Adder, 5x: 2 + 4

**Analog**

4.00V +
2.00V

6.00V ± noise = f(voltage, temp., manufacturing variations, ...)

**Digital**

4.00060
2.00000

6.00000

- Arbitrary precision

Rough book

1) System design (180, 170)

2) Logic design (18)

3) Circuit design (118, 118)

A) Combinational logic

B) Sequential Circuits (comb. + memory)

A) Combinational Logic

Def: Block whose outputs depend only on the present inputs
B) Sequential Circuits

Definition: Block whose outputs depend on present and past inputs

Require memory

Example: Microwave oven

Digital Systems are made of:
1) Memory - store data
2) Datapath - processes data (\( +, -, \times, \div, \text{compare} \))
3) Control
4) Misc supporting circuits

Number Systems

**Binary**
- Base 2
- Unsigned

<table>
<thead>
<tr>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>( \frac{1}{2} )</th>
<th>( \frac{1}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ 101101 \_{10} = \frac{8}{2} + \frac{2}{2} + \frac{1}{2} = 11.25 \_{10} \]
Hexadecimal
- base 16
- digits: 0 - 9, A - F

Bin/Hex

\[
\begin{array}{c}
10110101_2 \\
= B5_{16}
\end{array}
\]

Ex: Dec \rightarrow Bin

42.25_{10}

\[
\begin{array}{c}
\frac{1}{32} \frac{1}{16} \frac{1}{8} \frac{1}{4} \frac{1}{2} \frac{1}{4} \\
32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1
\end{array}
\]

10.25

2.25

Addition

Ex:

\[
\begin{array}{c}
1100_2 \\
+ 0110_2 \\
\hline
10010_2 = 18_{10}
\end{array}
\]

\[
\begin{array}{c}
2_{10} = 10_2 \\
3_{10} = 11_2
\end{array}
\]
Subtraction

1) Book: similar to decimal subtraction

2) \( a - b = a + (-b) \)

- done in HW

- must use "signed" numbers

Exs 1 - 2 = -1

Multiplication

\[ \begin{align*}
111_2 & = 7_{10} \\
\times 101_2 & = 5_{10} \\
\hline
111 & \\
1000 & \\
+ 111 & \\
\hline
100011_2 & = 35_{10} \checkmark
\end{align*} \]

Signed Number Formats

① Signed Magnitude

<table>
<thead>
<tr>
<th>Sign</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>000 - 1</td>
</tr>
</tbody>
</table>

0 = pos
1 = neg

Std. unsigned binary
- not so good for $\pm w$ for $\pm 1$-
- great for multi-
- two zeros

0 0 0 0 0 0 = +0
1 0 0 0 0 0 = -0

(2) Two's Complement

Def: Same as unsigned except the positional weight of the MSB is negative

\[
\begin{array}{ccccccc}
& & & & & & \\
\text{MSB} & \text{bit} & \text{bit} & \text{bit} & \text{bit} & \text{bit} & \text{LSB} \\
-2^3 & 2^2 & 2^1 & 2^0 & 2^{-1} & 2^{-2} \\
-8 & 4 & 2 & 1 & \frac{1}{2} & \frac{1}{4} & \left[-8.0 + \frac{3}{4}\right] \\
\end{array}
\]

-32 16 8 4 2 1

-4 1 1 0 0 0 0 0 = -4

\[
\begin{array}{c}
-8 + \Box = -4 \\
\end{array}
\]
\[-4 = \overline{00100000} \]
\[-4 = \overline{10101} \]

**Book:** \( n \)-bit number \( N \), \( \rightarrow \) \( -N = 2^n - N \)

**Complement**

Invert \( 2\)'s complement number:

"Flip bits and add 1"

\[+3 \quad 0011\]

\[1100 \quad \text{flip bit}\]
\[1 \quad \text{add 1}\]
\[1101 = -8 + 4 + 1 = -3 \checkmark\]

- MSB is a sign bit: \( 1 \) is negative,
- \( 0 \) = positive or zero

+ one zero \( 000000 \)

- All 1's = \(-1\):
  \[1111 = -1\]
  \[1111111 = -1\]

- We can always sign extend MSB bit by replicating it

\[000005\]
\[000005\]
Binary-Coded Decimal (BCD)

Every 4 bits = 1 decimal digit

Ex: $129_{10}$ = 0001 0010 1001 $_{BCD}$

$8_8, 4_8, 2_8, 1_8, 8_8, 4_8, 2_8, 1_8$