Word problems

1) Figure out inputs and outputs

2) Understand relationship between inputs and outputs

3) Write an expression for each output

- Each output is an independent problem

Ex: Thermostat for HVAC

1) AC on if "cool" on and Temp > 2° + setting

2) Fan/cool on if "heat" on "" ≤ 8° setting -2°

```
  "cool" switch → AC on
  "heat" switch → Fan/cool on
  Temp T
  2° (constant)
  Time of day
  Fire sensor
```

Methods to describe Boolean expressions:

1) Simple English (if A=1 and B=0, then out=1)

2) Circuit schematic
3) Truth Table
   Enumerate all input combinations

4) Expression
   \[ A + B \cdot C \quad \text{SOP, POS} \]

5) Min term

6) Max term

Min terms
SOP expression
Enumerate combinations of inputs which give the output to be 1

Diagram:
- Inputs
- Connections to XOR gates
- XOR gates
- OR gate
- Output

"Normally" zero

(AND)

(OR)
Truth table

<table>
<thead>
<tr>
<th>XYZ</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
</tr>
<tr>
<td>001</td>
<td>0</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
</tr>
<tr>
<td>011</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>101</td>
<td>0</td>
</tr>
<tr>
<td>110</td>
<td>0</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ W = m_0 + m_2 + m_3 + m_7 \]

\[ W = \sum m(0,2,3,7) \]
Maxterms

P.O.S. expression where each entry in a T.T. is a potential maxterm
\[ \begin{array}{c|cccc|c}
X & Y & Z & W \\
0 & 0 & 0 & 1 & X + Y + Z = M_0 \\
0 & 0 & 1 & 0 & X + Y + Z' = M_1 \\
0 & 1 & 0 & 1 & X + Y' + Z = M_2 \\
0 & 1 & 1 & 1 & X + Y' + Z' = M_3 \\
1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
\end{array} \]

\[ W = M_1 \cdot M_4 - M_5 \cdot M_6 \]

\[ = \prod M(1, 4, 5, 6) \]

\[ = (X + Y + Z') \cdot (X' + Y + Z) \cdot (X' + Y + Z') \cdot (X' + Y' + Z) \]

- No common terms between minterm & maxterm expansions of the same function
- All \(2^n\) (\(n = 4\) at inputs) terms in minterm or maxterm expansion.
\[
\begin{array}{c|c|c}
X & Y & Z \\
00 & 1 & 0 \\
01 & 1 & 0 \\
10 & 1 & 0 \\
11 & 0 & 1 \\
\end{array}
\]
\[
z = \Sigma m(0,1,2) = \Pi M(3)
\]
\[
z' = \Sigma m(3) = \Pi M(0,1,2)
\]

Incompletely-Specified Functions

"Don't Care" = X (~third value for variables)

Ex. 7-segment display

[0-15]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
A & B & C & D & S0 & S1 & S2 & S3 & S4 \\
0000 & 1 & 1 & 1 & 1 & 0 \\
0001 & 0 & 1 & 1 & 1 & 0 \\
\vdots \\
1111 & X & X & X & X & X \\
\end{array}
\]

7-segment decoder

A B C D | S0 S1 S2 S3 S4 S5 S6
0000 | 1 1 1 1 1 1 0
0001 | 0 1 1 1 0 0 0
\[ S_0 = \sum m(0, 2, 3, \ldots) + \sum d(10, 11, 12, 13, 14, 15) \]
\[ S_0 = \Pi M(1, 4, \ldots) \cdot \Pi D(10, 11, 12, 13, 14, 15) \]