Assignment 1
Due 23 January 2007 (in class)

Reading Assignment: Chapters 3 and 34 of CLRS (T. Cormen, C. Leiserson R. Rivest, C. Stein, ”Introduction to Algorithms”, Mc-Graw Hill and MIT Press, 2001). The chapters are on growth of functions, algorithm’s complexity, and NP-Completeness, respectively. We have not covered many of the topics in those chapters, however, an overview would be helpful.

Problem 1: A Vertex Cover is a subset of nodes of a graph, such that for any edge at least one of the two adjacent nodes are in the subset. Prove that finding the minimum size Vertex Cover is (technically, the corresponding decision problem) is NP-Complete.

Problem 2: Consider a company that has $n$ departments that work serially, i.e, dept 1 passes its output to dept 2, dept 2 passes its output to dept 3, \ldots, dept $n-1$ passes its output to dept $n$. The final product is the output of dept $n$.

Each dept has two working modes (bonus mode and laid-back mode). For example dept $i$ can finish its task (which can start only after dept $i-1$ has finished its own task) either in $d_{i,1}$ days or in $d_{i,2}$ days, where $d_{i,1} < d_{i,2}$ (all $d_i$’s are integers). The employee salaries are higher when they work hard. So, the company saves $d_{i,1}$ dollars in employee salaries, for operating dept $i$ in bonus mode, and saves $d_{i,2}$ dollars in laid-back mode.

The problem is to maximize company’s savings under product time-to-market constraint. That is, the product of the company has to be ready at most after $T$ days. Prove that for arbitrary $d_i$’s, the corresponding decision problem is in NP. (*hint: transform subset sum problem to this problem.*)

Problem 3: Formulate graph coloring as an Integer Linear Programming (ILP) problem. This problem was informally assigned in class on thursday 1/11.