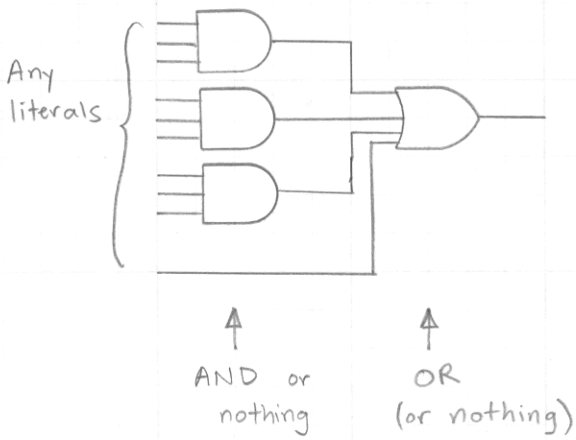
2.7 Multiplying Out and Factoring

Distributive laws can be used to <u>multiply out</u> Boolean expressions into a special form called <u>sum-of-products (SOP)</u>.



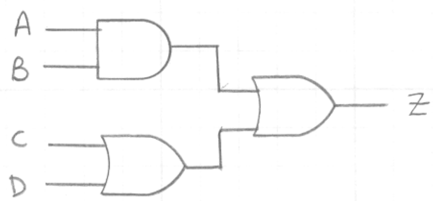
<u>Ex</u>: $A + BC' + D'E + F'G'H'$

Each product term is a product of only single variables.

<u>Ex</u>: $\underbrace{(A+C+D)}_{\text{not a single variable}}B + BE'$ <u>Not</u> SOP

We can convert any expression to sum-of-products form:

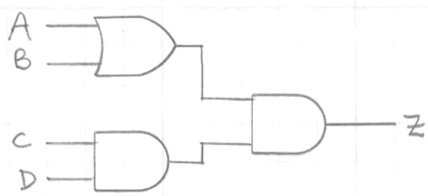
<u>Ex</u>:



$Z = (A \cdot B) + (C + D)$

$= AB + C + D$ ✓

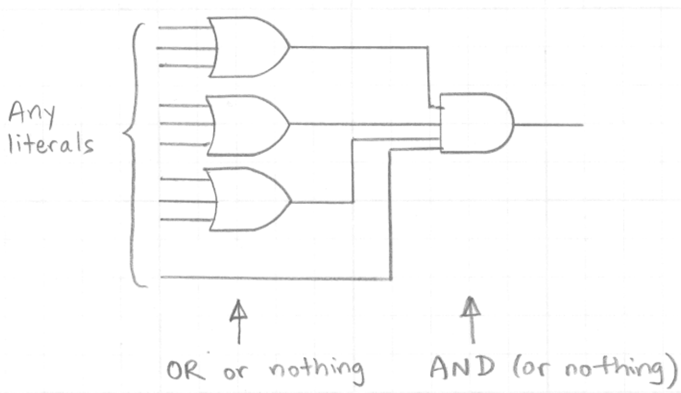
<u>Ex</u>:



$Z = (A + B) \cdot (C \cdot D)$

$= ACD + BCD$ ✓

Distributive laws can also be used to <u>factor</u> Boolean expressions into <u>product-of-sums (POS)</u> form.

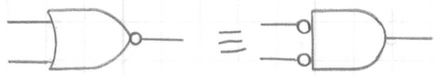


<u>Ex</u>: $(A + C + D) \cdot B \cdot (E' + F)$ POS

$(A + C)B + (E' + F)$ <u>Not</u> POS

2.8 DeMorgan's Laws

$(X+Y)' = X'Y'$



$(XY)' = X' + Y'$

<u>EX:</u> $(C + (A+B'))' = C' \cdot (A+B')'$

<u>Duality</u> To obtain the <u>dual</u> of an expression,

AND $\rightarrow$ OR
OR $\rightarrow$ AND
0 $\rightarrow$ 1
1 $\rightarrow$ 0

Literals remain unchanged. If the original expression is true, so is its dual.

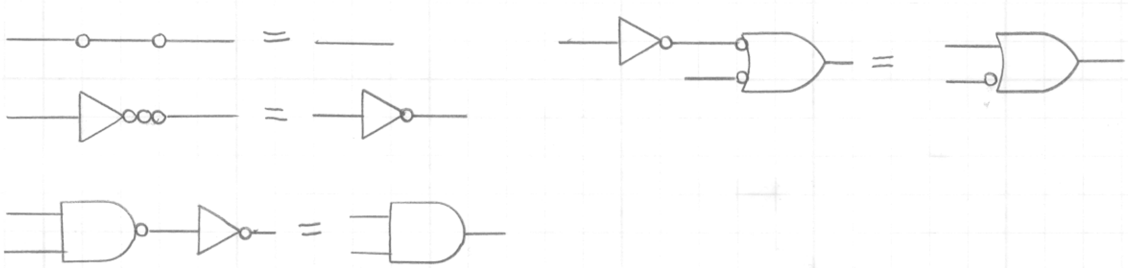
<u>Ex:</u> $X + 0 = X \xrightarrow[\text{dual}]{} X \cdot 1 = X$ Dual is also true (theorems with constant 1 or 0).

<u>Ex:</u> If $A = B + DE'$ is true, then $A = B \cdot (D + E')$ is also true.

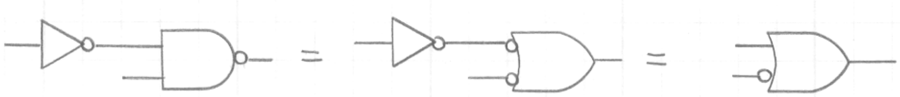
<u>Ex:</u> Dual of $(X'YZ)' + W'V = (X'YZ)' + (W'V) = (X' + Y + Z)' \cdot (W' + V)$

<u>Ex:</u> Find dual of $(X+Y)(X'+Z) = XY + X'Z$

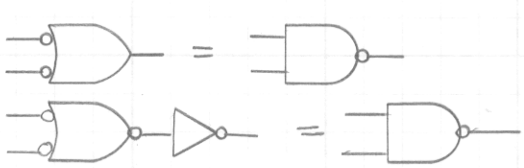
<u>Ex:</u> Re-forming circuits using $(X')' = X$

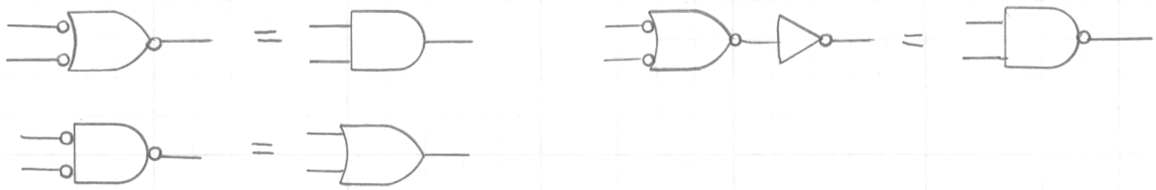


<u>Ex:</u>

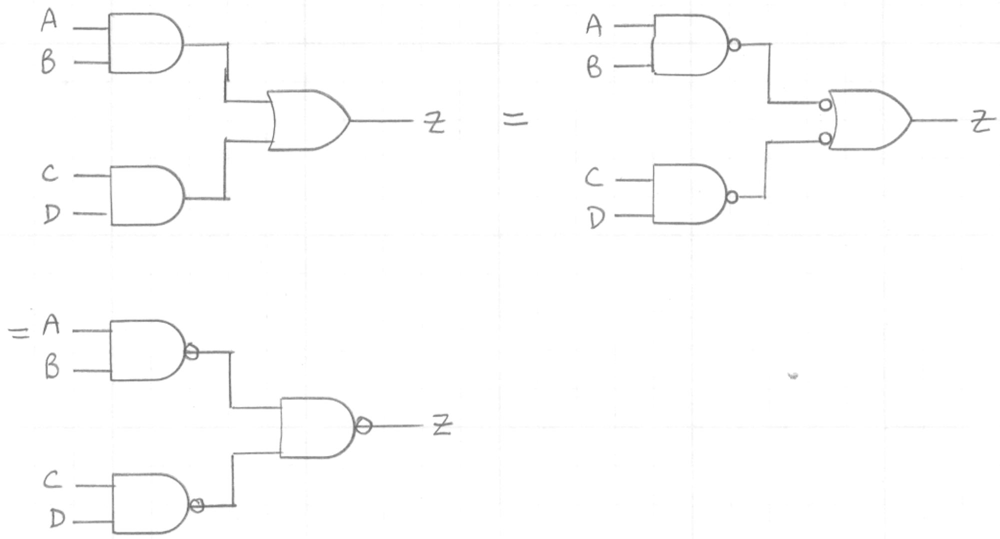


DeMorgan

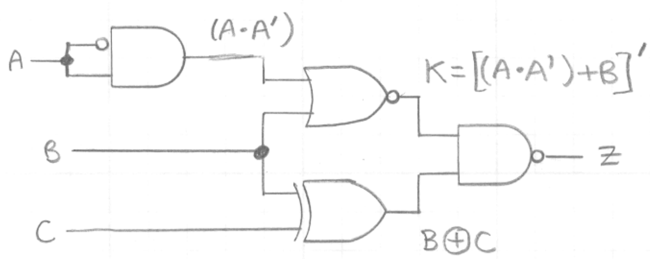




Ex: Sum-of-Products example: converting to NAND/NOR



Ex: Write equation for the following circuit:



$$Z = \Big[\big[(A\cdot A')+B\big]' \cdot (B\oplus C)\Big]'$$
$$= \Big[\big[0+B\big]' \cdot (B\oplus C)\Big]'$$
$$= \big[B' \cdot (B\oplus C)\big]' = B + (B\oplus C)'$$