d_k^* \right)^\gamma \right] \right\},$ (25)

where d_k^* , A_N^* , and $P_{i,j}^*$ are determined by the corresponding $\mathbf{d}^*(N)$ and $\mathbb{P}^*(N)$. Clearly, the optimal network size N^* depends on both event arrival rate λ and sensing power consumption P_s .

B. An Approximation Approach

For a given network size N, (21) provides an optimal solution to the lifetime maximization problem by jointly optimizing sensor placement and transmission structure. The complexity of (21), however, increases dramatically with N since the number of optimizing parameters d and \mathbb{P} increases in the order of N^2 . We thus propose an approximation approach which reduces the dimension of optimization in (21) by fixing the transmission structure \mathbb{P} .

We have shown that when E_{tc} and E_{rx} are negligible, the optimal transmission strategy is to transmit packets via multiple short hops, *i.e.*, each sensor should transmit its packets to its nearest left neighbor. We thus fix the transmission structure as (15) (see Fig. 2), and simplify



Fig. 2. Transmission structure in the approximation approach for linear WSN.

the optimization problem in (21) as

$$\min_{\mathbf{d}} \mathbb{E}[E_r^{(i)}]$$
subject to: $\mathbb{E}[E_r^{(1)}] = \ldots = \mathbb{E}[E_r^{(N)}],$

$$0 < d_1 \le R, \qquad (26)$$

$$0 < d_i \le 2R, \quad \text{for } 2 \le i \le N - 1,$$

$$0 < L - \sum_{j=1}^N d_j < R.$$

where the average energy consumption $\mathbb{E}[E_r^{(i)}]$ of s_i in a randomly chosen reporting process can be simplified as

$$\mathbb{E}[E_r^{(i)}] = \frac{E_{tc} + E_{rx} + \tilde{E}d_i^{\gamma}}{L} \sum_{j=i}^N A_j - \frac{E_{rx}}{L}A_i.$$
(27)

Note that the number of optimizing parameters in (26) increases in the order of N instead of N^2 . Since transmitting via multiple short hops is optimal in long-distance transmissions, the approximation given in (26) offers optimal performance for sparse networks where the distance between adjacent sensors is large.

Applying (27) to (25), we can obtain an approximation to the optimal network size N^* as:

$$N^* \approx \arg\min_{N} \left\{ NP_s + \frac{N\lambda A_N}{L} [E_{tc} + \tilde{E}d_N^{\gamma}] \right\}.$$
(28)

V. EXTENSION TO TWO-DIMENSIONAL WSNs

This section extends results obtained in Section IV to a two-dimensional WSN with grid structures. We propose an optimal approach and a heuristic approach with reduced complexity to network configuration for optimal utilization efficiency.

A. Network Model

Consider a two-dimensional WSN with N^2 sensors and a coverage area of $L \text{ km} \times L \text{ km}$. The event arrival process is assumed to be Poisson distributed with mean λ and the location of the event is uniformly distributed in the desired coverage area $[0, L] \times [0, L]$ of the network.



Fig. 3. A grid-based WSN.

Sensors are placed at the intersections of the grids, and the gateway node is located at the leftbottom corner of the square (see Fig. 3). Let $\{s_{ij}\}_{i,j=1}^N$ denote the (i, j)-th sensor in the network and $\mathbf{d} = [d_1, \ldots, d_N]$ the distance between two adjacent grids. Since the sensor closest to the event is responsible for initiating the reporting process, the Voronoi cell of s_{ij} is a rectangle with size

$$A_{ij} = A_i A_j, \tag{29}$$

where A_i is given in (2). The distance between s_{ij} and the corners of its rectangular Voronoi cell is given by $\sqrt{a_i^2 + a_j^2}$, $\sqrt{a_{i+1}^2 + a_j^2}$, $\sqrt{a_i^2 + a_{j+1}^2}$, and $\sqrt{a_{i+1}^2 + a_{j+1}^2}$, respectively, where

$$a_{i} = \begin{cases} d_{1}, & i = 1, \\ \frac{d_{i}}{2}, & 2 \leq i \leq N, \\ L - \sum_{j=1}^{N} d_{i}, & i = N + 1. \end{cases}$$
(30)

Hence, to ensure network coverage under a sensing range R, the grid-spacing d should satisfy the following constraint:

$$\left(\max\{a_i, a_{i+1}\}\right)^2 + \left(\max\{a_j, a_{j+1}\}\right)^2 \le R^2, \quad 1 \le i \le j \le N.$$
(31)

Let $P_{l,m}^{(i,j)}$ denote the probability that sensor s_{ij} sends its packet to sensor s_{lm} and $P_{0,0}^{(i,j)}$ the probability that sensor s_{ij} sends its packet directly to the gateway node. We will only consider energy-efficient transmission structures \mathbb{P} where $P_{i,j}^{(i,j)} = 0$ and $P_{l,m}^{(i,j)} = 0$ for $1 \le i < l \le N$ and $1 \le j < m \le N$, *i.e.*, sensors always transmit toward the gateway node. Thus, the transmission structure should satisfy

$$\begin{cases} 0 \le P_{l,m}^{(i,j)} \le 1, & \text{for } 1 \le l \le i \le N \text{ and } 1 \le m \le j \le N, \\ P_{i,j}^{(i,j)} = 0, P_{l,m}^{(i,j)} = 0, & \text{for } 1 \le i < l \le N \text{ and } 1 \le j < m \le N, \\ 0 \le P_{0,0}^{(i,j)} \le 1, & \text{for } 1 \le i, j \le N, \\ P_{0,0}^{(i,j)} + \sum_{l=1}^{i} \sum_{m=1}^{j} P_{l,m}^{(i,j)} = 1, & \text{for } 1 \le i, j \le N. \end{cases}$$

$$(32)$$

Our goal is to design the size of the network N, the grid-spacing d, and the transmission structure $P_{l,m}^{(i,j)}$ for optimal utilization efficiency.

Applying the lifetime formula (7), we obtain utilization efficiency of the grid-based WSN as

$$\eta = \frac{E_0 - \frac{1}{N^2} \mathbb{E}[E_w]}{N^2 P_s + \lambda \mathbb{E}[E_r]}$$
(33)

where $\mathbb{E}[E_r] = \sum_{i=1}^N \sum_{j=1}^N \mathbb{E}[E_r^{(i,j)}]$ is the average reporting energy consumption over the whole network in a randomly chosen reporting process, and $\mathbb{E}[E_r^{(i,j)}]$ is the average reporting energy of s_{ij} in a randomly chosen reporting process.

B. Network Configuration for Optimal Utilization Efficiency

Similar to (21), the optimal network configuration for the grid-based WSNs with a given number N^2 of sensors can be formulated as a multi-variant non-linear optimization problem with constraints on sensor placement (31) and transmission structure (32):

$$\min_{\mathbf{d},\mathbb{P}} \mathbb{E}[E_r^{(i,j)}]$$
subject to: $\mathbb{E}[E_r^{(1,1)}] = \dots = \mathbb{E}[E_r^{(N,N)}],$

$$(\max\{a_i, a_{i+1}\})^2 + (\max\{a_j, a_{j+1}\})^2 \le R^2, \quad 1 \le i \le j \le N,$$

$$0 \le P_{l,m}^{(i,j)} \le 1, \quad \text{for } 1 \le l \le i \le N \text{ and } 1 \le m \le j \le N,$$

$$P_{i,j}^{(i,j)} = 0, P_{l,m}^{(i,j)} = 0, \quad \text{for } 1 \le i < l \le N \text{ and } 1 \le j < m \le N,$$

$$0 \le P_{0,0}^{(i,j)} \le 1, \quad \text{for } 1 \le i, j \le N,$$

$$P_{0,0}^{(i,j)} \le 1, \quad \text{for } 1 \le i, j \le N,$$

$$P_{0,0}^{(i,j)} + \sum_{l=1}^{i} \sum_{m=1}^{j} P_{l,m}^{(i,j)} = 1, \quad \text{for } 1 \le i, j \le N.$$

$$(34)$$

Next, we derive $\mathbb{E}[E_r^{(i,j)}]$ for every s_{ij} . Let q_{ij} be the probability that s_{ij} relays the reporting packet generated by other sensors, which can be obtained as

$$q_{ij} = \begin{cases} 0, & i = j = N, \\ \sum_{l=i}^{N} \sum_{m=j}^{N} P_{i,j}^{l,m} (q_{lm} + p_{lm}), & \text{otherwise} \end{cases}$$
(35)

where p_{ij} is the probability that s_{ij} initiates the reporting process, which is given by

$$p_{ij} = \frac{A_{ij}}{L^2}.$$
(36)

The required energy for s_{ij} to transmit a reporting packet to s_{lm} is given by

1

$$E_{l,m}^{(i,j)} = \begin{cases} E_{tc} + \left[\left(\sum_{k=l+1}^{i} d_{k} \right)^{2} + \left(\sum_{k=m+1}^{j} d_{k} \right)^{2} \right]^{\frac{\gamma}{2}}, & 1 \le l \le i-1, 1 \le m \le j-1, \\ E_{tc} + \left(\sum_{k=m+1}^{j} d_{k} \right)^{\gamma}, & l = i, 1 \le m \le j-1 \\ E_{tc} + \left(\sum_{k=l+1}^{i} d_{k} \right)^{\gamma}, & 1 \le l \le i-1, m = j, \\ E_{tc} + \left[\left(\sum_{k=1}^{i} d_{k} \right)^{2} + \left(\sum_{k=1}^{j} d_{k} \right)^{2} \right]^{\frac{\gamma}{2}}, & l = m = 0. \end{cases}$$

$$(37)$$

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Hence, $\mathbb{E}[E_r^{(i,j)}]$ can be obtained as

$$\mathbb{E}[E_{r}^{(i,j)}] = (q_{ij} + p_{ij}) \left\{ \sum_{l=1}^{i-1} \sum_{m=1}^{j-1} P_{l,m}^{(i,j)} E_{l,m}^{(i,j)} + \sum_{l=1}^{i-1} P_{l,j}^{(i,j)} E_{l,j}^{(i,j)} + \sum_{m=1}^{j-1} P_{i,m}^{(i,j)} E_{i,m}^{(i,j)} + P_{0,0}^{(i,j)} E_{0,0}^{(i,j)} \right\} - p_{ij} E_{rx}.$$
(38)

Solving (34), we obtain the optimal grid-spacing $d^*(N)$ and transmission structure $\mathbb{P}^*(N)$ for every network size N. Using the upper bound on utilization efficiency, we can approximate the optimal network size N^2 as

$$N^* \approx \arg\min_{N} \left\{ N^2 P_s + \lambda N^2 \mathbb{E}[E_r^{(N,N)}] \right\},\tag{39}$$

where $\mathbb{E}[E_r^{(N,N)}]$ can be computed by substituting the optimal grid-spacing $\mathbf{d}^*(N)$ and transmission structure $\mathbb{P}^*(N)$ into (38).

Notice that the optimizing parameters d and \mathbb{P} increases in the order of N^4 , which makes (34) computationally prohibitive. Next, we propose a heuristic approach with reduced complexity to configure the grid-based WSN. The basic idea is to reduce the dimension of optimization by fixing the transmission structure \mathbb{P} . One possible transmission structure is given by (see Fig. 4)

$$P_{l,m}^{(i,j)} = \begin{cases} \frac{1}{2}, & l = i - 1 \& m = j \text{ or } l = i \& m = j - 1, \\ 1, & l = i = 1 \& m = j - 1 \text{ or } l = i - 1 \& m = j = 1, \\ 0, & \text{otherwise.} \end{cases}$$
(40)

That is, each sensor $s_{i,j}$ transmit its packets to its nearest vertical neighbor $s_{i-1,j}$ toward the gateway node with probability 0.5 and to its nearest horizontal neighbor $s_{i,j-1}$ toward the gateway node with probability 0.5. If a sensor does not have a vertical (horizontal) neighbor, it forwards all packets to its horizontal (vertical) neighbor. The above transmission structure balances the traffic load of sensors located at the same distance from the gateway node. Unfortunately, with fixed transmission structure, there is no feasible solution to (34) because the number of optimizing parameters is much less than the number of valid equations in the constraints. It is impossible to find a grid placement with which the average energy consumption of each sensor is the same. Notice that the network lifetime is limit by the minimum lifetime of the sensors. Our heuristic approach, referred to as minimax, minimizes the maximum energy consumption of sensors.



Fig. 4. Transmission structure in the heuristic approach for grid-based WSN.

Specifically,

$$\min_{\mathbf{d}} \max_{1 \le i,j \le N} \{ \mathbb{E}[E_r^{(i,j)}] \}$$
subject to: $(\max\{a_i, a_{i+1}\})^2 + (\max\{a_j, a_{j+1}\})^2 \le R^2, \quad 1 \le i \le j \le N.$

$$(41)$$

Notice that the number of optimizing parameters in (41) increases in the order of N instead of N^4 as in (34).



Fig. 5. Utilization efficiency of optimal and approximation approaches. $\lambda = 0.1$, R = 0.2 km, $\gamma = 2$, $E_0 = 50$, L = 2 km.

VI. NUMERICAL AND SIMULATION EXAMPLES

This section provides some numerical and simulation examples to study the optimal network configuration in terms of utilization efficiency. Utilization efficiency achieved by the uniform network configuration where sensors are equally-spaced is also plotted for comparison. In all the figures, we normalize the energy and power quantities by the energy \tilde{E} required to transmit one packet over a distance of 1 km. We assume that the energy consumed to receive a reporting packet is $E_{rx} = 1.35 \times 10^{-2}$, and the transmitter circuitry energy consumption is $E_{tc} = 4.5 \times 10^{-3}$ per transmission [20]. The constant sensing power consumption is assumed to be $P_s = 5 \times 10^{-3}$ [20].

We first consider a linear WSN. Fig. 5 compares the utilization efficiency achieved by the optimal configuration (21) and the approximation approach (26). The approximation approach



Fig. 6. Sensor placement for different sensing range. $R = \{1, 2, 3\}$ km, $\gamma = 2$, N = 15, $E_0 = 20$, L = 10 km.

has near optimal performance when the network size N is small. When the network size N is large, however, the approximation approach performs worse than the optimal approach because transmitting via multiple short hops is not optimal in dense networks.

Since the approximation approach provides nearly optimal performance but with less complexity, we focus on the study of the approximation approach given in (26). Figs. 6 and 7 show, respectively, the effect of the sensing range R and the path loss exponent γ on the sensor placement. Recall that sensors closer to the gateway node carry more payloads than those further away. To balance the energy consumption of each sensor (23), we need to assign shorter relay distance to those sensors that are closer to the gateway node. As expected, the distance d_i between adjacent sensors increases with the index of sensor s_i . We find that it is always desirable to place the last sensor s_N as close to the gateway node as possible in order to reduce the distance



Fig. 7. Sensor placement for different path loss exponents. $\gamma = \{2, 4\}, R = 1$ km, $N = 15, E_0 = 20, L = 10$ km.

between adjacent sensors and consequently the reporting energy consumption. Due to the limit of its sensing range, the last sensor is usually placed L - R km away from the gateway node. We also find that as the pass loss exponent γ increases, sensors are placed more uniformly. This agrees with our expectation that when γ is large, the d_i^{γ} term dominates the energy consumption $\mathbb{E}[E_r^{(i)}]$ of each sensor (23) and thus a more uniform placement is desired to balance $\mathbb{E}[E_r^{(i)}]$.

Fig. 8 compares the utilization efficiency achieved by the approximation and the uniform approaches. The uniform configuration employs the same transmission structure as the approximation approach, where sensors transmit their packets via multiple short hops. Unlike network lifetime which increases with network size N [14], utilization efficiency increases when N is small and decreases when N is large. It diminishes for extremely large or extremely small number of sensors. Since network lifetime decreases with the event arrival rate λ for each

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Fig. 8. Utilization efficiency of approximation and uniform approaches. $\lambda = \{0.05, 0.08, 0.1\}, R = 1 \text{ km}, \gamma = 2, E_0 = 20, L = 10 \text{ km}.$

N, utilization efficiency η also decreases with λ . The approximation approach outperforms the uniform placement. We also notice that when λ is large, the utilization efficiency η curves are more flat; however, when λ is small, the η curves change widely. This observation agrees with our expectation: since λ appears in the denominator of η (see (8)), η is more sensitive to small λ .

In Tables I-II, we demonstrate the effect of event arrival rates λ and sensing power consumption P_s on the optimal network size N^* . We also compare the numerical approximation N_a^* given in (28) with the simulation result N^* . We can see that the numerical approximate N_a^* is very close to the simulation result N^* . The optimal number of sensors increases with λ , but the rate of increasing diminishes. As P_s increases, the optimal network size decreases and so does its

TABLE I

The optimal network size N^* and its approximate N_a^* given in (28) for different event arrival rates λ ,

$$P_s = 5 \times 10^{-3}, E_0 = 5, L = 2 \text{ KM}$$

	$\lambda = 0.05$	$\lambda = 0.08$	$\lambda = 0.1$	$\lambda = 0.2$
N^*	22	26	28	33
N_a^*	19	24	26	33

TABLE II

The optimal network size N^st and its approximate N_a^st given in (28) for different sensing power

Consumption P_s . $\lambda = 0.05, E_0 = 5, L = 2$ km.

	$P_s = 10^{-3}$	$P_s = 5 \times 10^{-3}$	$P_s = 10^{-2}$
N^*	38	22	16
N_a^*	36	19	14

rate. The above observations agree with our intuitions. When the event arrival rate λ is large, more reporting processes are required. Hence, deploying more sensors is desirable in order to reduce the energy consumption in each reporting process by reducing the transmission distance. On the other hand, when the sensing power consumption P_s is large, deploying less sensors is preferred in order to reduce the total energy consumed in sensing.

Fig. 9 compares utilization efficiency of the minimax approach (41) and the uniform configuration in a grid-based two-dimension WSN. The uniform configuration employs the same transmission structure as the minimax approach. The minimax approach outperforms the uniform configuration. Similar to Fig. 8, when the number N of sensors is large, the performance gain achieved by the minimax approach diminishes.

VII. CONCLUSION

In this paper, we introduced the performance measure of utilization efficiency, which is defined as network lifetime per unit deployment cost. We proposed an optimal approach and an approximation approach with reduced complexity to optimize network size, sensor placement, and transmission structure for optimal utilization efficiency. The approximation approach has



Fig. 9. Utilization efficiency of heuristic and uniform approaches. $\lambda = \{0.05, 0.08, 0.1\}, R = 1 \text{ km}, \gamma = 2, E_0 = 20, L = 10 \text{ km}.$

nearly optimal performance for sparse networks. We studied the effect of sensing range, channel path loss exponent, event arrival rate, and sensing power consumption on the optimal network configuration. We observed the following.

- When the event arrival rate is large, more sensors should be deployed to reduce the reporting energy consumption. On the other hand, when the sensing energy consumption is large, a smaller WSN is preferred to reduce the total (network-level) energy consumed in sensing.
- Sensors should be placed as close to the gateway node as possible to reduce the reporting energy consumption under the network coverage constraint. Hence, when sensing range is large, sensors are placed more compact and closer toward the gateway node. When the channel path loss exponent is large, sensors should be placed more evenly in the desired

coverage area of the network.

• The optimal transmission structure depends on the node density and sensor radio model. Specifically, in relatively sparse WSN, the transmission energy dominates; transmitting data via multiple short hops is optimal. In dense WSNs, however, energy consumed in transceiver circuitry and data reception can be significant; relaying data over longer hops may be desirable.

The above observations provide general design guidelines for network configuration under the performance measure of utilization efficiency.

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