SIGNALS

CONTINUOUS/DISCRETE TIME/FREQUENCY DOMAINS

Continuous-Time Signals

- Signals in the "real world"
- Frequency content (spectrum) for finite-time signals theoretically extends to infinity
- Ref: Discrete-Time Signal Processing, Oppenheim & Schafer

Euler's Formula

$$e^{ix} = \cos(x) + i\sin(x)$$

$$e^{i2\pi xs} = \cos(2\pi xs) + i\sin(2\pi xs)$$

$$e^{-i2\pi xs} = \cos(2\pi xs) - i\sin(2\pi xs)$$

- Called "the most remarkable formula in mathematics" by Richard Feynman *[The Feynman Lectures on Physics, page 22-10]*
- e^{ix} is also called a "complex sinusoid"
- e^{ix} is the kernel of the Fourier Transform

Continuous Fourier Transform

- Fourier transform pair
  - *f(x)* is the "time domain" representation
  - *F(s)* is the "frequency domain" representation

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs} dx$$

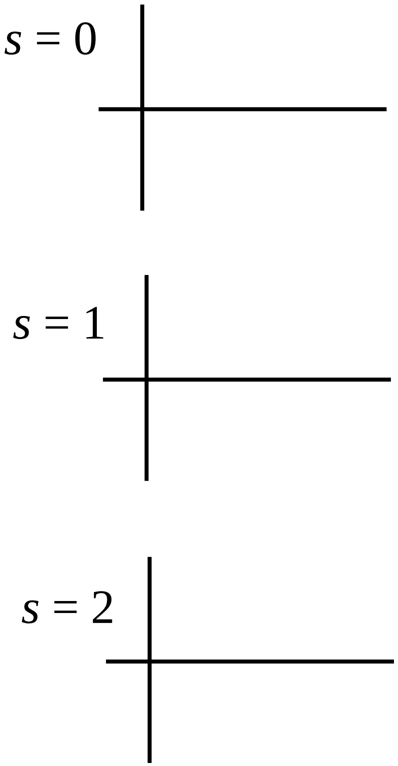
$$f(x) = \int_{-\infty}^{\infty} F(s) e^{i2\pi xs} ds$$

Time and Frequency Domains

- Although we defined
  - $f(x)$ as the "time domain"
  - $F(s)$ as the "frequency domain"
  - this is not necessary always true
- Ex: A signal could be a function of linear distance
  - The Fourier transform could be called the "spatial frequency" (cycles per meter)
- Ex: Multi-dimensional signals
  - Signal could be an image, CAT scan or MRI or radar electromagnetic signal

Fourier Transform

- Inputs may be complex
- Outputs may be complex
- Consider the $e^{i2\pi xs}$ kernel of the transform
  - For a particular value of s, the kernel is a complex sinusoid of “frequency” s
  - All $e^{i2\pi xs}$ complex sinusoids are orthogonal
- The Fourier transform “decomposes” or “describes” a signal as a sum of complex sinusoids



Discrete-Time Signals

- Signal levels specified only at discrete time intervals (t_{sample})
- Sampling frequency = $1/ t_{sample}$
- Frequency content extends to infinity but repeats every 2π radians
  - Ex: DC content value is at …, -4π, -2π, 0, 2π, 4π, …
  - Ex: Highest unambiguously-representable content value is at …, -3π, $-\pi$, π, 3π, …
- Nyquist frequency = f_{Nyq}

  $= 1 / (2 * T_s)$

  $= (1/2)\, f_{sample}$

Discrete Fourier Transform (DFT)

- $X(k)$ is the DFT of the N-point sequence $x(n)$

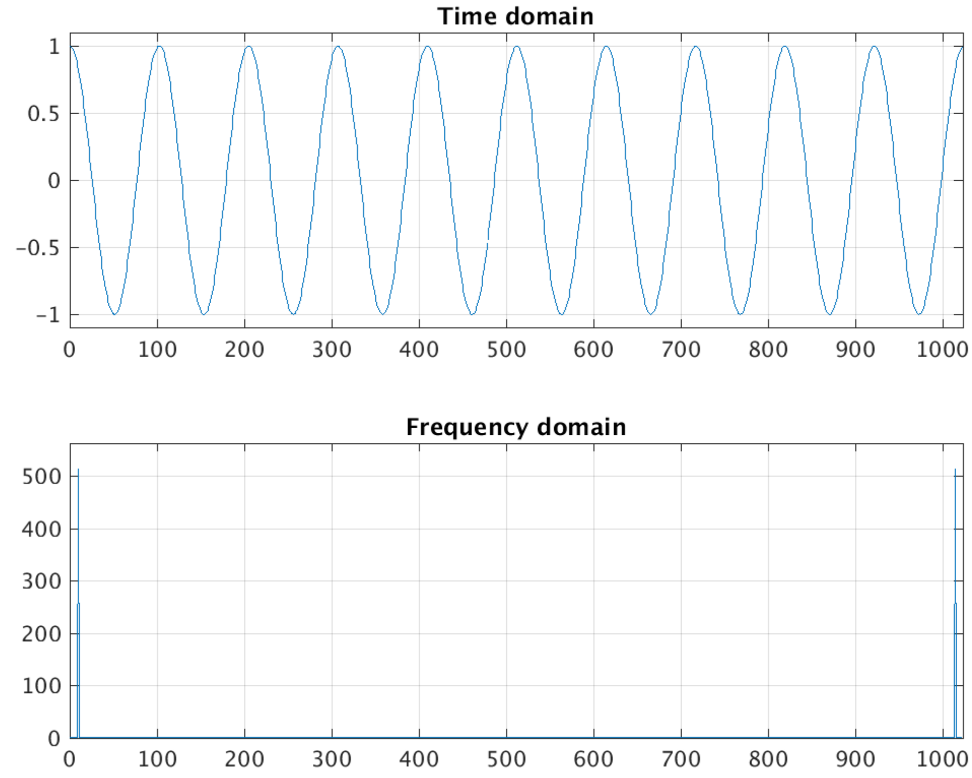
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i2\pi nk/N}, k = 0,1,\ldots, N-1$$

- And the Inverse DFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{i2\pi nk/N}, n = 0,1,\ldots, N-1$$

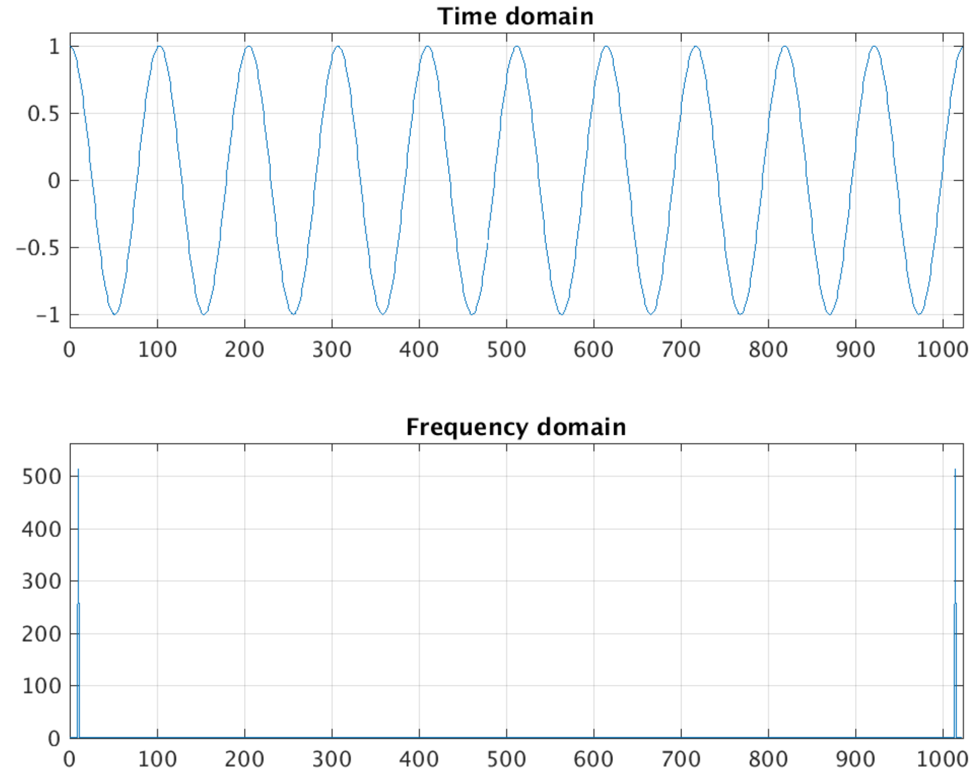
cos/impulse Pair

- Ref: *The Fourier Transform and Its Applications*, Ron Bracewell
- cos( ) and its Fourier transform
- cos and sin are “pure tones” of a single frequency



cos/impulse Pair

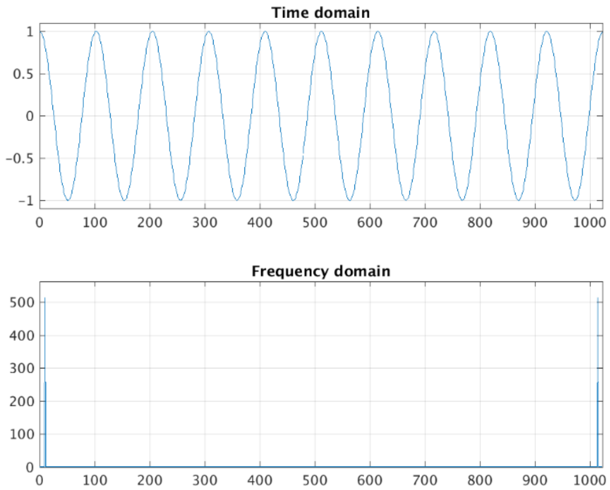
- Example:
  - length = 1024
  - top plot: cos(2π*10*t/length)
  - bottom plot: abs(fft(top_waveform))



cos/impulse Pair

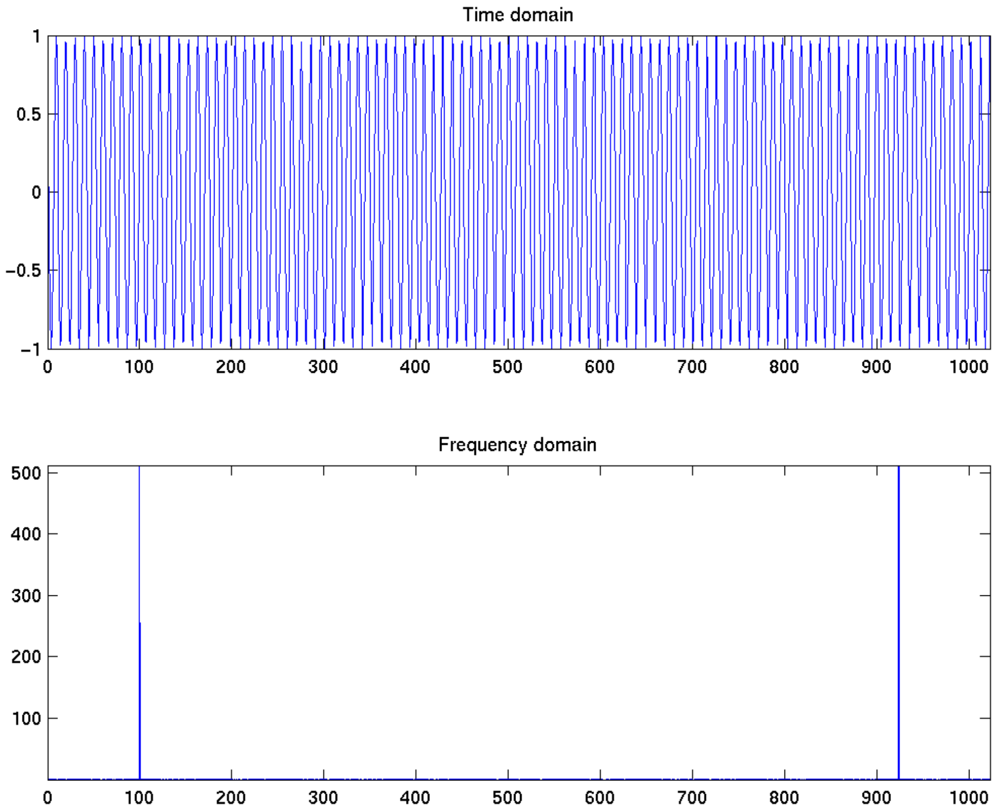
- matlab code for the adjacent plots

```
clear;
length          = 1024;
x               = 0:(length-1);

%--- cos(10)
freq = 10;

t = cos(2*pi*x*freq/length);
f = abs(fft(t));

figure(1); clf;

subplot(2,1,1);
plot(x,t);
title('Time domain');
axis([0 (length-1) -1.1 +1.1]);
grid on;

subplot(2,1,2);
plot(x,f);
title('Frequency domain');
axis([0 (length-1) 0 (length/2*1.1)]);
grid on;

print -dtiff 1.tiff
```



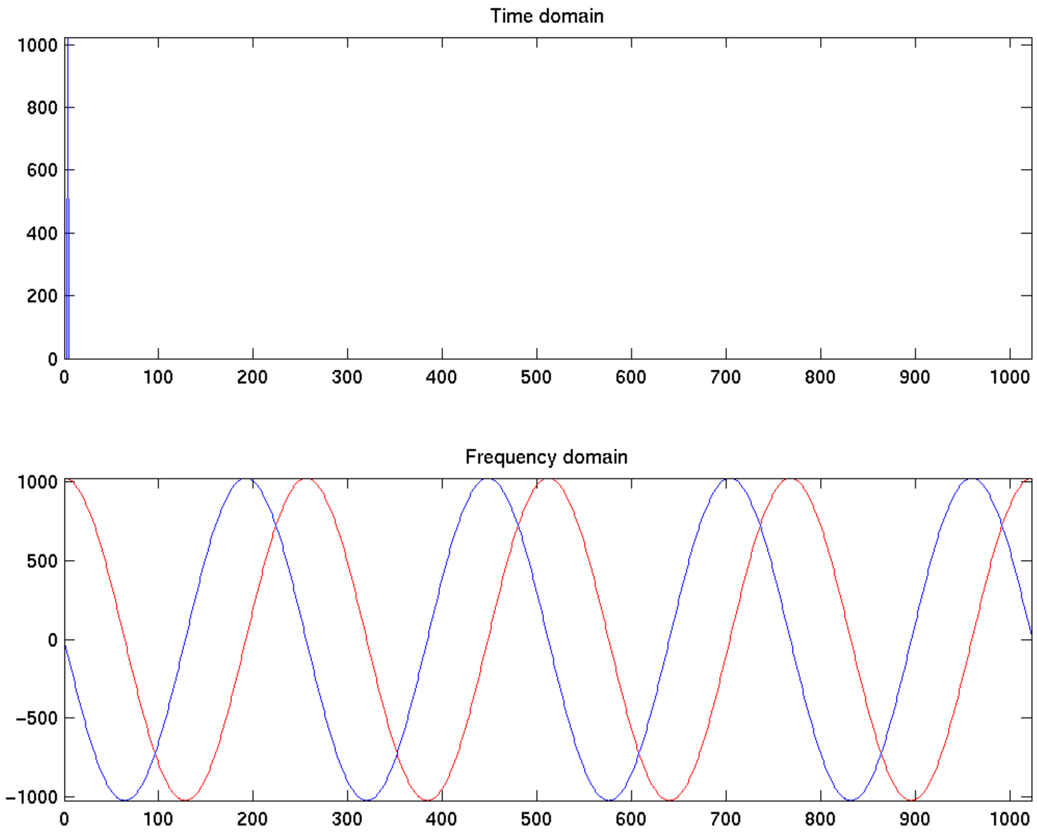
cos/impulse Pair

- A higher frequency cos( )



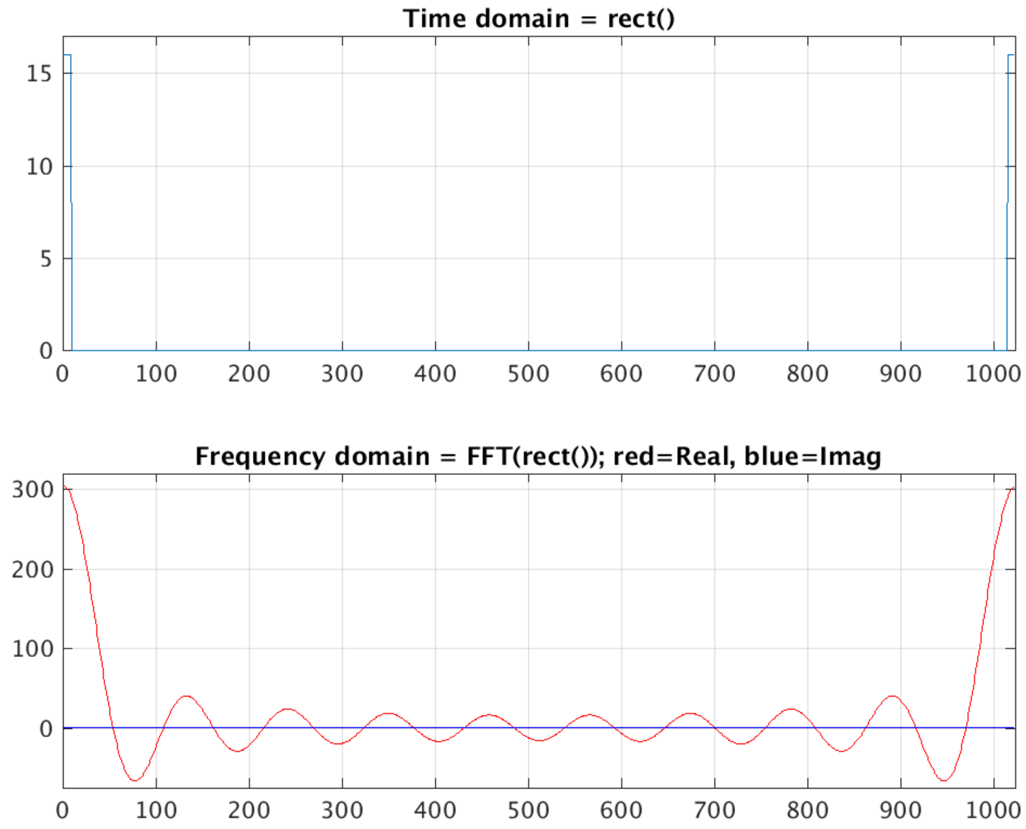
impulse/sinusoid Pair

- There is a symmetry with the previous transform pairs in that an impulse in the “time” domain produces sin/cos in the “frequency” domain
- A real one-sided impulse in the time domain results in complex sinusoids in the frequency domain



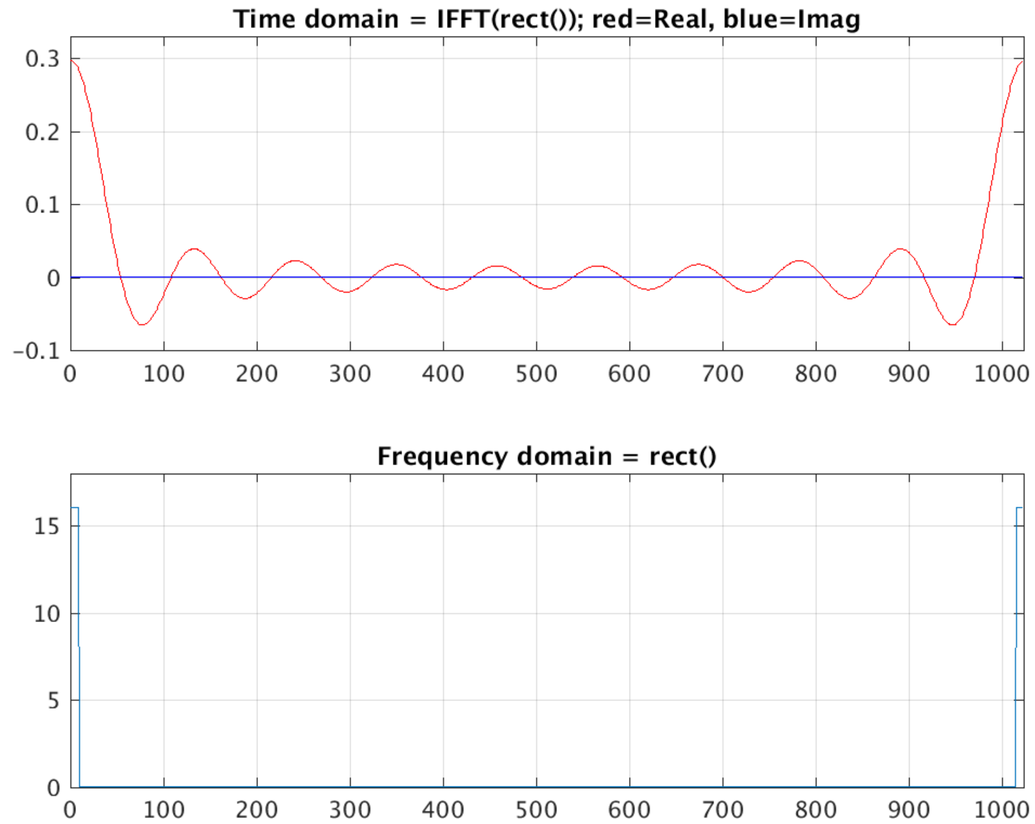
rect/sinc Pair

- The rect(x) function transforms into a sinc(x) function
- rect(x) is a low-pass filter
- sinc(x) = sin(x)/x
- Can also view sinc(x) as sin(x) within a 1/x envelope



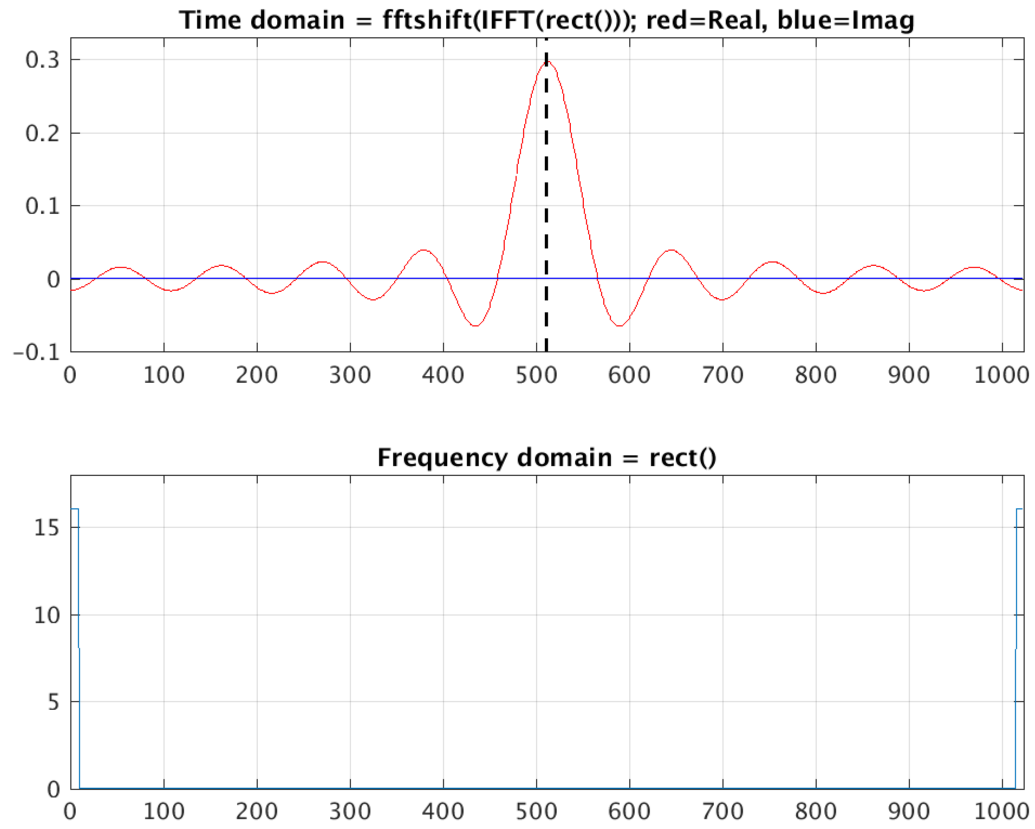
rect/sinc Pair

- Similarly, the rect(x) function in the "frequency" domain transforms into a sinc(x) function in the "time" domain
- A processor that implements a rect(x) in the frequency domain is a widely-used low-pass filter (LPF)



rect/sinc Pair

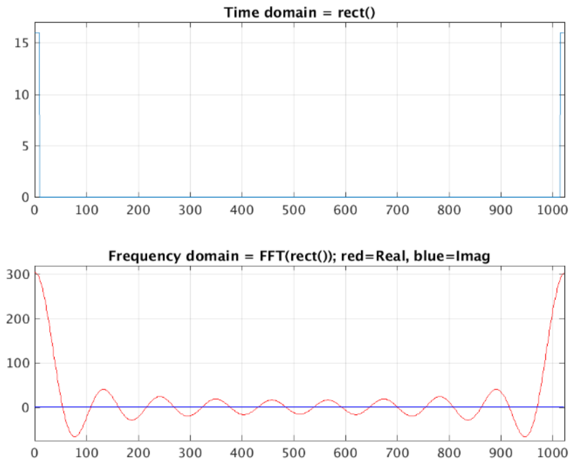
- To get a better look at the time-domain version of the LPF, we swap the two halves of the waveform using fftshift() in matlab, which is effectively viewing the filter's coefficients shifted by π to the right or left
- The sinc() shape of a filter's time-domain coefficients is a tell-tale sign of a low-pass filter



rect/sinc Pair

- matlab code for the rect() / sinc() plots

```
clear;
length  = 1024;
x       = 0:(length-1);

t = zeros(1,length);
t(1:10) = 16;
t((length-8):length) = 16;   % "time" domain
f = fft(t);                          % "frequency" domain

figure(5); clf;

subplot(2,1,1);
plot(x,t);
title('Time domain = rect()');
axis([0 (length-1) 0 +17]);
grid on;

subplot(2,1,2);
plot(x,real(f),'r');
hold on;
plot(x,imag(f),'b');
title('Frequency domain = FFT(rect()); red=Real, blue=Imag');
axis([0 (length-1) -75 320]);
grid on;

print -dtiff 5.tiff
```



cos/impulse Pair

```
%--- rect 10 in frequency
clear;
length  = 1024;
x       = 0:(length-1);

f = zeros(1,length);
f(1:10) = 16;
f((length-8):length) = 16;   % "frequency" domain
t = ifft(f);                 % "time" domain

figure(7); clf;

subplot(2,1,1);
plot(x,real(t),'r');
hold on;
plot(x,imag(t),'b');
title('Time domain = IFFT(rect()); red=Real, blue=Imag');
axis([0 (length-1) -0.1 0.33]);
grid on;

subplot(2,1,2);
plot(x,f);
title('Frequency domain = rect()');
axis([0 (length-1) 0 18]);
grid on;

print -dtiff 7.tiff


%--- fftshift of rect 10 in frequency
t = fftshift(t);

figure(8); clf;

subplot(2,1,1);
plot(x,real(t),'r');
hold on;
plot(x,imag(t),'b');
plot([511 511], [-1 +1], 'k--', 'linewidth', 1.5);
title('Time domain = fftshift(IFFT(rect())); red=Real, blue=Imag');
axis([0 (length-1) -0.1 0.33]);
grid on;

subplot(2,1,2);
plot(x,f);
title('Frequency domain = rect()');
axis([0 (length-1) 0 18]);
grid on;

print -dtiff 8.tiff
```