BINARY MULTIPLICATION

Multipliers

- Multiplies are widely used in digital signal processing, generally more so than in general-purpose workloads
- Major categories of multiplier types
  - *Unsigned* × *Unsigned*
    Also very useful for sign-magnitude data
  - *Signed 2's complement* × *Signed 2's complement*
    Very useful for fixed-point 2's complement data
- Hardware is typically built in a manner broadly similar to how you would do it with paper and pencil
- The naming convention is somewhat unfortunate:

$$
\begin{array}{rr}
 & \textit{multiplicand} \\
\times & \textit{multiplier} \\
\hline
\end{array}
$$

Multipliers

- Example: 4-bit unsigned *multiplicand "a"* times 4-bit *multiplier "b"*
- b could be signed or unsigned
- $p_{xy} = a_x \times b_y$
  $= a_x$ AND b_y

$$
\begin{array}{ccccccccc}
 & & & a_3 & a_2 & a_1 & a_0 & & \\
 & & \times & b_3 & b_2 & b_1 & b_0 & & \\
\hline
 & & & p_{30} & p_{20} & p_{10} & p_{00} & & \leftarrow b_0 \\
 & & p_{31} & p_{21} & p_{11} & p_{01} & 0 & & \leftarrow b_1 \\
 & p_{32} & p_{22} & p_{12} & p_{02} & 0 & 0 & & \leftarrow b_2 \\
p_{33} & p_{23} & p_{13} & p_{03} & 0 & 0 & 0 & & \leftarrow b_3
\end{array}
$$

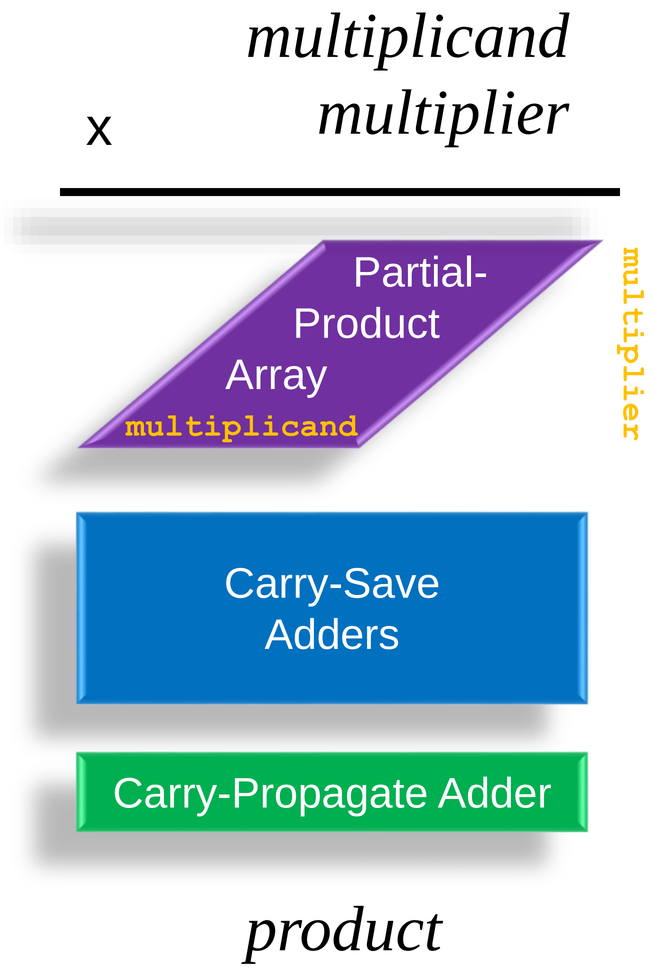
Multipliers

- Example: 4-bit signed 2's complement *multiplicand "a"* times 4-bit *multiplier "b"*
- b could be signed or unsigned
- s = partial product sign extension bits
- $p_{xy} = a_x \times b_y$
  $= a_x$ AND b_y

					a_3	a_2	a_1	a_0	
				$\times$	b_3	b_2	b_1	b_0	
s	s	s	s		p_{30}	p_{20}	p_{10}	p_{00}	$\leftarrow b_0$
s	s	s	p_{31}		p_{21}	p_{11}	p_{01}	0	$\leftarrow b_1$
s	s	p_{32}	p_{22}		p_{12}	p_{02}	0	0	$\leftarrow b_2$
s	$\overline{p_{33}}$	$\overline{p_{23}}$	$\overline{p_{13}}$		$\overline{p_{03}}$	0	0	0	$\leftarrow b_3$
					1				

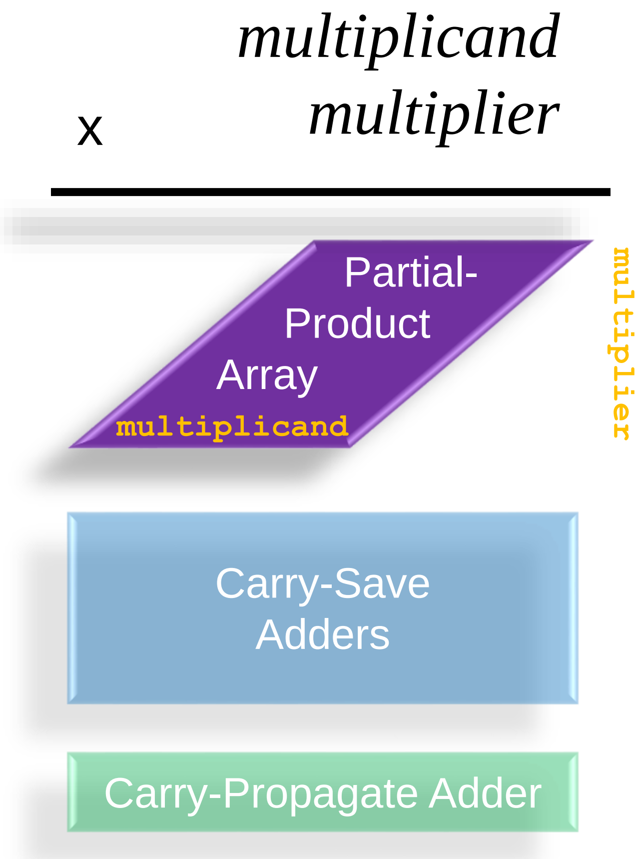
3 Main Steps in Every Multiplier

1) Generation of partial products
2) Reduction or "compression" of the partial product array (normally using carry-save addition) so that the product is composed of two words
   - Linear array addition
   - Tree addition (Wallace tree)
3) Final adder: Carry-propagate adder (CPA)
   - Converts the product in carry-save form into a single word form
   - Any style of CPA is fine though we probably favor faster ones



Straight-forward Partial Product Generation

- This is the simplest method to generate partial products
- Hardware looks at one bit of the *multiplier* (Y_i) at a time
- Partial products are copies of the *multiplicand* AND'd by bits of the *multiplier*
- Number of bits in the *multiplier*
  = Number of partial products
  = Number of terms/words/rows that must be added



Y_i	Partial product
0	0
1	+x (= *multiplicand*)

Straight-forward Partial Product Generation

- There are only two possible partial product results
- Two reasonable hardware solutions are:
  - a row of 2:1 muxes with zeros on one input
  - a row of AND gates (this should be more efficient)

$+x$ *multiplicand*

0 0

Y_i	Partial product
0	0
1	$+x$ (= *multiplicand*)

Example unsigned 4-bit × 4-bit multiplication

- Example: 4-bit unsigned *multiplicand "a"* 1100 times 4-bit *multiplier "b"* 1010
- 1100 × 1010 = 12 × 10 = 120
- 1100 × 1010 = (12 × 8) + (12 × 0) + (12 × 2) + (12 × 0) = 120
- 1111000 = 64 + 32 + 16 + 8 = 120 ☺

			1	1	0	0	
		×	1	0	1	0	
			0	0	0	0	← 0
		1	1	0	0	0	← 1
	0	0	0	0	0	0	← 0
1	1	0	0	0	0	0	← 1
1	1	1	1	0	0	0	

Example 2's complement 4-bit × 4-bit multiplication

- Example: 4-bit signed 2's complement *multiplicand "a"* 1100 times 4-bit *multiplier "b"* 1010
- 1100 × 1011 = –4 × –5 = +20
- 1100 × 1011 = (–4 × –8) + (–4 × 0) + (–4 × 2) + (–4 × 1) = +20
- 00010100 = 16 + 4 = +20 ☺

				1	1	0	0	
			×	1	0	1	1	
1	1	1	1	1	1	0	0	← 1
1	1	1	1	1	0	0	0	← 1
0	0	0	0	0	0	0	0	← 0
0	$\overline{1}$	$\overline{1}$	$\overline{0}$	$\overset{1}{\overline{0}}$	0	0	0	← 1
0	0	0	1	0	1	0	0	