Filtering
- Remove parts of a signal
- Enhance
- Sharpen spectrum

Often specified in the frequency domain

Notation:
- $x(t)$: Signal
- $X(s)$: Its Laplace transform
- $H(s)$: Filter
- $h(t)$: Inverse Laplace transform of $H(s)$

$y(t) = x(t) * h(t)$ (Convolution)
$Y(s) = X(s)H(s)$ (Multiplication)

Diagram:
- Input $x(t)$
- Filter $H(s)$
- Output $y(t)$
I. Filter in Frequency domain
   term-by-term mult.
      * in matlab

II. Filter in Time domain

   \[ y(n) = \sum_{k = -\infty}^{\infty} x(k) h(n-k) \]

   \( N \) is length of \( h \) (filter)

   \[ x(n) \quad \ldots \quad h_3 \quad h_2 \quad h_1 \quad h_0 \quad \]
   \[ h(n) \quad y(n) \]

   Each output requires \( N \) mults and \( N-1 \) adds

   Matlab: \( \text{out} = \ttt \text{filter} \left( \text{coeffs}, 1, \text{in} \right) \);

   \[ y \quad h \quad x \]

   Want real \( h(n) \)

   Normally choose real even \( H \), and \( h \)

   \[ \Rightarrow H(s) = H(-s) \]
When $x(t)$ is in the time domain, two ways to find $y(t)$:

1) Convolve $x(t) * h(t)$

2) a) Transform $h(t) \rightarrow H(s)$ — done only once (2)
    b) Transform $x(t) \rightarrow X(s)$
    c) Multiply $X(s) \cdot H(s) \rightarrow Y(s)$
    d) Inverse transform $Y(s) \rightarrow y(t)$

Digital Filters
- Butterworth
- Chebyshev

Books - McClellan method to generate coefficients
- Published in early 70s
- Iterative
- Works by specify:
  1) Length of Filter
  2) Frequency/magnitude pairs
mag. often in dB

Example:

- Low pass filter
- Max passband ripple of ± 0 dB
- Sampling freq of 100 MHz = f_s
- Passband: 0 C - 12.5 MHz
- Stopband: Min attenuation of 20 dB

\[
\text{from } 19 \text{ MHz} \rightarrow 50 \text{ MHz}
\]

\[
0.35 \pi \quad \frac{\pi}{2}
\]

\[
f [0 \ 0.25 \ 0.38 \ 1]
\]

\[
\text{mag } [1 \ 1 \ 0 \ 0]
\]

- remez ( )
See the freq response of a filter

Method 1 - `frez()` in matlab
  + Exact freq response
  + Very fast

Ideal specifications

![Diagram](image-url)
Method 2

1) Make a flat (white) spectrum input signal

2) Put signal into filter
   
   - Need many samples $10^5$?
   - Much slower than #1
   
   + Sometimes the only way
     - rounding
     - saturation

1) abs (fft (x))

2) psd ( )

3) spectrum ( ) → pwelch ( )