SENSOR PLACEMENT FOR MAXIMIZING LIFETIME PER UNIT COST IN WIRELESS SENSOR NETWORKS

Yunxia Chen, Chen-Nee Chuah, and Qing Zhao
Department of Electrical and Computer Engineering
University of California, Davis, CA

ABSTRACT

Lifetime per unit cost, defined as the network lifetime divided by the number of sensors deployed in the network, can be used to measure the utilization efficiency of sensors in a wireless sensor network (WSN). Analyzing the lifetime per unit cost of a linear WSN, we find that deploying either an extremely large or an extremely small number of sensors is inefficient in terms of lifetime per unit cost. We thus seek answers to the following questions: how many sensors should be deployed and how to deploy them to maximize the lifetime per unit cost. Numerical and simulation results are provided to study the optimal sensor placement and the optimal number of deployed sensors.

I. INTRODUCTION

Wireless sensor networks (WSNs) have captured great attention recently due to their enormous potential for both commercial and military applications. A WSN consists of a large number of low-cost, low-power, energy-constrained sensors with limited computation and communication capability. Sensors are responsible for monitoring certain phenomenon within their sensing ranges and reporting to gateway nodes where the end-user can access the data.

In WSNs, sensors can be deployed either randomly or deterministically [1]. Generally, fewer sensors are required to perform the same task in a deterministic deployment than a random deployment. Research efforts have been made to design optimal sensor placement schemes under different performance metrics. For example, Dhillon and Chakrabarty [2] propose two algorithms to optimize the sensor placement with a minimum number of sensors for effective coverage and surveillance purposes under the constraint of probabilistic sensor detections and terrain properties. Ganesan *et. al.* [3] jointly optimize the sensor placement and the transmission structure in a one-dimensional data-gathering WSN. Their approach is aimed at minimizing the total power consumption under distortion constraints. Kar and Banerjee [4] address the optimal sensor placement to ensure connected coverage in WSNs. Sensor placement schemes that maximize network lifetime have also been addressed for different WSNs. For example, Dasgupta *et. al.* [5] propose an algorithm to find the optimal placement and role assignment to maximize the lifetime of a WSN which consists of two types of nodes: sensor nodes and relay nodes. Hou *et. al.* [6] address the energy provisioning and relay node placement in a two-tiered WSN. In [7], the placement of the gateway node is studied to maximize the lifetime of a two-tiered WSN. In [8], a greedy sensor placement that minimizes and balances the average energy consumption of each sensor is proposed to maximize the lifetime of a linear WSN.

While many published papers focus on optimizing sensor placement for lifetime maximization, this paper aims at maximizing the utilization efficiency of sensors in an event-driven linear WSN. In most WSNs, the network lifetime increases with the number of deployed sensors, but the rate of increasing diminishes. We propose a new performance metric, called lifetime per unit cost, to measure the utilization efficiency of sensors. We define the lifetime per unit cost as the network lifetime divided by the number of deployed sensors. We find that deploying either an extremely large or an extremely small number of sensors leads to low lifetime per unit cost. We are thus motivated to optimize both the number of sensors and their placement for maximizing the lifetime per unit cost. Our approach is carried out in two steps. First, we apply a greedy strategy to optimize the sensor placement. Second, we propose a numerical approximation to determine the optimal number N^* of sensors. We find that sensors should be placed more uniformly as their sensing range or the path loss exponent increases, and more sensors should be deployed as the event arrival rate increases or the sensing power consumption decreases.

II. NETWORK MODEL AND LIFETIME DEFINITION

Consider an event-driven linear WSN with N sensors, each powered by a non-rechargeable battery with initial energy E_0. Sensors are responsible for monitoring the event of interest and reporting it to the gateway node where the end-user can access. Due to the power limit and hardware constraint, every sensor has a sensing range of

R km and a communication range of $2R$ km. Sensors are placed in sequence along a straight line of length L km with the gateway node at the left end (see Fig. 1). Let s_i denote the i-th sensor in the network where s_1 is closest to the gateway node and s_N is the furthest, and d_i the distance between adjacent sensors s_i and s_{i-1}. To ensure the coverage of the network, a sensor placement $\{d_i\}_{i=1}^N$ should satisfy the following constraint:

$$0 < d_1 \le R, \tag{1a}$$
$$0 < d_i \le 2R, \qquad \text{for } 2 \le i \le N-1, \tag{1b}$$
$$0 < L - \sum_{j=1}^{N} d_j < R. \tag{1c}$$

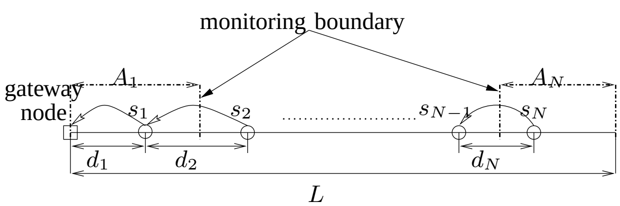

Fig. 1. A linear WSN.

When an event of interest occurs, the sensor that is closest to the event will initiate the reporting process by generating an equal-sized packet and sending it to its nearest left neighbor. It is equivalent to allowing the sensor with the strongest sensed signal to report since the strength of the sensed signal decreases with the sensing distance. Opportunistic carrier sensing [9], [10] can thus be employed to determine which sensor should report. Specifically, each sensor that detects the event maps the strength of its sensed signal to a backoff time based on a predetermined strictly decreasing function and then listens to the channel. Sensor will transmit with its chosen backoff delay if and only if no one transmits before its backoff time expires. When the propagation delay is negligible, the sensor with the strongest sensed signal and hence closest to the event will initiate the reporting process. As a concequence, sensor s_i is responsible for reporting the event that occurs in its Voronoi cell with size A_i given by (see Fig. 1)

$$A_i = \begin{cases} d_1 + \dfrac{d_2}{2}, & i = 1, \\ \dfrac{d_i + d_{i+1}}{2}, & 2 \le i \le N-1, \\ L - \displaystyle\sum_{j=1}^{N-1} d_j - \dfrac{d_N}{2}, & i = N. \end{cases} \tag{2}$$

The reporting packet is then relayed sequentially to the gateway node. For example, the packet from s_i will be relayed via $s_{i-1}, s_{i-2}, \ldots, s_1$ to the gateway node. We assume that the event arrival process is Poisson distributed with mean λ and the location of the event is uniformly distributed in the desired coverage area $[0, L]$ of the network.

Let $\tilde{E}$ denote the energy required to transmit one reporting packet over the distance of 1 km. The energy consumed to transmit one packet over a distance of d km can be modelled as

$$E_{tx}(d) = E_{tc} + \tilde{E} d^{\gamma} \tag{3}$$

where E_{tc} is the energy consumed in the transmitter circuitry and $2 \le \gamma \le 4$ is the path loss exponent. Notice that the transmission energy consumption $E_{tx}(d)$ increases super-linearly with the transmitting distance d. Let P_s denote the sensing power consumption of each sensor and E_{rx} the energy consumed to receive one packet.

For our network setting, we define the network lifetime as the amount of time until any sensor runs out of energy [8], which is equivalent to the minimum lifetime of the sensors, *i.e.*,

$$\mathbb{E}[\mathcal{L}] = \mathbb{E}[\min_i(\mathcal{L}_i)] \tag{4}$$

where $\mathcal{L}_i$ is the lifetime of s_i.

III. Lifetime per Unit Cost Analysis

To measure the utilization efficiency of sensors, we define the lifetime per unit cost η as the network lifetime $\mathcal{L}$ divided by the number of deployed sensors N, *i.e.*,

$$\eta = \frac{\mathbb{E}[\mathcal{L}]}{N}. \tag{5}$$

Lifetime per unit cost shows the rate at which the network lifetime $\mathcal{L}$ increases with the number N of sensors. In this section, we derive the lifetime per unit cost of the linear WSN and analyze its asymptotic behavior.

In [11], a general formula has been derived for the lifetime of any WSN, which holds independently of the underlying network model and the definition of network lifetime. Applying this lifetime formula to our network setting, we obtain the lifetime per unit cost as:

$$\eta = \frac{E_0 - \frac{1}{N}\mathbb{E}[E_w]}{NP_s + \lambda \mathbb{E}[E_r]}, \tag{6}$$

where $\mathbb{E}[E_w]$ is the expected wasted energy (the unused energy of sensors when the network dies) over the whole network and $\mathbb{E}[E_r]$ is the expected reporting energy (the energy consumed over the whole network to report an event) in a randomly chosen reporting process, which can be obtained as (see Appendix A):

$$\begin{aligned}\mathbb{E}[E_r] =& \frac{E_{tc}+E_{rx}}{L}\sum_{i=1}^{N} iA_i - E_{rx} \\ &+ \frac{\tilde{E}}{L}\sum_{i=1}^{N}\left(\sum_{j=i}^{N} A_j\right) d_i^{\gamma}.\end{aligned} \tag{7}$$

Equation (6) shows that the lifetime per unit cost η depends on not only the energy model of the network, the event arrival rate λ, and the sensing power consumption P_s, but also the number N of deployed sensors and the sensor placement $\{d_i\}_{i=1}^N$. We aim to seek the answers to the following questions: how many sensors should be deployed and how to deploy them to maximize the lifetime per unit cost.

Noticing that $\mathbb{E}[E_w] \geq 0$, we derive an upper bound for the lifetime per unit cost (6) as

$$\eta \leq \frac{E_0}{NE_s + \lambda \mathbb{E}[E_r]}. \tag{8}$$

The upper bound (8) is tight when the wasted energy $\mathbb{E}[E_w]$ in the network is negligible compared to the network initial energy NE_0. From (8), we find that as the number N of deployed sensors goes to infinity, the lifetime per unit cost approaches 0:

$$\lim_{N\to\infty} \eta = 0. \tag{9}$$

Hence, deploying an extremely large number N of sensors in the network is inefficient in terms of lifetime per unit cost. On the other hand, careful inspection of (6) reveals that deploying an extremely small number N of sensors reduces the sensing power consumption NP_s at the expense of increasing the distance d_i between adjacent sensors which causes more reporting energy consumption $\mathbb{E}[E_r]$. Hence, the number N of sensors and the sensor placement $\{d_i\}_{i=1}^N$ should be carefully chosen for maximizing the lifetime per unit cost of a WSN.

IV. Sensor Placement for Lifetime per Unit Cost Maximization

In the last section, we have shown that deploying either an extremely large or an extremely small number of sensors leads to low lifetime per unit cost. In this section, we apply a greedy approach to optimize the sensor placement $\{d_i\}_{i=1}^N$ and propose a numerical approximation to compute the optimal number N of sensors for maximizing the lifetime per unit cost. Our solution can be carried out in two steps. First, fix the number N of deployed sensors and optimize the sensor placement $\{d_i\}_{i=1}^N$ for network lifetime maximization. Second, apply the optimal sensor placement to optimize the number N^* of sensors for lifetime per unit cost maximization.

A. *Optimize Sensor Placement*

From (6), we find that to maximize the lifetime per unit cost for a fixed number N of sensors, the optimal sensor placement should minimize both the wasted energy $\mathbb{E}[E_w]$ and the reporting energy $\mathbb{E}[E_r]$. With this goal in mind, we apply a greedy strategy [8] which minimizes the reporting energy consumption $\mathbb{E}[E_r]$ over the whole network under the constraint that the average energy consumption $\mathbb{E}[E_r^{(i)}]$ of each sensor is the same. The greedy sensor placement can be formulated as

$$\begin{aligned}&\min_{\{d_i\}_{i=1}^N} \mathbb{E}[E_r] \\ &\text{subject to: } \mathbb{E}[E_r^{(1)}] = \ldots = \mathbb{E}[E_r^{(N)}] \\ &\qquad\qquad \text{and the coverage constraint (1).}\end{aligned} \tag{10}$$

To solve (10), we derive the average energy consumption $\mathbb{E}[E_r^{(i)}]$ of s_i in a randomly selected reporting process as

$$\begin{aligned}\mathbb{E}[E_r^{(i)}] =& \frac{E_{tc}+\tilde{E}d_i^{\gamma}}{L}\sum_{j=i}^{N} A_j + \frac{E_{rx}}{L}\sum_{j=i+1}^{N} A_j \\ =& \frac{E_{tc}+E_{rx}+\tilde{E}d_i^{\gamma}}{L}\sum_{j=i}^{N} A_j - \frac{E_{rx}}{L}A_i\end{aligned} \tag{11}$$

Combining (7) and (11) yields the relation between $\mathbb{E}[E_r]$ and $\mathbb{E}[E_r^{(i)}]$:

$$\mathbb{E}[E_r] = \sum_{i=1}^{N} \mathbb{E}[E_r^{(i)}]. \tag{12}$$

With (11) and (12), the greedy sensor placement problem reduces to a multi-variant non-linear optimization problem, which can be solved numerically. We find that the greedy sensor placement $\{d_i\}_{i=1}^N$ depends on not only the underlying energy model but also the sensing region R and the pass loss exponent γ. We also notice that for a given N, the greedy sensor placement is independent of the event arrival rate λ and the sensing power consumption P_s. It, however, should be mentioned that both λ and P_s play important roles in the lifetime per unit cost of the network and the selection of optimal number of sensors.

B. *Optimize the Number of Sensors*

With the numerical solution $\{d_i\}_{i=1}^N$ to (10), we are ready to optimize the number N^* of sensors for maximizing the lifetime per unit cost η, which is given by

$$N^* = \arg\max_N \frac{E_0 - \frac{1}{N}\mathbb{E}[E_w]}{NE_s + \lambda\mathbb{E}[E_r]}. \tag{13}$$

Unfortunately, the calculation of the average wasted energy $\mathbb{E}[E_w]$ is usually intractable. We thus propose a numerical approximation to calculate (13) by using the upper bound (8) of the lifetime per unit cost (6). Since the greedy sensor placement $\{d_i\}_{i=1}^N$ is designed to balance the energy consumption of sensors, the wasted energy of the network is negligible and (8) is tight. Hence, we can approximate N^* as

$$N^* \approx \arg\max_N \frac{E_0}{NE_s + \lambda\mathbb{E}[E_r]} \tag{14}$$

where $\mathbb{E}[E_r]$ can be readily obtained by substituting the optimal placement $\{d_i\}_{i=1}^N$ into (7).

V. Numerical and Simulation Examples

This section provides some numerical and simulation examples to study the greedy sensor placement $\{d_i\}_{i=1}^N$ and the optimal number N^* of sensors, and compare the lifetime per unit cost η of the greedy sensor placement and the uniform sensor placement where sensors are equally-spaced. In all the figures, we normalize the energy and power quantities by the energy $\tilde{E}$ required to transmit one packet over the distance of 1 km. The initial energy of each sensor is $E_0 = 20$. We assume that the energy consumed to receive a reporting packet is $E_{rx} = 1.35 \times 10^{-2}$, and the transmitter circuitry energy consumption is $E_{tc} = 4.5 \times 10^{-3}$ per transmission. The sensing power consumption is assumed to be $P_s = 5 \times 10^{-3}$. The network coverage area is $L = 10$ km.

Figs. 2 and 3 show the effect of the sensing range R and the path loss exponent γ on the greedy sensor placement. Recall that sensors closer to the gateway node carry more payloads than those further away. To balance the energy consumption of each sensor (11), we need to assign shorter relay distance to those sensors that are closer to the gateway node. As expected, the distance d_i between adjacent sensors increases with the index of sensor s_i. We find that it is always desired to place the last sensor s_N as close to the gateway node as possible in order to reduce the distance between adjacent sensors and the reporting energy consumption. Due to the limit of its sensing range, the last sensor is usually placed $L - R$ km away from the gateway node. We also find that as the pass loss exponent γ increases, sensors are placed more uniformly. This agrees with our expectation that when γ is large, the d_i^γ term dominates the energy consumption of each sensor $\mathbb{E}[E_r^{(i)}]$ (11) and thus a more uniform placement is desired to balance $\mathbb{E}[E_r^{(i)}]$.

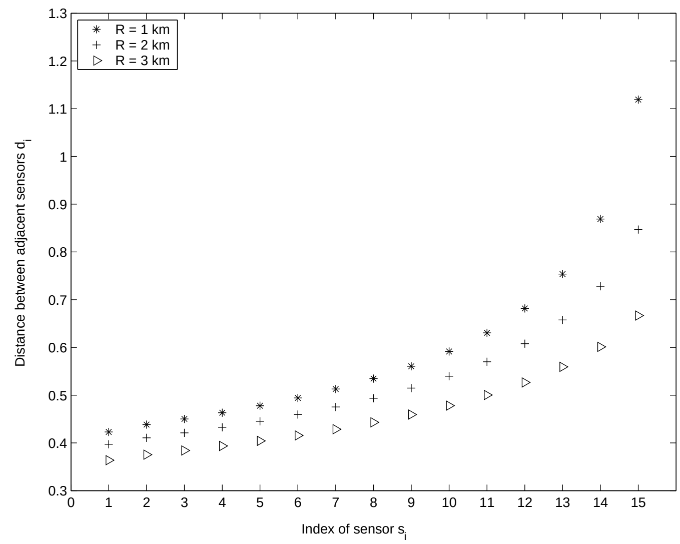

Fig. 2. Greedy sensor placement for different maximum sensing region. $R = \{1, 2, 3\}$ km, $\gamma = 2$, $N = 15$.

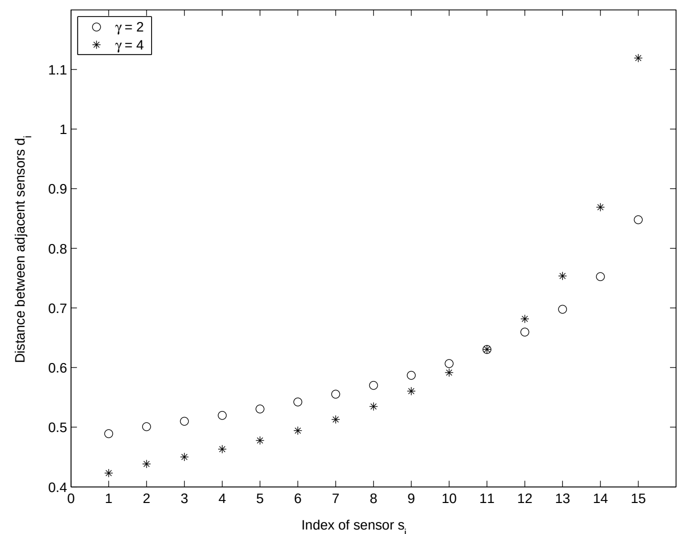

Fig. 3. Greedy sensor placement for different path loss exponents. $\gamma = \{2, 4\}$, $R = 1$ km, $N = 15$.

Fig. 4 compares the lifetime per unit cost of the greedy and the uniform sensor placement schemes. Unlike the network lifetime which increases with the number N of sensors [8], the lifetime per unit cost increases when N is small and decreases when N is large. The lifetime per unit cost diminishes for extremely large or extremely small number of sensors. Since the network lifetime decreases with the event arrival rate λ for each N, the lifetime per unit cost η also decreases with λ. The greedy sensor placement outperforms the uniform placement. We also

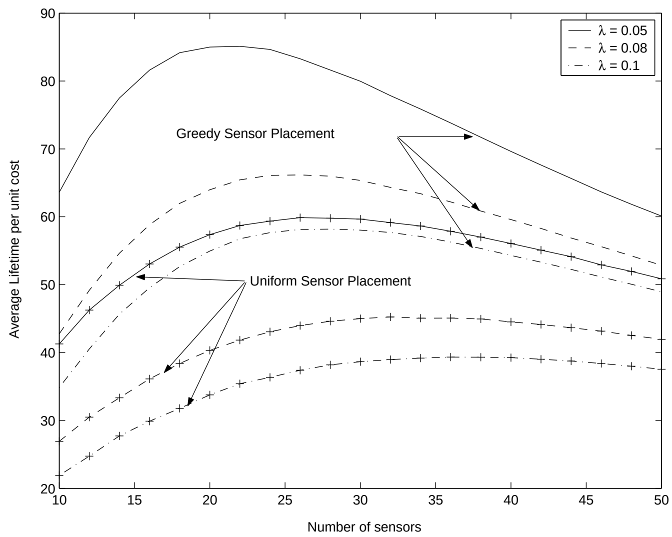

Fig. 4. Average lifetime per unit cost of greedy and uniform sensor placement schemes. $\lambda = \{0.05, 0.08, 0.1\}$, $R = 1$ km, $\gamma = 2$.

TABLE I
THE OPTIMAL NUMBER N^* OF SENSORS (13) AND ITS APPROXIMATE N_a^* (14) FOR DIFFERENT EVENT ARRIVAL RATES λ, $P_s = 5 \times 10^{-3}$.

	$\lambda = 0.05$	$\lambda = 0.08$	$\lambda = 0.1$	$\lambda = 0.2$
N^*	22	26	28	33
N_a^*	19	24	26	33

notice that when λ is large, the lifetime per unit cost η curves are more flat; however, when λ is small, the η curves change widely. This agrees with our expectation that since λ appears in the denominator of η (6), η is more sensitive to small λ.

To efficiently utilize sensors, we seek the optimal number N^* of sensors for maximizing the lifetime per unit cost and investigate the effect of event arrival rates λ and sensing power consumption P_s on N^*. In Tables I-II, N^* is obtained via simulation while N_a^* is obtained numerically (14). The approximate N_a^* is very close to the simulation result N^*. We can see that the optimal number of sensors increases with λ, but the rate of increasing diminishes. As P_s increases, the optimal number of sensors decreases and so does its rate. The above observations also agree with our intuitions. When the event arrival rate λ is large, more reporting processes are required. Hence, deploying more sensors is desired in order to reduce the energy consumption in each reporting process by reducing the transmission distance. However, when the sensing power consumption P_s is large, deploying less sensors is desired in order to reduce the energy wasted in sensing.

TABLE II
THE OPTIMAL NUMBER N^* OF SENSORS (13) AND ITS APPROXIMATE N_a^* (14) FOR DIFFERENT SENSING POWER CONSUMPTION P_s. $\lambda = 0.05$.

	$P_s = 10^{-3}$	$P_s = 5 \times 10^{-3}$	$P_s = 10^{-2}$
N^*	38	22	16
N_a^*	36	19	14

VI. CONCLUSION

In this paper, we analyzed the lifetime per unit cost of an event-driven linear WSN. We found that deploying either an extremely large or an extremely small number of sensors is inefficient in terms of lifetime per unit cost. We thus optimize the number of sensors to be deployed and their placement for maximizing lifetime per unit cost. We found that the last sensor should be placed as close to the gateway node as possible to reduce the reporting energy consumption. As the path loss exponent increases, the distance between adjacent sensors approaches uniform. We also found that the optimal number of deployed sensors increases with the event arrival rate and decreases with the sensing power consumption. Note that similar analysis and results can be developed for the linear WSN where the sensor closest to the gateway node is responsible for reporting.

APPENDIX A: DERIVATION OF (7)

In a randomly chosen reporting process, the probability that the event occurs in Voronoi cell of s_i is

$$p_i = \frac{A_i}{L}. \tag{15}$$

According to the transmission pattern specified in Section 2, s_i generates a reporting packet which will be relayed by $\{s_j\}_{j=1}^{i-1}$ to the gateway node. Hence, during this reporting process, the energy consumed by each sensor s_j is given by

$$E_r^{(j)} = \begin{cases} E_{tx}(d_j) + E_{rx}, & 1 \le j \le i-1, \\ E_{tx}(d_i), & j = i, \\ 0, & j > i. \end{cases} \tag{16}$$

Combining (3) and (15) with (16) yields the average energy consumed in a randomly chosen reporting process as

$$\begin{aligned}\mathbb{E}[E_r] &= \sum_{i=1}^{N} p_i \sum_{j=1}^{i} E_r^{(j)} \\ &= \frac{E_{tc} + E_{rx}}{L} \sum_{i=1}^{N} iA_i - E_{rx} + \frac{\tilde{E}}{L} \sum_{i=1}^{N} A_i \sum_{j=1}^{i} d_j^{\gamma}\end{aligned} \tag{17}$$

which is equivalent to (7) after some algebras.

REFERENCES

[1] J. Carle and D. Simplot-Ryl, "Energy-efficient coverage problems in wireless ad hoc sensor networks," 2004, to appear in Journal of Computer Communications on Sensor Networks.

[2] S. S. Dhillon and K. Chakrabarty, "Sensor placement for effective coverage and surveillance in distributed sensor networks," in *Proc. of IEEE Wireless Communications and Networking Conference*, vol. 3, March 2003, pp. 1609 – 1614.

[3] D. Ganesan, R. Cristescu and B. Beferull-Lozano, "Power-efficient sensor placement and transmission structure for data gathering under distortion constraints," in *Proc. of Third International Symposium on Information Processing in Sensor Networks (IPSN '04)*, Berkeley, Apr. 2004, pp. 142 – 150.

[4] K. Kar and S. Banerjee, "Node placement for connected coverage in sensor networks," in *Proc. of Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, 2003.

[5] K. Dasgupta, M. Kukreja and K. Kalpaki, "Topology-aware placement and role assignment for energy-efficient information gathering in sensor networks," in *Proc. of Eighth IEEE International Symposium on Computer and Communication*, June - July 2003, pp. 341 – 348.

[6] Y. T. Hou, Y. Shi, H. D. Sherali, and S. F. Midkiff, "On energy provisioning and relay node placement for wireless sensor networks," to appear in IEEE Trans. Wirel. Commun.

[7] J. Pan, Y. T. Hou, L. Cai, Y. Shi, and S. X. Shen, "Topology control for wireless video surveillance networks," in *Proc. of Ninth Annual International Conference on Mobile Computing and Networking*, San Diego, CA, 2003, pp. 286 – 299.

[8] P. Cheng, C. -N. Chuah and X. Liu, "Energy-aware node placement in wireless sensor network," in *Proc. of IEEE Global Telecommunications Conference*, vol. 5, Nov. - Dec. 2004, pp. 3210 – 3214.

[9] Q. Zhao and L. Tong, "Quality-of-Service Specific Information Retrieval for Densely Deployed Sensor Network," in *Proc. 2003 Military Communications Intl Symp.*, (Boston, MA), Oct. 2003.

[10] Q. Zhao and L. Tong, "Opportunistic Carrier Sensing for Energy Efficient Information Retrieval in Sensor Networks," *to appear in EURASIP Journal on Wireless Communications and Networking*, 2005.

[11] Y. Chen and Q. Zhao, "On the Lifetime of Wireless Sensor Networks," 2005, to appear in IEEE Commun. Lett.