# EEC180A DIGITAL SYSTEMS I Winter, 2006.

## Solutions for Homework # 4

**8.A**

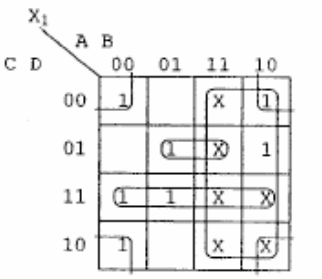
| $ABCD$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ |
|---|---|---|---|---|---|---|---|
| 0000 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0001 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0010 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0011 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0100 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0101 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0110 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0111 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1000 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1001 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |

$X_1 = B'D' + B\,D + A + C\,D = \underline{B'D'} + \underline{BC'D} + A + \underline{CD}$ (used in circuit)
$X_1 = B'D' + B\,D + A + B'C$
$X_2 = B' + \underline{C'D'} + \underline{C\,D}$
$X_3 = C' + D + B$
$X_4 = \underline{B'D'} + \underline{B'C} + \underline{B\,C'D} + \underline{C\,D'}$
$X_5 = \underline{B'D'} + \underline{C\,D'}$
$X_6 = \underline{C'D'} + \underline{B\,C'} + B\,D' + A$
$X_7 = \underline{B'C} + \underline{B\,C'} + A + \underline{C\,D'}$ (used in circuit)
$X_7 = B'C + B\,C' + A + B\,D'$

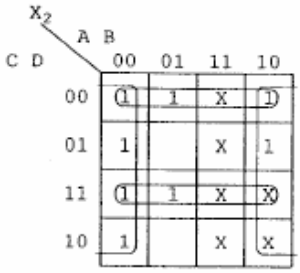
This solution uses 15 gates and 41 gate inputs.

Students are allowed to use a maximum of 18 gates.

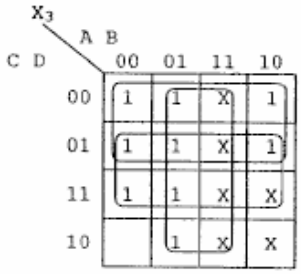
**8.A, continued**



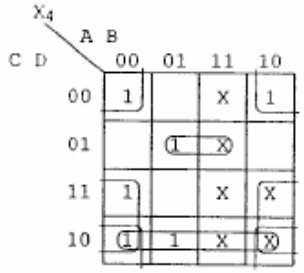

$X_1 = B'D' + B\,C'D + A + C\,D$



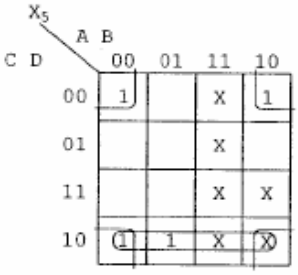

$X_2 = B' + C'D' + C\,D$



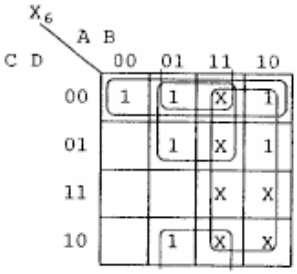

$X_3 = C' + D + B$



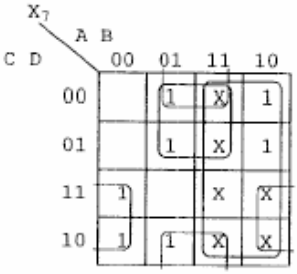

$X_4 = B'D' + B'C + B\,C'D + C\,D'$



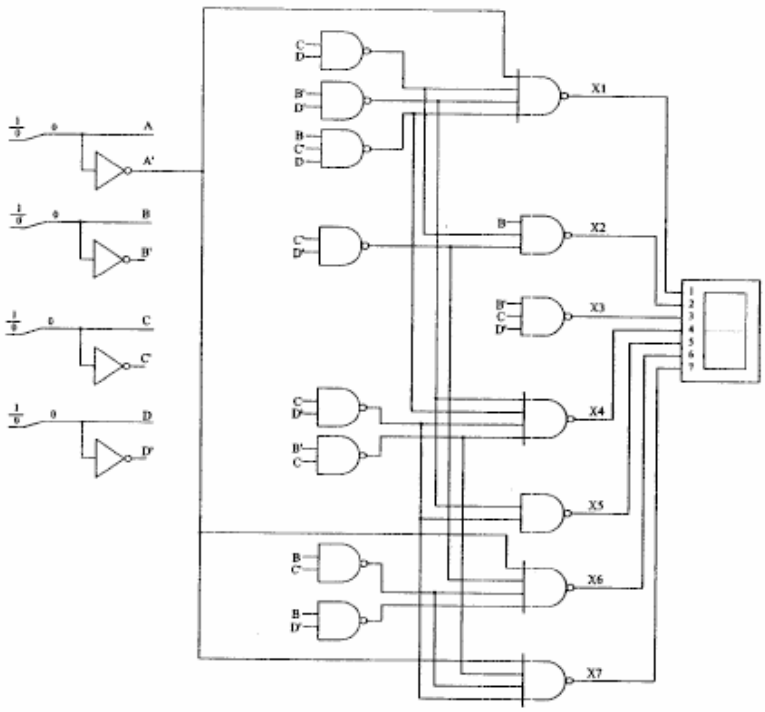

$X_5 = B'D' + C\,D'$




$X_6 = C'D' + B\,C' + B\,D' + A$




$X_7 = B'C + B\,C' + A + B\,D'$



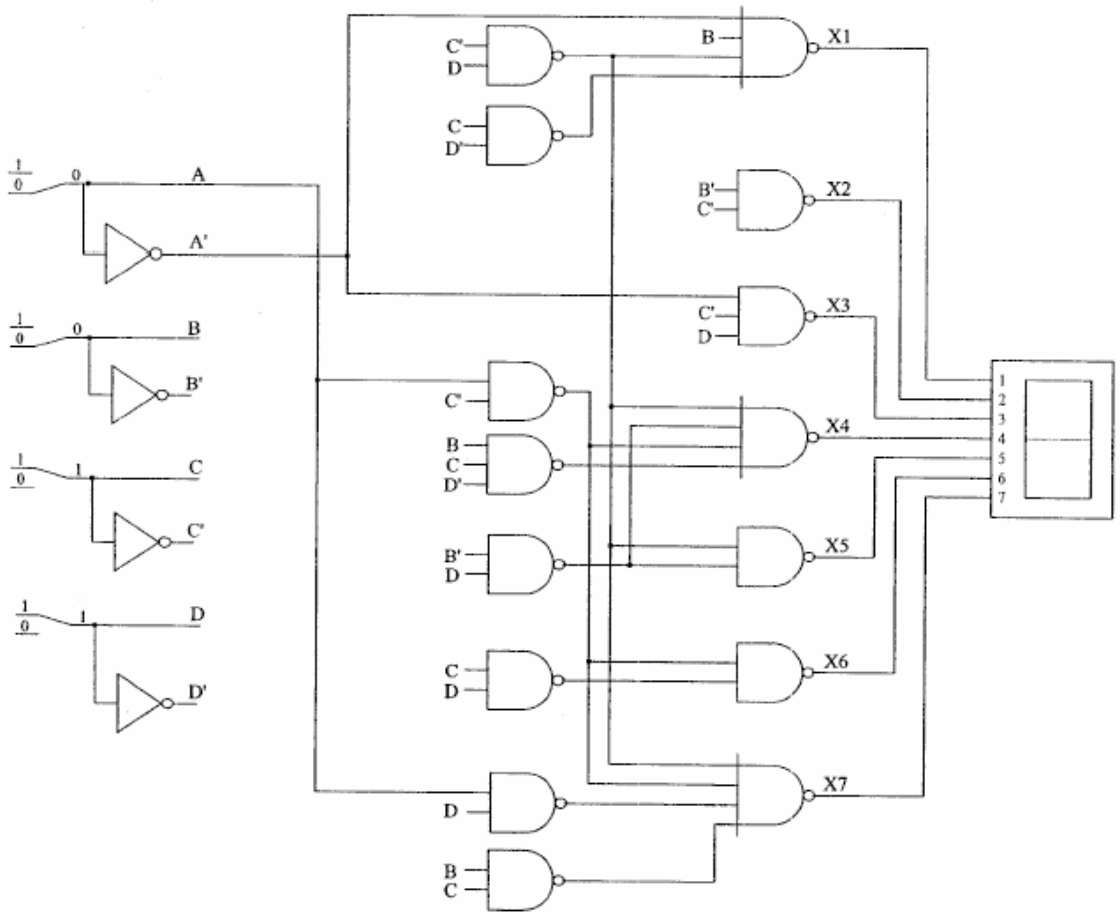

**8.B**

| *ABCD* | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ |
|---|---|---|---|---|---|---|---|
| 0000 | X | X | X | X | X | X | X |
| 0001 | X | X | X | X | X | X | X |
| 0010 | X | X | X | X | X | X | X |
| 0011 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0100 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0101 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0110 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0111 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1000 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1001 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1010 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1011 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1100 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

$X_1 = B' + \underline{C'D} + C\,D' + A$
$X_2 = C + B$
$X_3 = D' + C + A$ (used in circuit)
$X_3 = D' + C + B'$
$X_4 = \underline{C'D} + \underline{B'D} + B\,C\,D' + \underline{A\,C'}$ (used in circuit)
$X_4 = C'D + A'C\,D' + B'D + A\,C'$
$X_5 = \underline{C'D} + \underline{B'D}$
$X_6 = C\,D + \underline{A\,C'}$
$X_7 = A\,D + B\,C + \underline{C'D} + \underline{A\,C'}$ (used in circuit)
$X_7 = A\,D + B\,C + A\,C' + B\,D$

This solution uses 15 gates and 38 gate inputs.

Students are allowed to use a maximum of 16 gates.




**8.O**

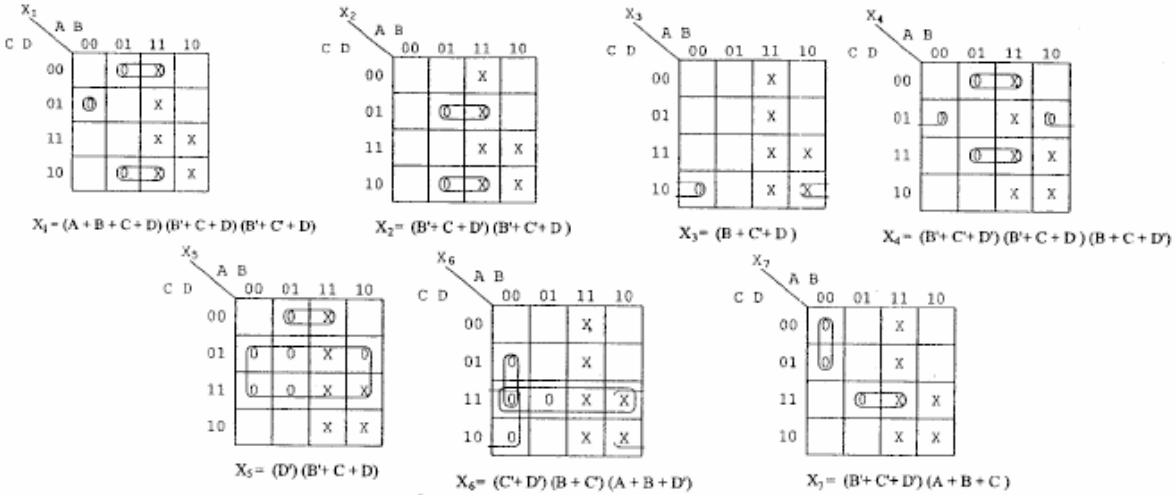
| *ABCD* | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ |
|---|---|---|---|---|---|---|---|
| 0000 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0001 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0010 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0011 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0100 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0101 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0110 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0111 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1000 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1001 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |

$X_1 = (A + B + C + D')(B' + D) = (A + B + C + D')\underline{(B' + C' + D)}\underline{(B' + C + D)}$
$X_2 = (B' + C + D')\underline{(B' + C' + D)}$
$X_3 = (B + C' + D)$
$X_4 = (B + C + D')\underline{(B' + C + D)}\underline{(B' + C' + D')}$
$X_5 = D'(B' + C) = D'\underline{(B' + C + D)}$
$X_6 = (A + B + D')(B + C')(C' + D')$
$X_7 = (A + B + C)\underline{(B' + C' + D')}$

This solution uses 18 gates and 48 gate inputs.

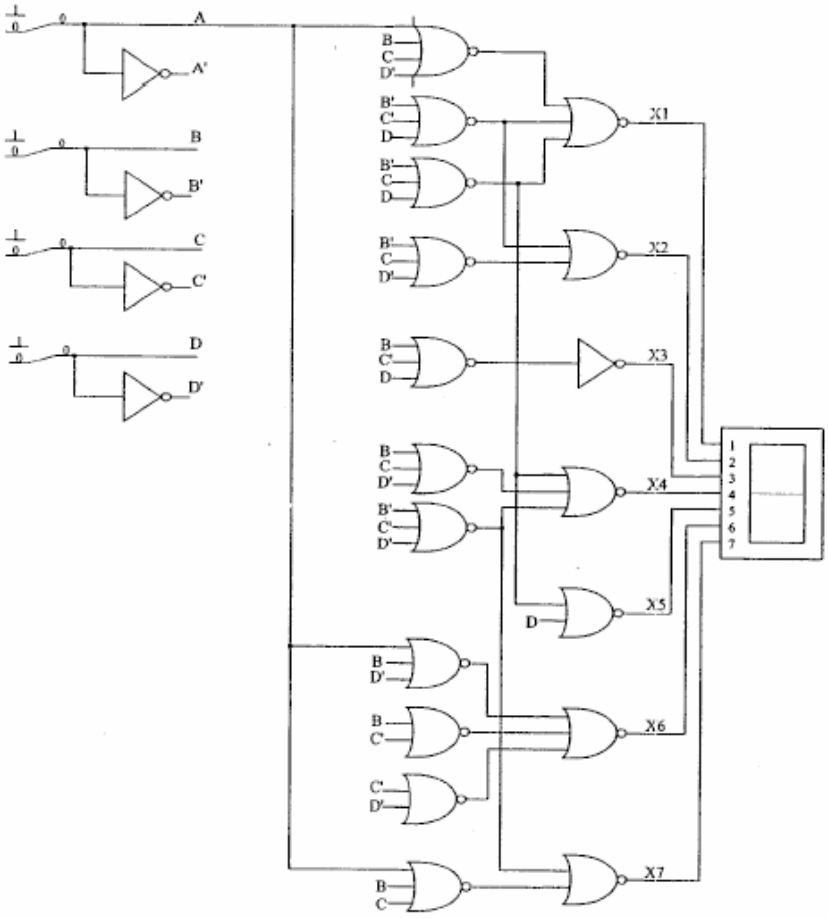
Students are allowed to use a <u>maximum</u> of 19 gates.

**8.O, continued**

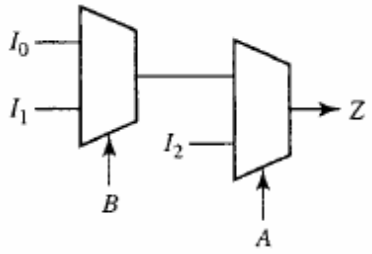



To save one gate, use:

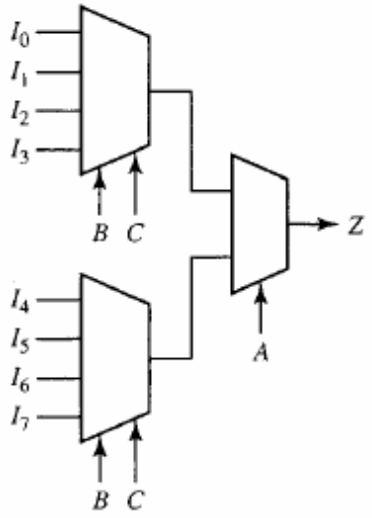
$X_6 = (B' + C' + D')(B + C')(A + B + D')$



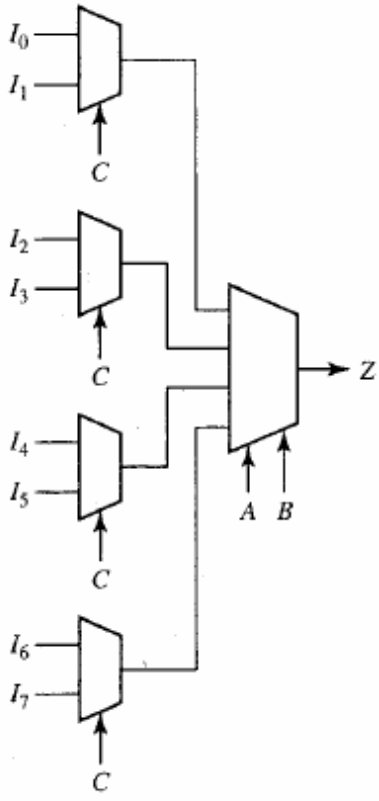
9.1 (a)


$I_0$
$I_1$
$I_2$
B
A
Z

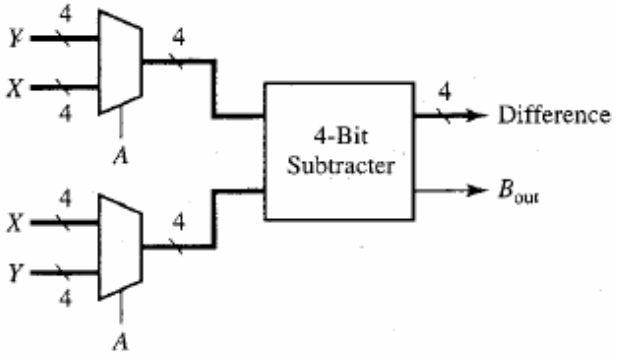
(b)


$I_0$
$I_1$
$I_2$
$I_3$
B C
$I_4$
$I_5$
$I_6$
$I_7$
B C
A
Z

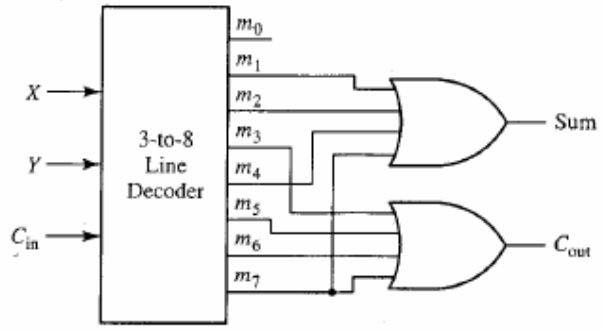
(c)


$I_0$
$I_1$
C
$I_2$
$I_3$
C
$I_4$
$I_5$
C
$I_6$
$I_7$
C
A B
Z

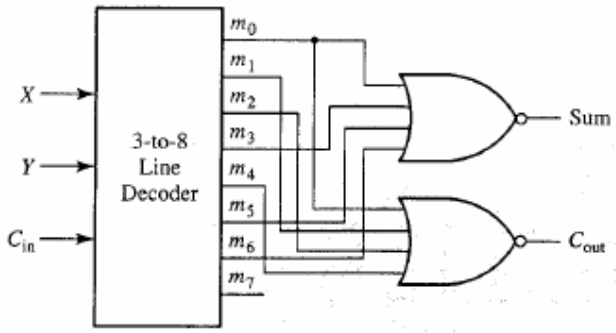
9.2


Y
X
4
4
A
4
4-Bit
Subtracter
Difference
$B_{out}$
X
Y
A

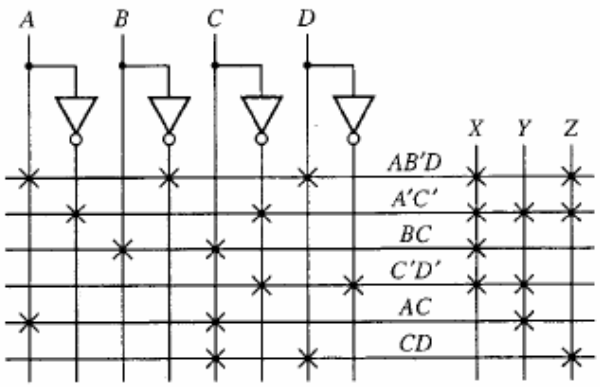
9.4 (a)


X
Y
$C_{in}$
3-to-8
Line
Decoder
$m_0$
$m_1$
$m_2$
$m_3$
$m_4$
$m_5$
$m_6$
$m_7$
Sum
$C_{out}$

(b)


X
Y
$C_{in}$
3-to-8
Line
Decoder
$m_0$
$m_1$
$m_2$
$m_3$
$m_4$
$m_5$
$m_6$
$m_7$
Sum
$C_{out}$

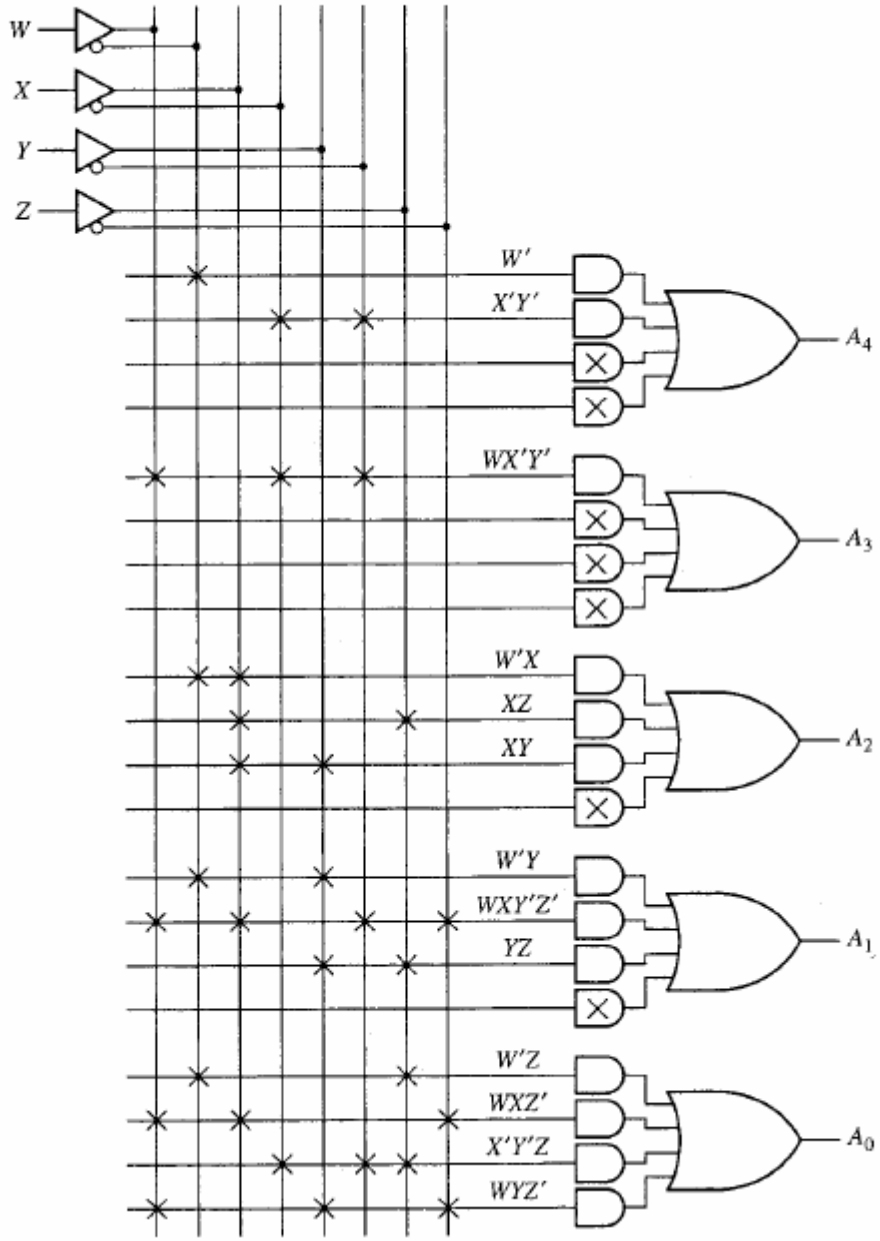
9.8 (a)




(b) Truth Table for the ROM

| A | B | C | D | X | Y | Z |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

9.10 (a) $A_4 = W' + X'Y' \quad A_3 = WX'Y' \quad A_2 = W'X + XZ + XY$
$A_1 = W'Y + WXY'Z' + YZ \quad A_0 = W'Z + WXZ' + X'Y'Z + WYZ'$



(b)

| W | X | Y | Z | $A_4$ | $A_3$ | $A_2$ | $A_1$ | $A_0$ |
|---|---|---|---|---|---|---|---|---|
| 0 | - | - | - | 1 | 0 | 0 | 0 | 0 |
| - | 0 | 0 | - | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | - | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | - | - | 0 | 0 | 1 | 0 | 0 |
| - | 1 | - | 1 | 0 | 0 | 1 | 0 | 0 |
| - | 1 | 1 | - | 0 | 0 | 1 | 0 | 0 |
| 0 | - | 1 | - | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| - | - | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | - | - | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | - | 0 | 0 | 0 | 0 | 0 | 1 |
| - | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | - | 1 | 0 | 0 | 0 | 0 | 0 | 1 |

**9.25 (a)**

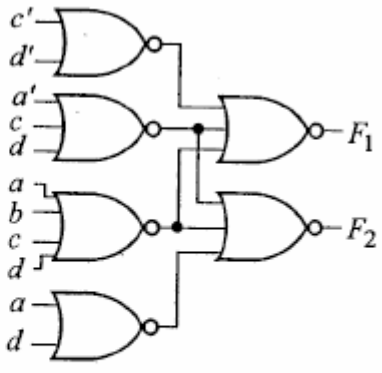
| C D \ A B | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) | 1 | 1 | 1 |
| 01 | 1 | 1 | (0) | (0) |
| 11 | (0) | (0) | (0) | (0) |
| 10 | 1 | 1 | 1 | 1 |

$F_1 = (A + B + C + D)(A' + C + D')(C' + D')$

**9.25 (a) (contd)**

| C D \ A B | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) | 1 | 1 | 1 |
| 01 | (0) | (0) | (0) | (0) |
| 11 | (0) | (0) | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 |

$F_2 = (A + B + C + D)(A' + C + D')(A + D')$




Alternate solution:

$F_1 = (a + b + c + d)\ (a + c' + d')\ (a' + d')$

$F_2 = (a + b + c + d)\ (a + b' + d')\ (c + d')$

**9.25 (b)**

| | $a\ b\ c\ d$ | $F_1F_2$ |
|---|---|---|
| $(cd')$ | - - 1 0 | 1 1 |
| $(bd')$ | - 1 - 0 | 1 1 |
| $(ad')$ | 1 - - 0 | 1 1 |
| $(ac)$ | 1 - 1 - | 0 1 |
| $(a'c'd)$ | 0 - 0 1 | 1 0 |