VITERBI DECODER PROCESSING

Viterbi References

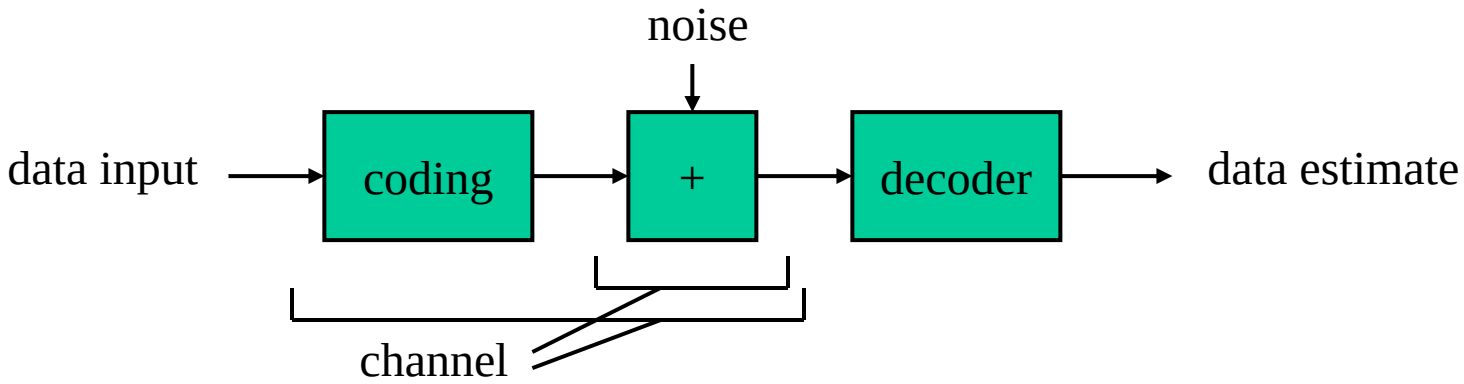
- Suggested references:
  - J. G. Proakis and M. Salehi, *Fundamentals of Communica-tion Systems*, Prentice Hall, 2005. [coding and viterbi]
  - A. J. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm," *IEEE Trans. on Information Theory,* April 1967.
  - G. D. Forney, "Maximum-Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference," *IEEE Trans. on Information Theory,* May 1972.
  - H.-L. Lou, "Implementing the Viterbi Algorithm," *IEEE Signal Processing Magazine,* Sept. 1995.
  - P. J. Black, "Algorithms and Architectures for High Speed Viterbi Decoding," Ph.D. dissertation, March 1993.

Viterbi Algorithm

- Widely used in communication systems to decode a data sequence that has been encoded by a "finite-state" process
  - Ex: ethernet receiver
  - Ex: hard disk read electronics
- A very common demanding DSP task
  - Ex: viterbi building blocks are in every serious DSP benchmarking suite
  - Ex: The Texas Instruments C64x DSP processor has a dedicated on-chip viterbi decoder. The viterbi decoder and a "turbo decoder" occupy about 8% of the chip's area

Viterbi Algorithm

- Involves coding data, adding noise, and decoding
  - Deliberate encoding: convolutional or trellis codes
  - Unintentional encoding: intersymbol interference
- Output is an *estimate* of the original data
- Viterbi algorithm is optimal in the maximum likelihood sense—it finds the input that is most likely, given the observed channel output

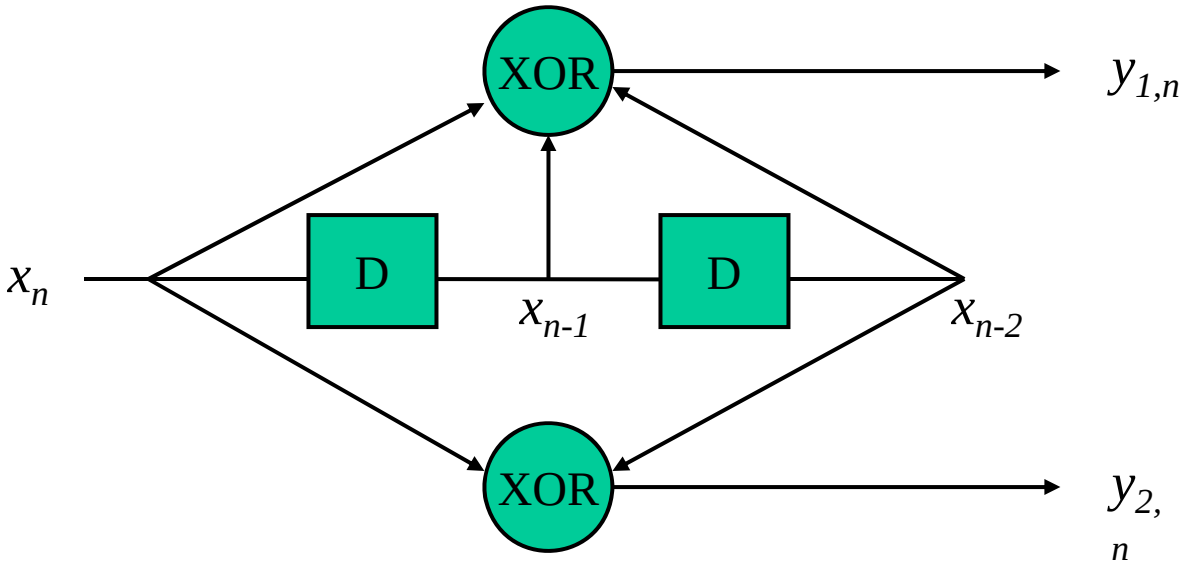


Convolutional Coding Example

- Encoding is a process which adds redundancy to data to reduce the probability of errors or increase the level of acceptable noise in the channel
- The constraint length (commonly called k) is a measure of the complexity of the code
  - Ex: $k = 7$ → 2^{k-1} states = 2^6 states = 64 states (in Wi-Fi)
  - Ex: $k = 9$ → 256 states (a challenging design)
- Ex: a 4-state, Rate = ½ convolutional encoder
  - input data x_n
  - encoded symbols y_1 and y_2
  - $y_{1,n} = x_n$ XOR x_{n-1} XOR x_{n-2}
  - $y_{2,n} = x_n$ XOR x_{n-2}
  - Two data bits output by decoder for each input bit (rate = ½)
  - "Double rate" output bits typically sent serially into one "channel"

Convolutional Coding Example

- Present state : x_{n-2} x_{n-1}
- Next state: x_{n-1} x_n (input)



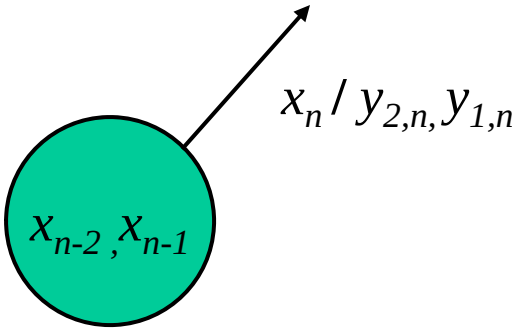
Convolutional Coding Example

- 8-entry truth table
  - inputs: three x's
  - outputs: two y's

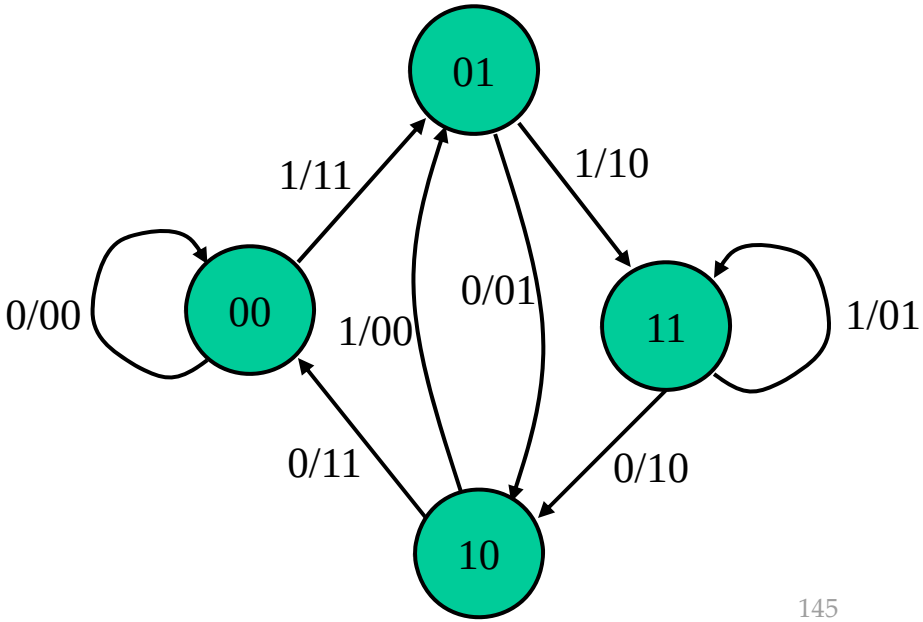
x_n, x_{n-1}, x_{n-2}	$y_{1,n}$	$y_{2,n}$
000	0	0
001	1	1
010	1	0
011	0	1
100	1	1
101	0	0
110	0	1
111	1	0

Convolutional Coding Example

- Each state can transition to only 2 other states
  - These two states are reached by either $x_n = 0$ or 1
- Next state found by: x_{n-1} x_n (input)

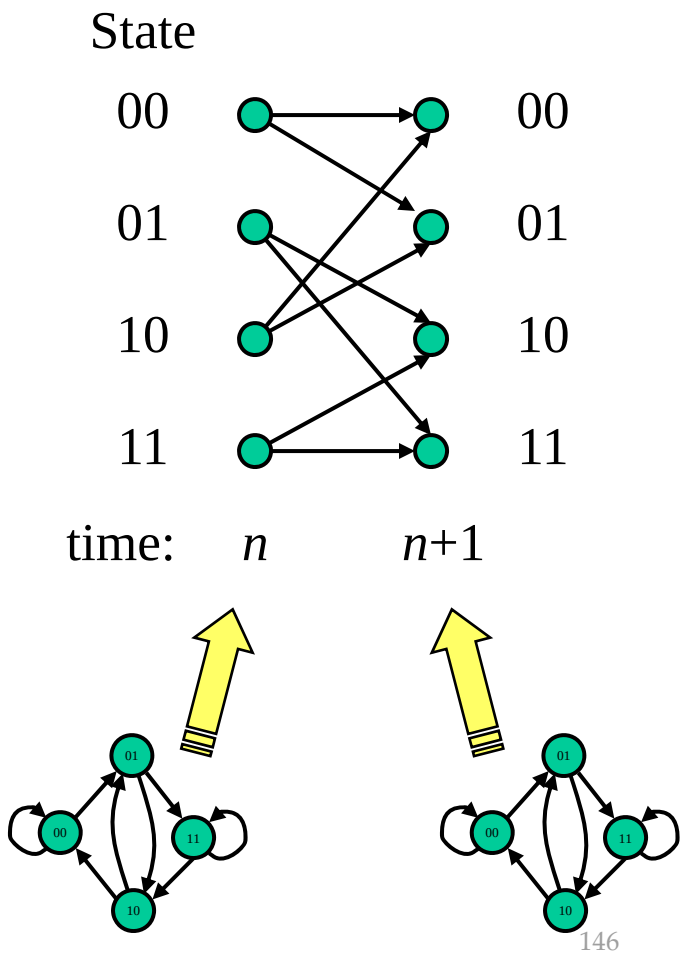


Notation used in State Graph



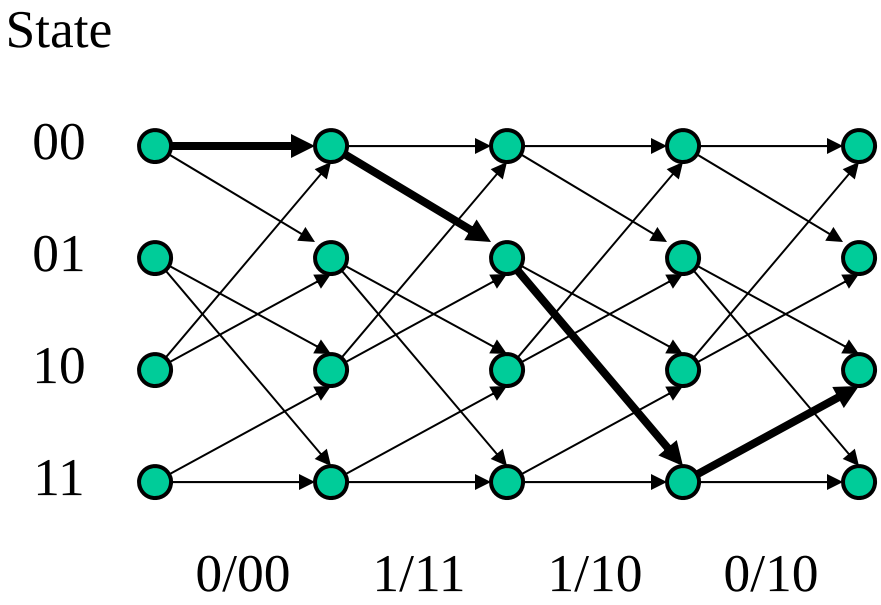
Trellis

- The *trellis* is a time indexed version of the state diagram
- Each state transition (input bit into the encoder) corresponds to a forward step in the trellis



Trellis

- Given a starting state, every possible input sequence corresponds to a unique path through the trellis
- Example:
  - Assume start in state 00
  - Inputs x_n = 0, 1, 1, 0
  - Outputs = {0,0}, {1,1}, {1,0}, {1,0}



Viterbi Decoding

- The Viterbi decoder calculates a semi-brute-force estimate of the likelihood for each path through the trellis
- Key point: Once the estimates for all states in a step/iteration of the trellis have been calculated, the probabilities for all previous steps/iterations can be discarded; only the most likely entry to a state must be remembered
- It comprises four basic steps:
  1. Calculate the trellis
     a) Weight the trellis branches by calculating branch metrics
     b) Compute the minimum weight path to time n+1 in terms of the minimum weight path to time n. Retain step decisions. Uses *add-compare-select (ACS)* algorithm.
  2. Find the last state of the minimum weight path.
  3. Find the entire minimum weight path. Also called survivor path decode or *traceback*.
  4. Reorder bits into correct forward ordering.

1. Calculating the Trellis

- Each branch is assigned a weight, called a *branch metric*
- The branch metric is a measure of the likelihood of the transition given the noisy observations
- Likelihood of a transition is given by an appropriate measure of the "distance" between an ideal encoder output and the actual received signal
- The overall goal is to find the minimum weight path (often called *shortest path*) through the trellis

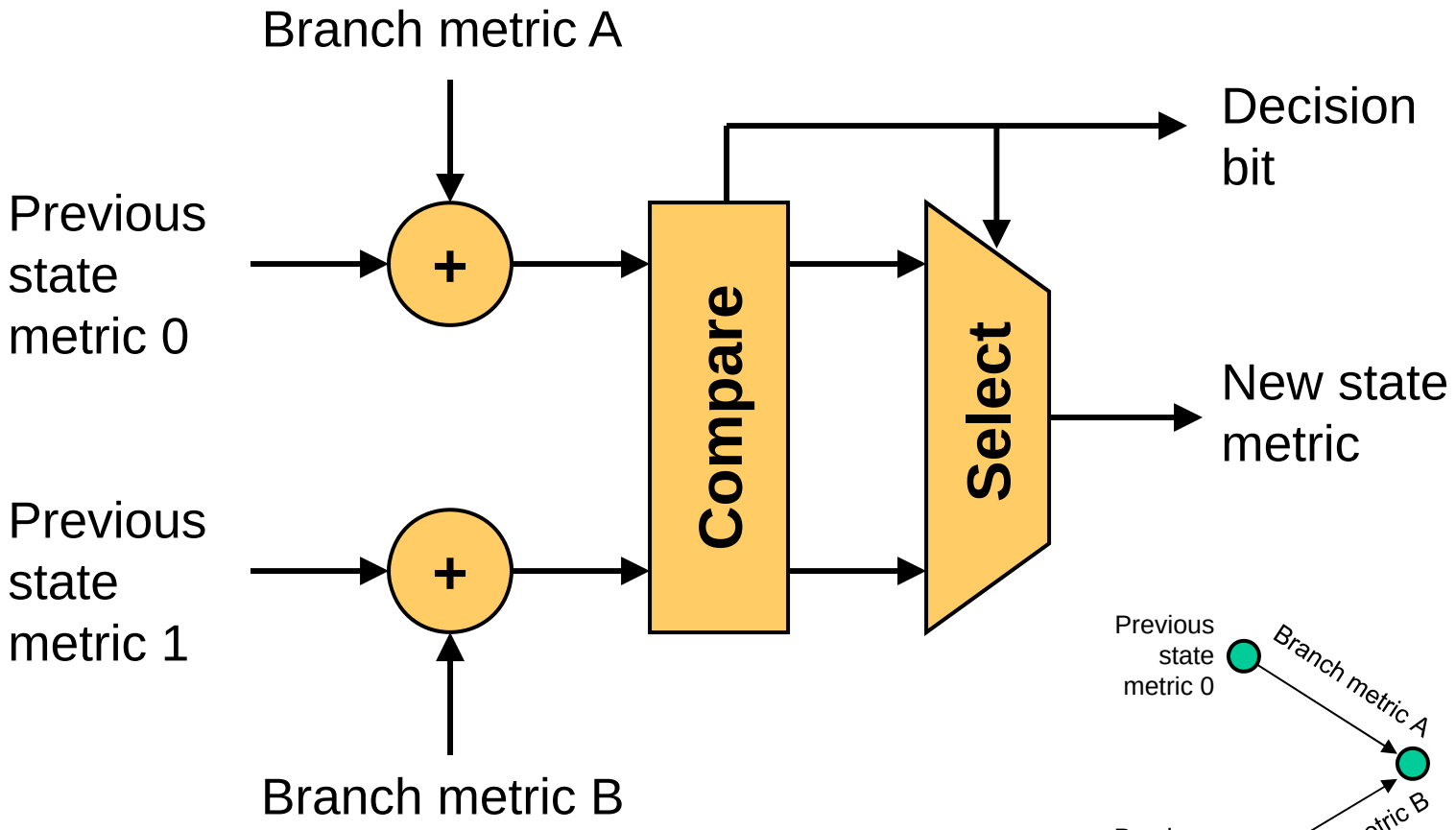
1a) Branch Metric Calculations

- More likely transitions receive *lower* weights
- Example:
  - Receive: {0.13, 0.89}
  - Branch with {0,1} outputs receives a lower weight (because it was the more likely transmitted pair) than the {1,0} branch
- A very simple implementation would add just the differences between received values, for example:
  - Starting state: 0,1 — Notice from State Graph 0/01 and 1/10 are only options
  - Receive: {0.13, 0.89}
  - Branch {0,0} — Impossible, not present on State Graph
  - Branch {0,1} — Branch metric = $|0–0.13| + |1–0.89| = 0.24$
  - Branch {1,0} — Branch metric = $|1–0.13| + |0–0.89| = 1.76$
  - Branch {1,1} — Impossible, not present on State Graph
- Higher performance decoders would use more complex mapping functions between received values and the branch metric weights

1b) ACS

- Goal is to find the most likely entry path into a state
- ADD previous state metric to current branch metric
  - for first branch
  - for second branch
- COMPARE which incoming branch produces the lower metric (higher probability)
- SELECT the minimum and save the branch
- Fully parallel viterbi decoders use many ACS datapaths to calculate an entire trellis state update in one cycle

ACS Hardware

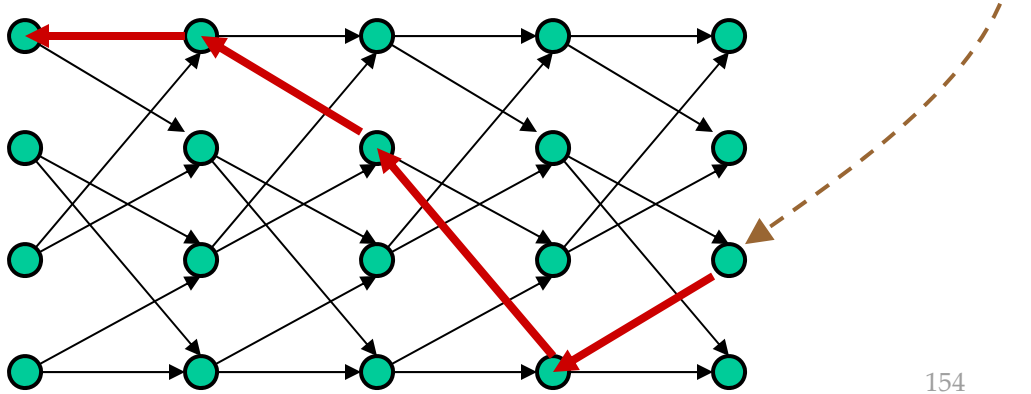


2. Finding The Most Likely Path

- After all data have been processed, we must find the most likely path through the trellis
- Done by searching through the final "column" of states and selecting the most likely (lowest weight)
- Operation done infrequently compared to other calculations

3. Traceback

- The goal is to read out the best estimate for all decoded bits
- The process begins at the most likely state and it follows this procedure for each state in the trellis
  1) Read stored decision bit from ACS
  2) Output this "decoded bit"
  3) Use the decision bit to calculate the previous state in the trellis
  4) <repeat>



4. Reorder Output Bits

- Bits output from the traceback operation are in reverse order (last bit out corresponds to first received)
- Need logic and memory to buffer and reverse order
- Fundamental to the algorithm, cannot be avoided
- Summary
  - Viterbi is data and memory intensive
    - *Forward* ACS
    - *Reverse* traceback
    - *Forward* re-ordering