SIGN EXTENSION
Sign Extension

- Needed for 2’s complement addition
- Consider case of adding two numbers of different widths

\[
\begin{align*}
1 & 0 & 1 & 1 & \quad \text{−5} \\
0 & 1 & 0 & 0 & 1 & 0 & \quad +18 \\
\hline
& & & & \\
0 & 1 & 1 & 1 & 0 & 1 & \quad +29!
\end{align*}
\]
Sign Extension

- Rule #1: 2’s complement input and output operands must be the same word-width because of implied zeros

\[
\begin{array}{cccc}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\end{array}
\begin{array}{cccc}
1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 \\
\end{array}
= -5 + 18 = 29
\]
Sign Extension

- Rule #2: Despite a fundamental change to the number’s definition, the value of a 2’s complement number will never change due to any amount of sign extension—regardless of whether the value is positive or negative

\[
\begin{align*}
1011 &= -8 + 2 + 1 \\
11011 &= -16 + 8 + 2 + 1 \\
111011 &= -32 + 16 + 8 + 2 + 1
\end{align*}
\]
Sign Extension

• Procedure:
  1) Calculate the necessary minimum width of the sum so that it contains all input possibilities
     • It’s up to you to make sure the output range is sufficient
  2) Extend the inputs’ sign bits to the width of the answer
  3) Add as usual
  4) Ignore bits that ripple to the left of the answer’s MSB

|   1 1 1 1 0 1 1   | -5     | [-8,+7]        |
| 0 0 1 0 0 1 0   | +18    | [-32,+31]      |
-----------------|--------|----------------|
  x 0 0 0 1 1 0 1 | +13    | [-40,+38]      |

Choose 7-bit [-64,+63]
Sign Extension

• Ignore carry bits
  – Do not spend any hardware calculating any bits to the left of the answer’s MSB

\[
\begin{array}{c}
\begin{array}{ccccccc}
1 & 1 & 1 & 0 & 1 & 1 & -5 \\
\hline
1 & 1 & 0 & 0 & 1 & 0 & -14 \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{ccccccc}
x & 1 & 0 & 1 & 1 & 0 & 1 & -19 \cr
\end{array}
\end{array}
\]

ignore all bits to the left of the MSB
Sign Extension In Verilog

• Adding signed variables in verilog requires one of two methods:
  – Using *signed* variables: always works in verilog simulators but in rare cases these variables do not work as expected with some CAD tools
  – Declare all variables normally as *regs* and *wires* and perform sign extension manually. This method can be tedious but will *always always* work as expected.
Sign Extension In Verilog

- These cases use no sign extension at all
- Two cases work correctly, one does not

```verilog
reg [3:0] m, n;
reg [4:0] sum;

initial begin
    m = 4'b0000;
    n = 4'b0000;
    sum = m + n;
    #10;
    $write("m = %b, n = %b, sum = %b\n", m, n, sum);

    m = 4'b1111;
    n = 4'b1111;
    sum = m + n;
    #10;
    $write("m = %b, n = %b, sum = %b\n", m, n, sum);

    m = 4'b1111;
    n = 4'b0000;
    sum = m + n;
    #10;
    $write("m = %b, n = %b, sum = %b # ERROR: -1 + 0 = +15\n", m, n, sum);
end

m = 0000, n = 0000, sum = 00000
m = 1111, n = 1111, sum = 11110 # Lucky!
# 1111, n = 0000, sum = 01111 # ERROR: -1 + 0 = +15
```
Sign Extension In Verilog

- Writing the sign extension manually is the most robust method.
- Both inputs are sign extended to the same width as the sum—five bits in this case.

```verilog
reg [3:0] m, n;
reg [4:0] sum;

initial begin
  m = 4'b1111;
  n = 4'b0000;
  sum = {m[3],m} + {n[3],n};
  #10;
  $write("m = %b, n = %b, sum = %b # OK!\n", m, n, sum);
end

m = 1111, n = 0000, sum = 11111 # OK!
```