

# Unit 5: Karnaugh Maps

EEC180A

# 5.1 Minimum Forms of Switching Functions

Find a minimum sum-of products expression for:

$$F(a,b,c) = \sum m(0,1,2,5,6,7)$$

Note: Use  $XY' + XY = X$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$F = a'b' + b'c + bc' + ab$$

None of these terms can be eliminated

However, if we combine in a different way.

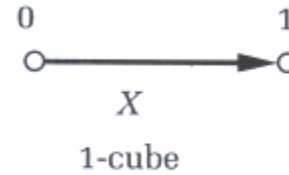
$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$F = a'b' + bc' + ac$$

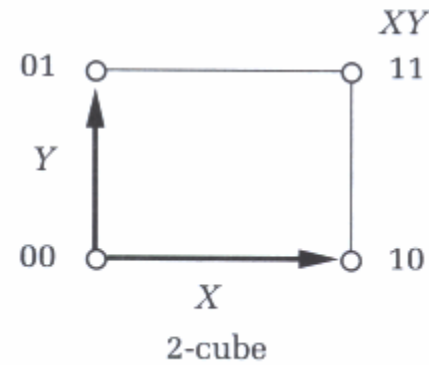
# 5.2 Karnaugh Maps

We can represent a 1 and 2-input truth table as 1-D and 2-D cube

X	F
0	
1	

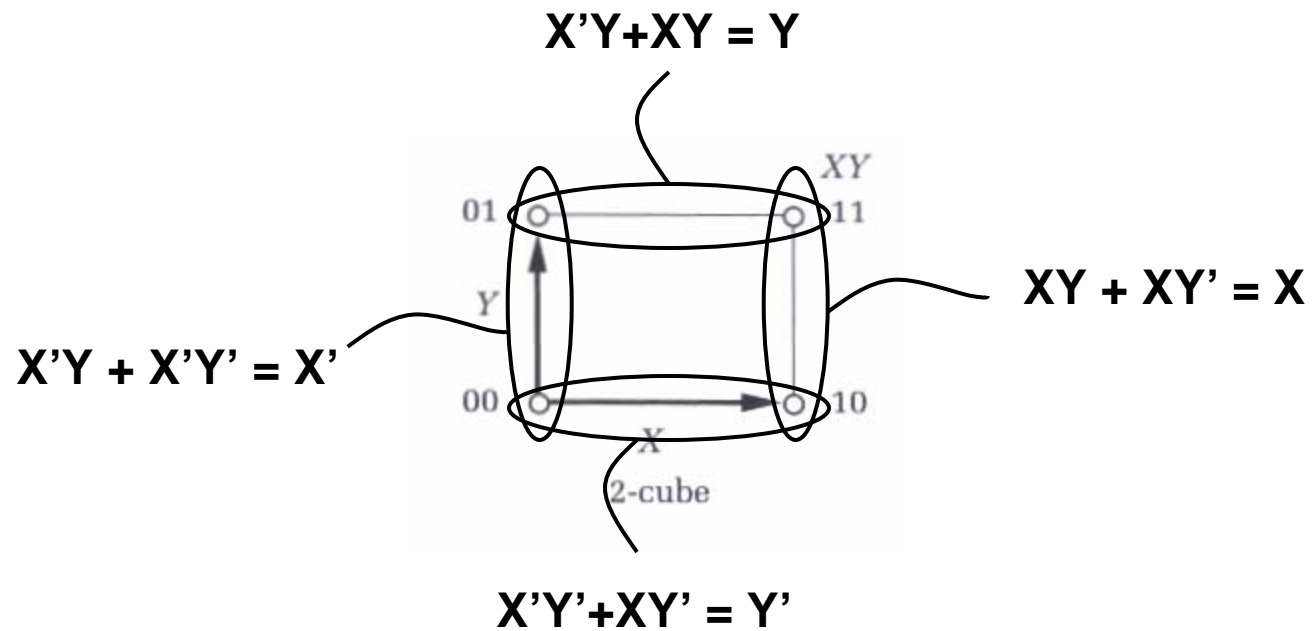


XY	F
00	
01	
10	
11	

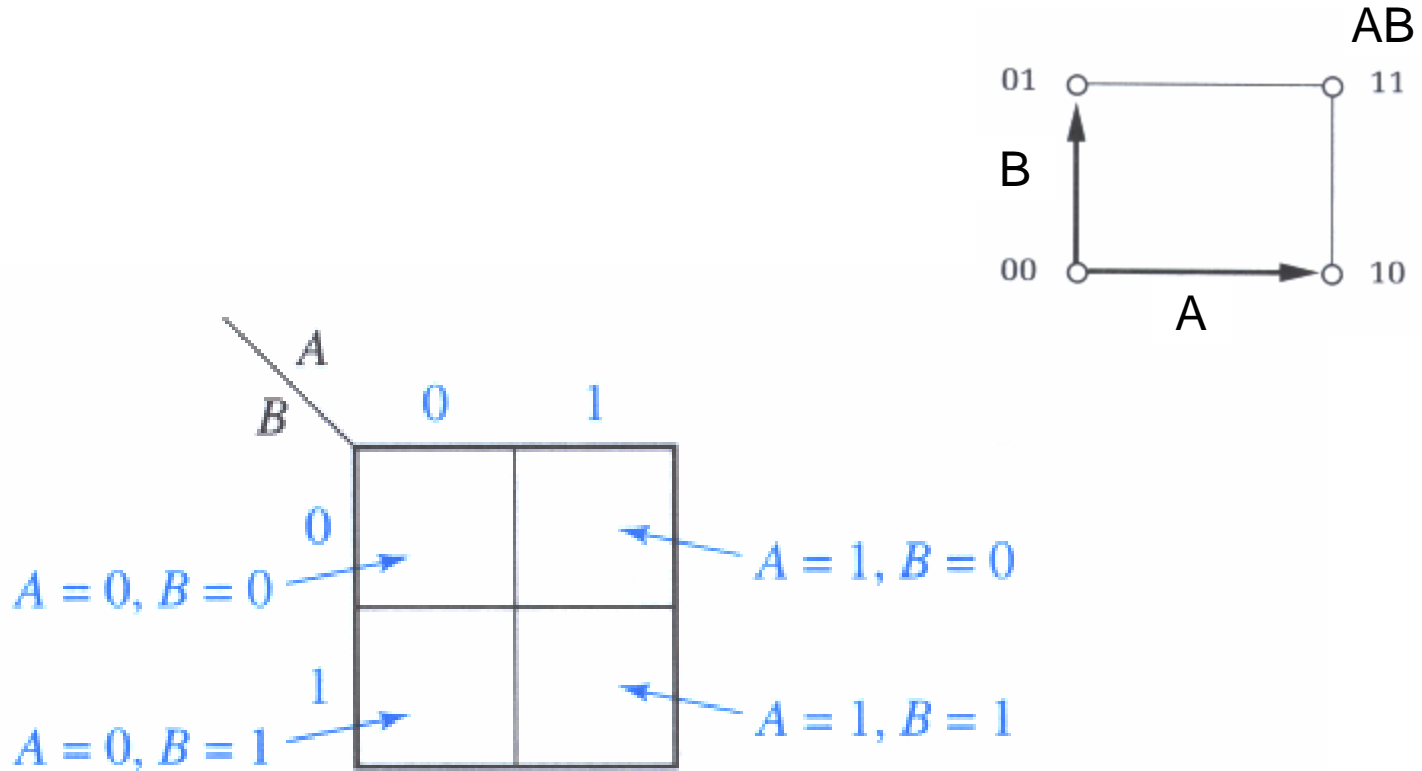


## 5.2 Karnaugh Maps

Allows for easy application of  $XY + XY' = X$



# 5.2 Two-Variable Karnaugh Maps

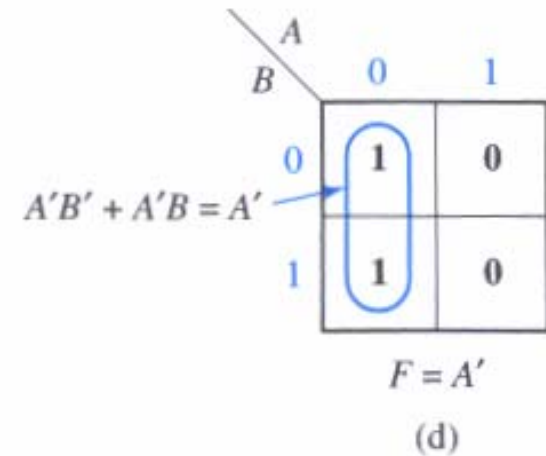
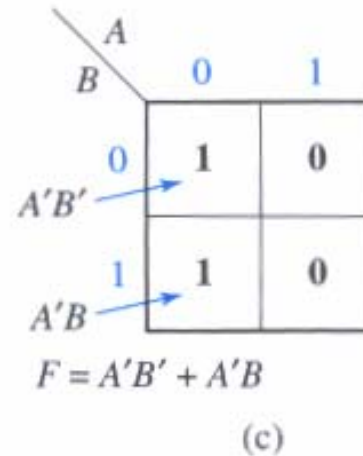
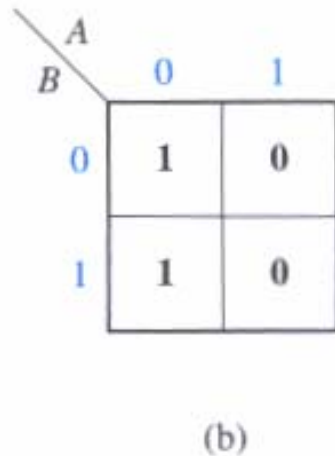


# 5.2 Two-Variable Karnaugh Maps

Two Variable Karnaugh Map Example:

$AB$	$F$
00	1
01	1
10	0
11	0

(a)

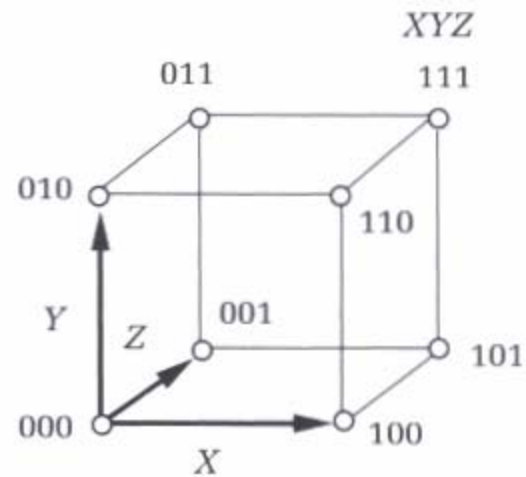


- **Minterms in adjacent squares on the map can be combined since they differ in only one variable (i.e.  $XY' + XY = X$ )**

## 5.2 Three-Variable Karnaugh Maps

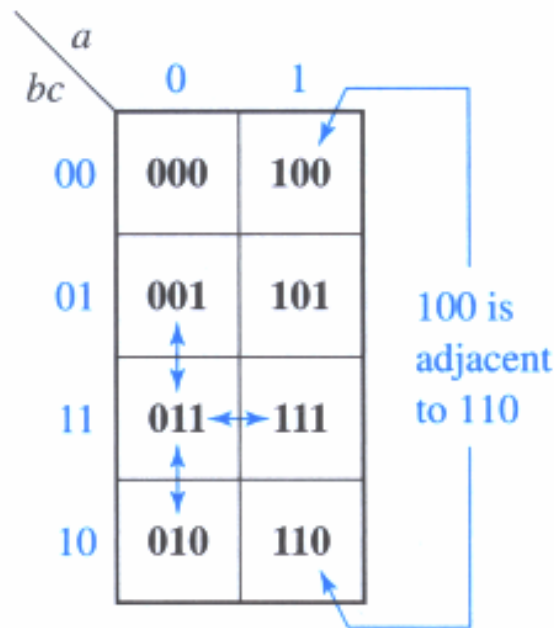
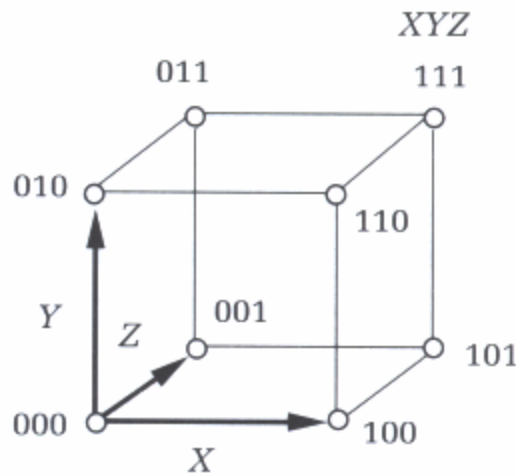
**We can represent a 3-input truth table as a 3-D cube**

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

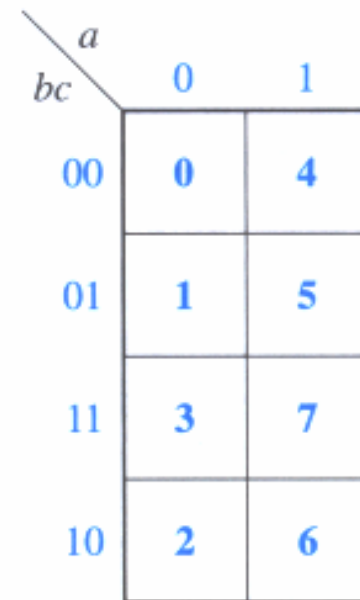


# 5.2 Three-Variable Karnaugh Maps

## Location of Minterms on a Three Variable Karnaugh Map



(a) Binary notation



(b) Decimal notation



## 5.2 Three-Variable Karnaugh Maps

Truth Table and resulting Karnaugh Map for Three-Variable Function

<i>ABC</i>	<i>F</i>
000	0
001	0
010	1
011	1
100	1
101	0
110	1
111	0

(a)

<i>A</i> \ <i>BC</i>	0	1
00	0	1
01	0	0
11	1	0
10	1	1

*F*

*ABC = 001, F = 0* →

← *ABC = 110, F = 1*

(b)

## 5.2 Three-Variable Karnaugh Maps

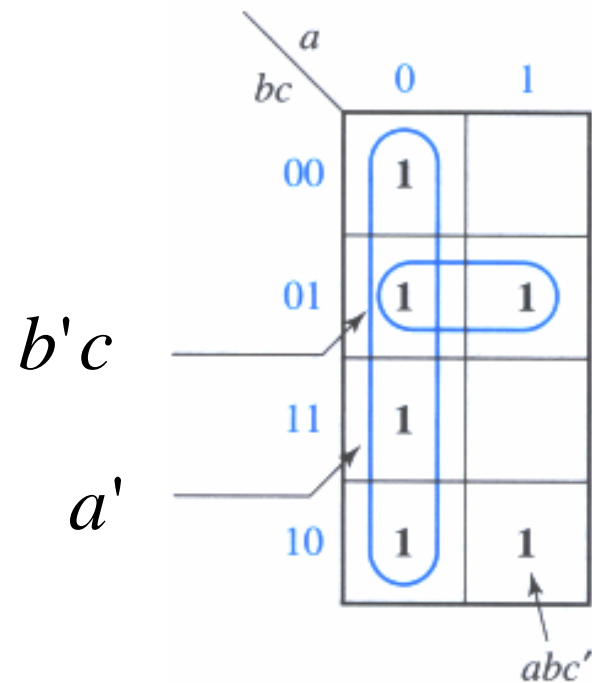
Location of Minterms on a Three Variable Karnaugh Map

$$F(a,b,c) = \sum m(1,3,5) = \prod (0,2,4,6,7)$$

<i>a</i>	<i>bc</i>	
	0	1
00	0 0	0 4
01	1 1	1 5
11	1 3	0 7
10	0 2	0 6

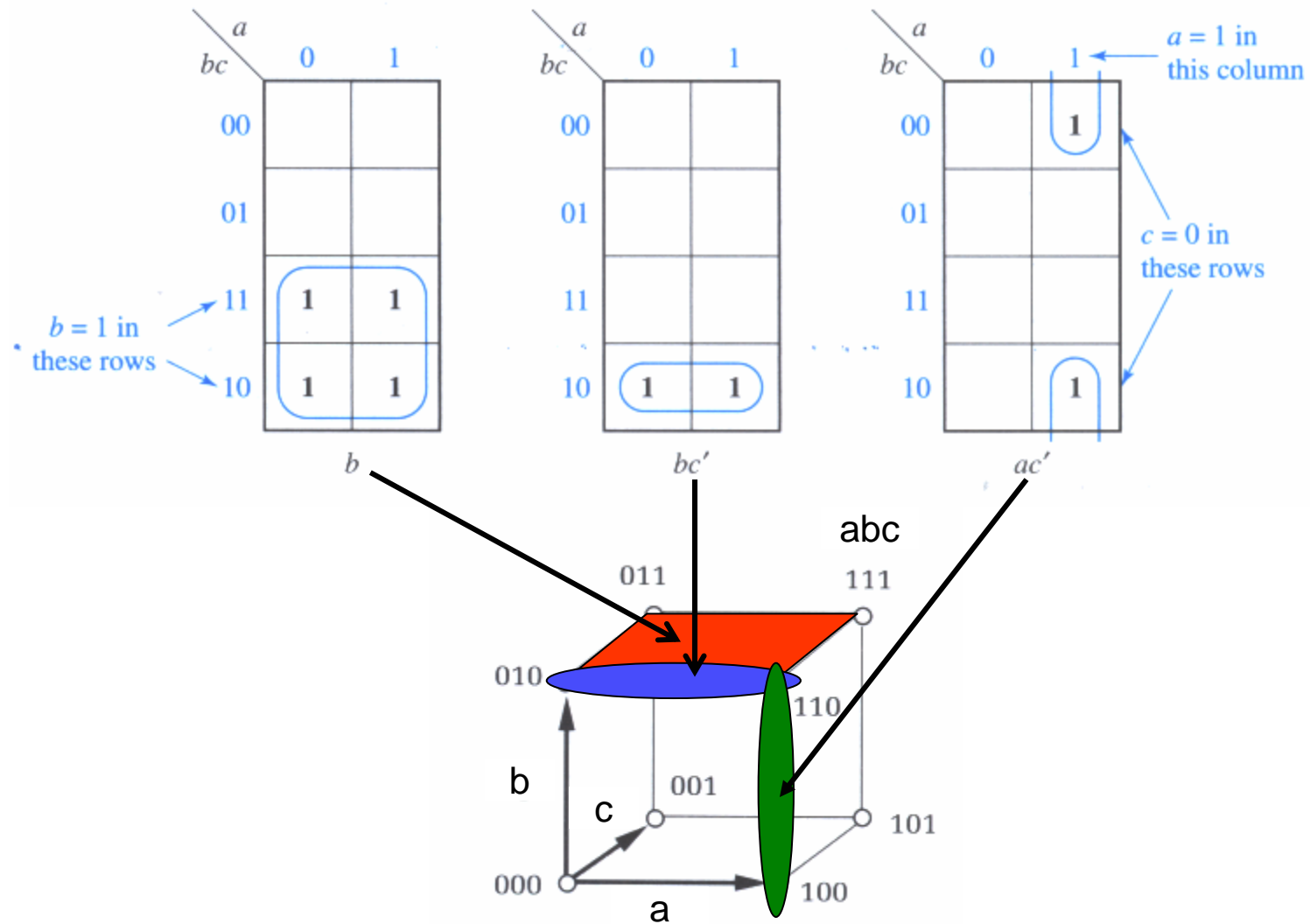
## 5.2 Three-Variable Karnaugh Maps

Karnaugh Map for  $F = abc' + b'c + a'$



# 5.2 Three-Variable Karnaugh Maps

## Karnaugh Maps for Product Terms



## 5.2 Three-Variable Karnaugh Maps

### Simplification of a Three-Variable Function

$$F = \sum m(1,3,5)$$

$a \backslash bc$	0	1
00		
01	1	1
11	1	
10		

$a \backslash bc$	0	1
00		
01	1	1
11	1	
10		

$T_1 = a'b'c + a'bc = a'c$

$T_2 = a'b'c + ab'c = b'c$

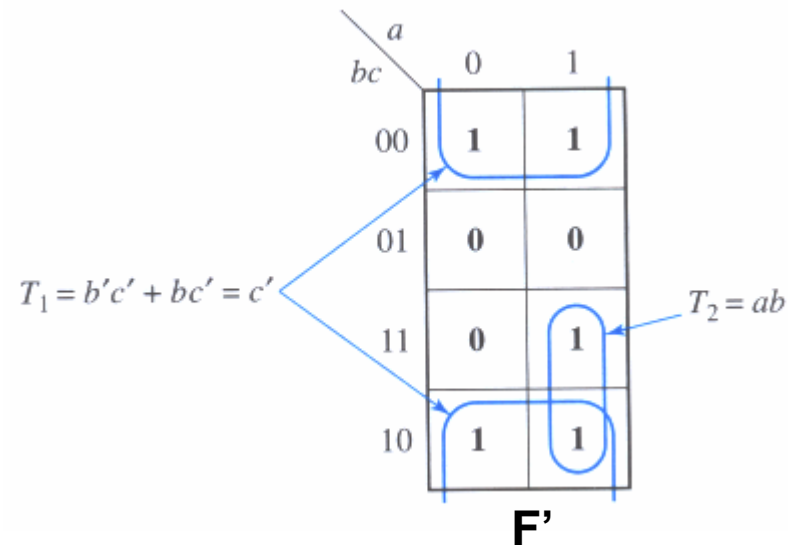
$$F = a'c + b'c$$

## 5.2 Three-Variable Karnaugh Maps

Simplification of  $F'$

$$F = \sum m(1,3,5)$$

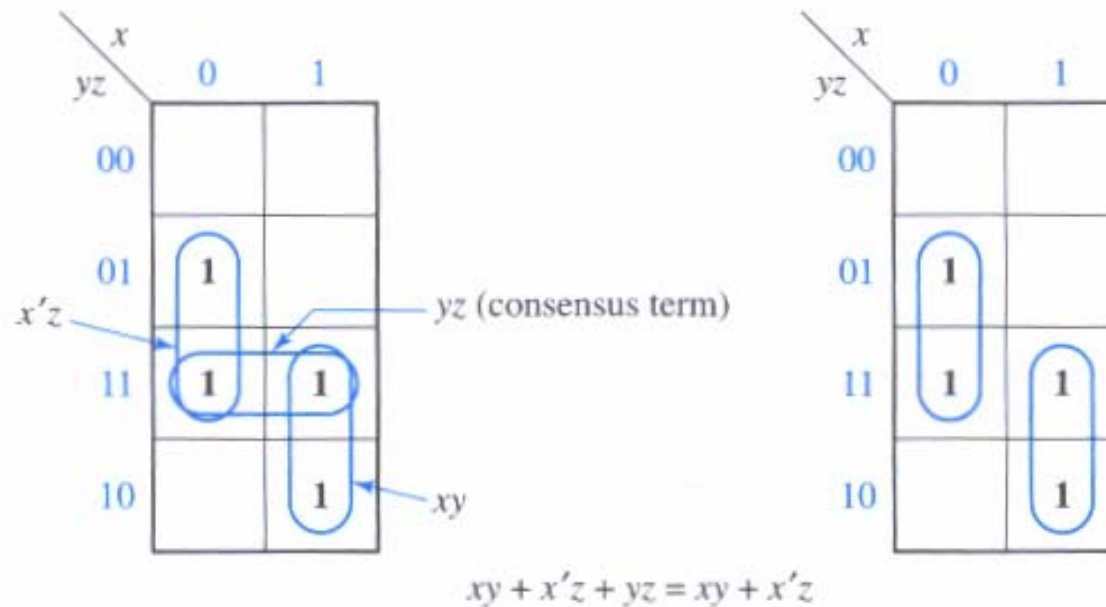
$$F' = \sum m(0,2,4,6,7)$$



$$F' = c' + ab$$

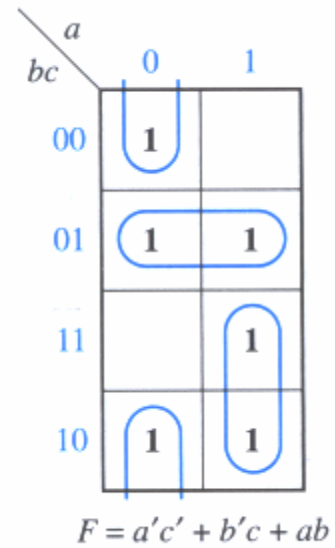
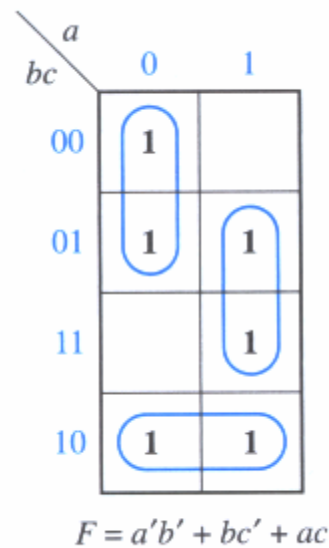
## 5.2 Three-Variable Karnaugh Maps

Karnaugh Maps which illustrate the Consensus Theorem



## 5.2 Three-Variable Karnaugh Maps

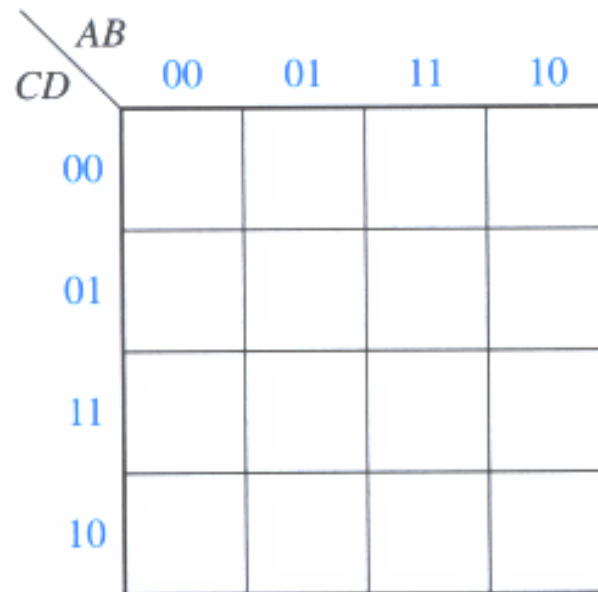
Function with Two Minimum Forms





## 5.3 Four-Variable Karnaugh Maps

Adjacent squares should differ by only one variable



## 5.3 Four-Variable Karnaugh Maps

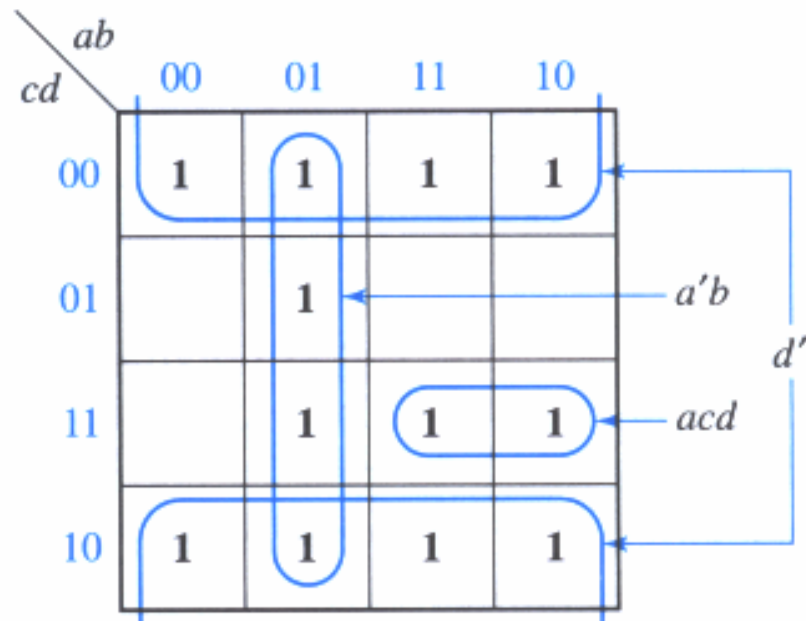
Location of Minterms on a Four-Variable Karnaugh Map

<i>CD</i> \ <i>AB</i>	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

## 5.3 Four-Variable Karnaugh Maps

Sample 4-variable Karnaugh Map

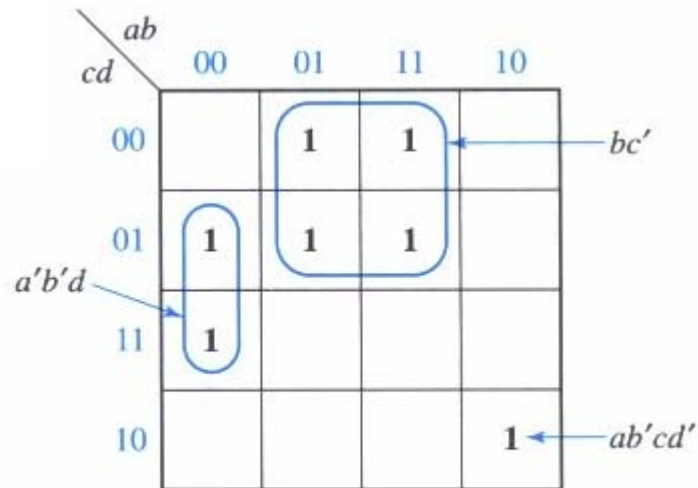
$$F = acd + a'b + d'$$



## 5.3 Four-Variable Karnaugh Maps

Simplification of Four-Variable Functions

$$F = \sum m(1,3,4,5,10,12,13)$$

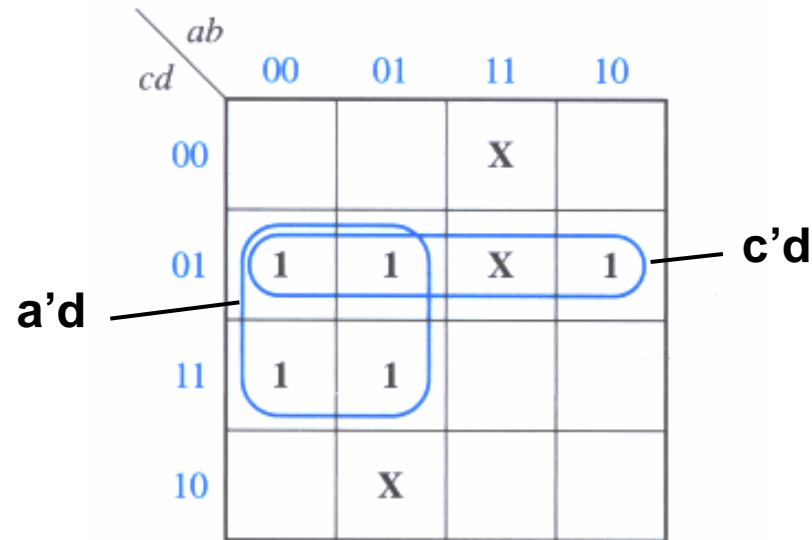


$$F = bc' + a'b'd + ab'cd'$$

## 5.3 Four-Variable Karnaugh Maps

Simplification of Incompletely Specified Function

$$F = \sum m(1,3,5,7,9) + \sum d(6,12,13)$$

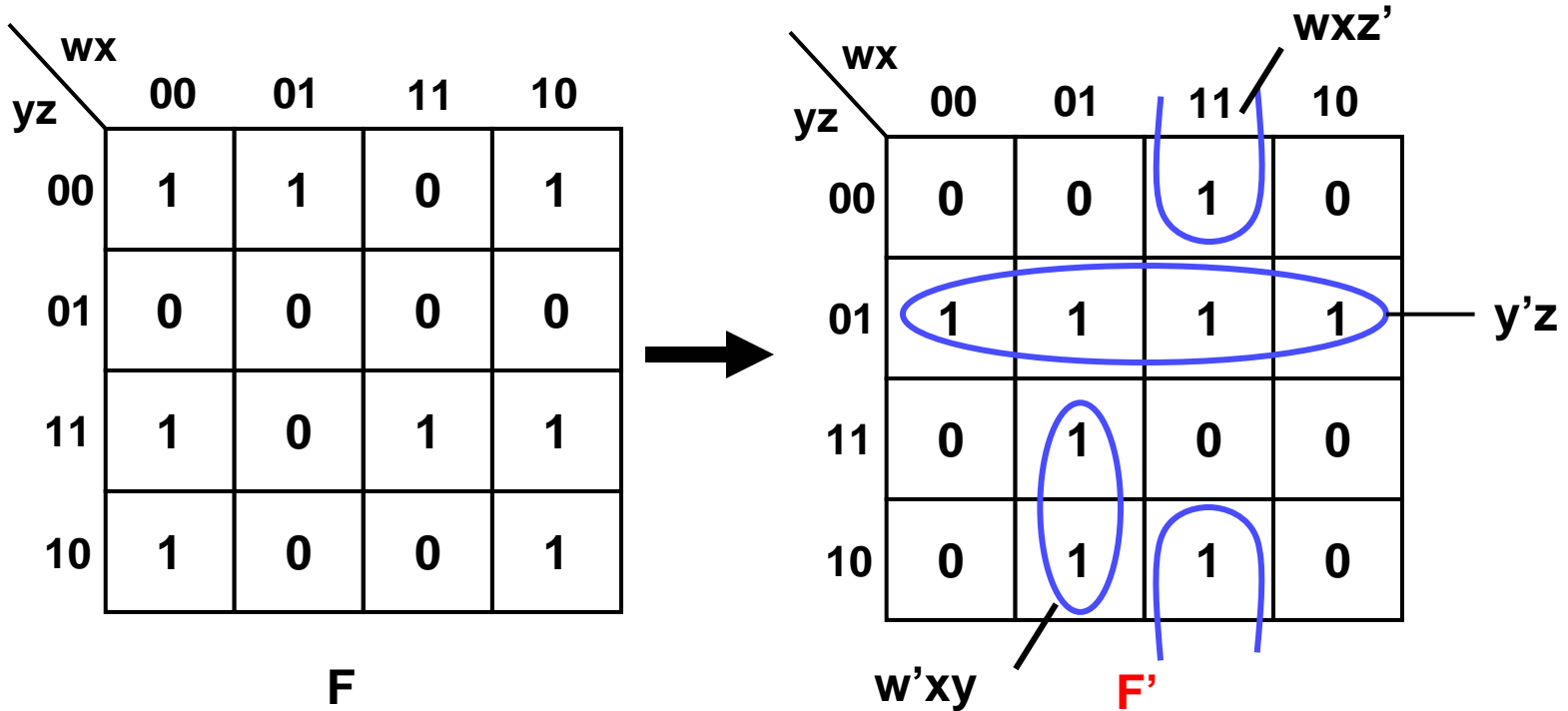


$$F = a'd + c'd$$

# 5.3 Four-Variable Karnaugh Maps

Finding Minimum Product of Sums from Karnaugh Maps

$$F = x'z' + wyz + w'y'z' + x'y$$



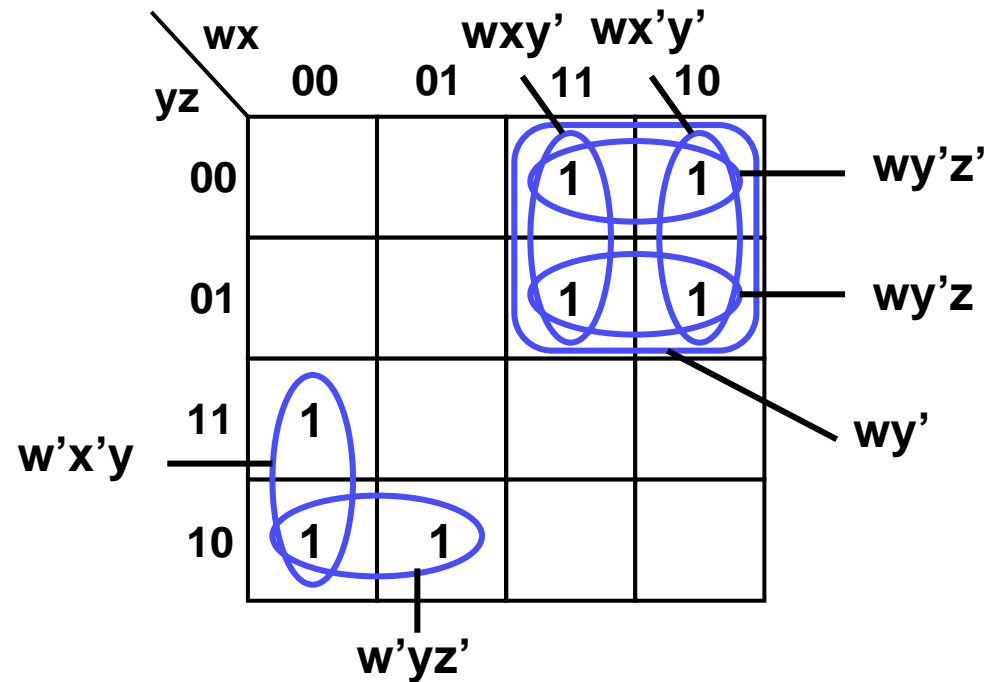
$$F' = y'z + wxz' + w'xy$$

Using DeMorgan's

$$F = (y + z')(w' + x' + z)(w + x' + y')$$

## 5.4 Determination of Minimum Expressions

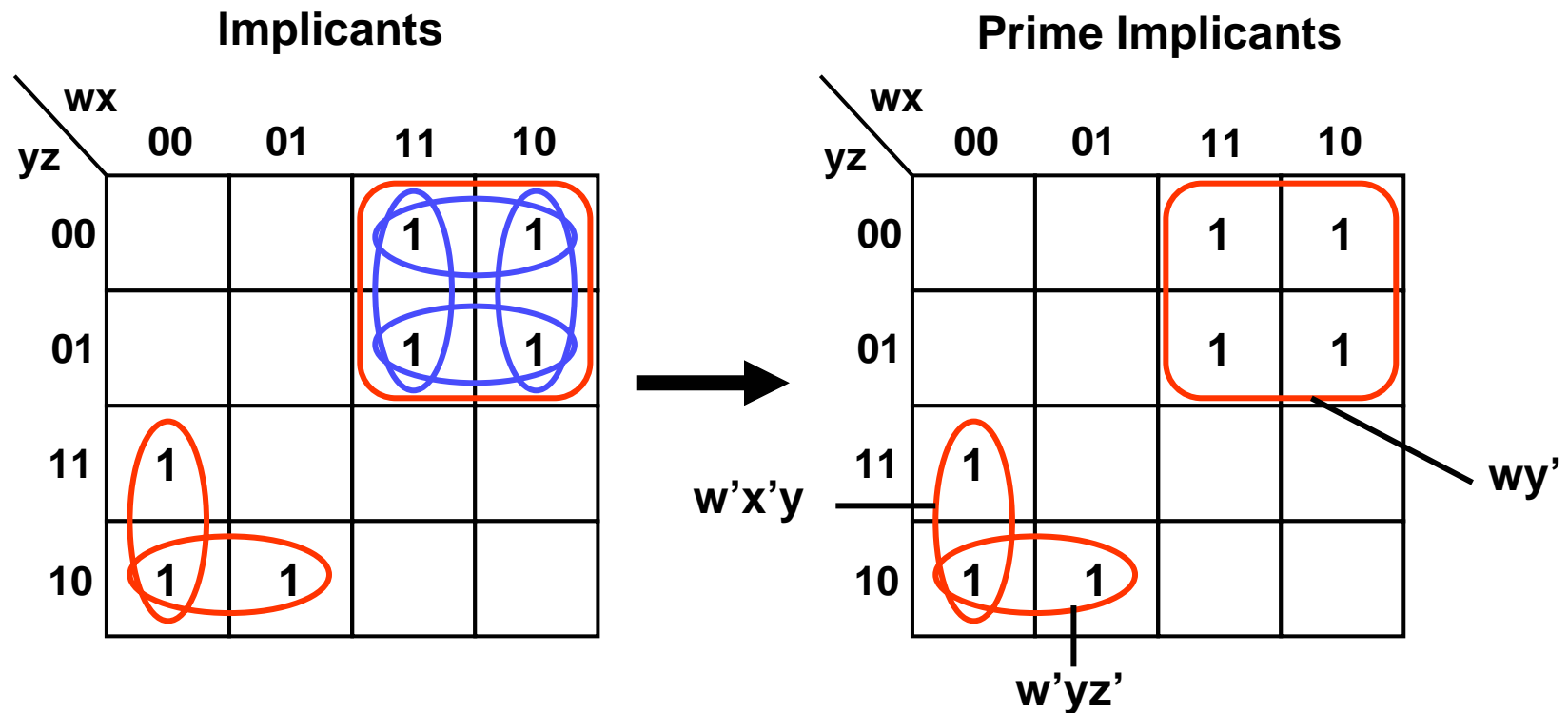
*Implicant* – any single 1 or any group of 1's which can be combined together on a map of the function F



*List of Implicants* –  $wxy'$ ,  $wx'y'$ ,  $wy'z'$ ,  $wy'z$ ,  $wy'$ ,  $w'x'y$ ,  $w'yz'$   
and all single 1's

## 5.4 Determination of Minimum Expressions

*Prime Implicant* – an implicant which can not be combined with another term to eliminate a variable.



List of Prime Implicants:  $w'x'y$ ,  $w'yz'$ ,  $wy'$



## 5.4 Determination of Minimum Expressions


Find the prime implicants:

cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1

## 5.4 Determination of Minimum Expressions

Find the prime implicants:

cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1



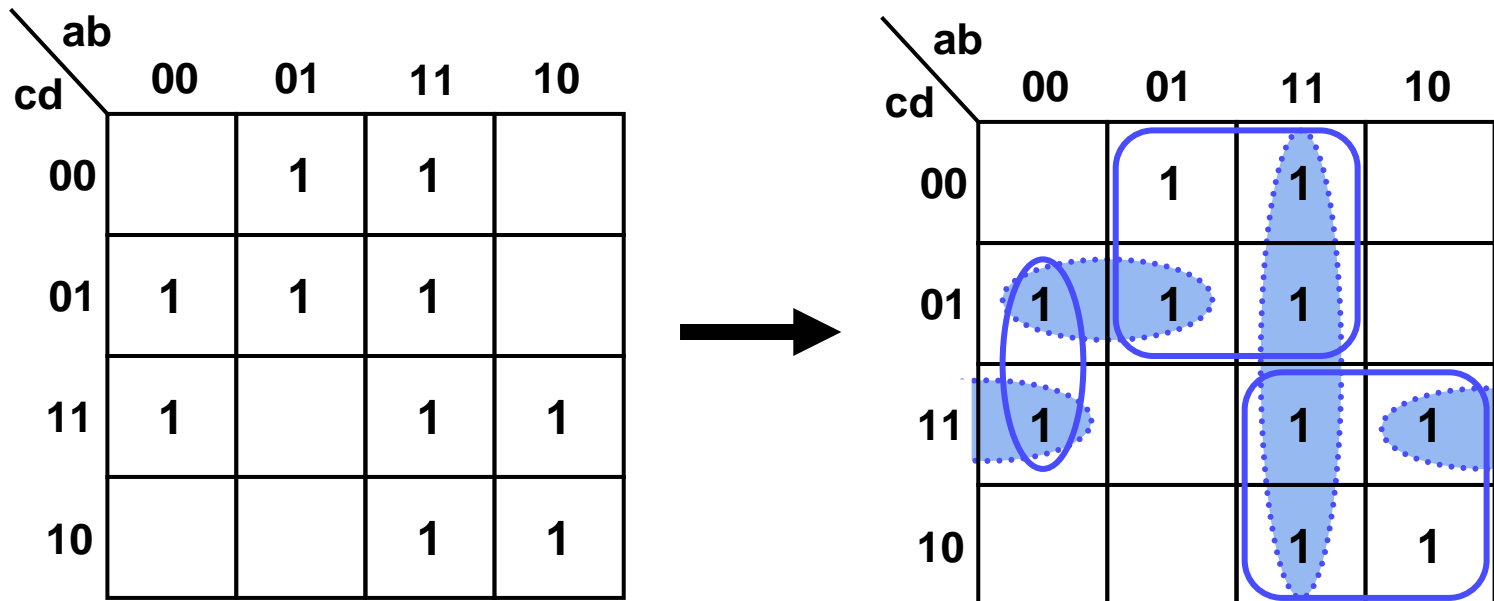
cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1

The second Karnaugh map shows the same data as the first, but with blue circles and lines highlighting the prime implicants. The circles group the 1s in the following ways: a vertical circle around the 1s in the 11 column (covering cells (00,11), (01,11), (11,11), (10,11)); a horizontal circle around the 1s in the 01 row (covering cells (01,01), (11,01)); a horizontal circle around the 1s in the 11 row (covering cells (01,11), (11,11)); a horizontal circle around the 1s in the 10 row (covering cells (11,10), (10,10)); a vertical circle around the 1s in the 00 column (covering cells (01,00), (11,00)); a vertical circle around the 1s in the 10 column (covering cells (11,10), (10,10)); a horizontal circle around the 1s in the 00 row (covering cells (01,00), (11,00)); a horizontal circle around the 1s in the 01 row (covering cells (00,01), (01,01)); a horizontal circle around the 1s in the 11 row (covering cells (00,11), (01,11)); and a horizontal circle around the 1s in the 10 row (covering cells (00,10), (01,10)).

All prime implicants:  $a'b'd$ ,  $bc'$ ,  $ac$ ,  $a'c'd$ ,  $ab$ ,  $b'cd$

## 5.4 Determination of Minimum Expressions

Minimum Solution might not utilize all prime implicants



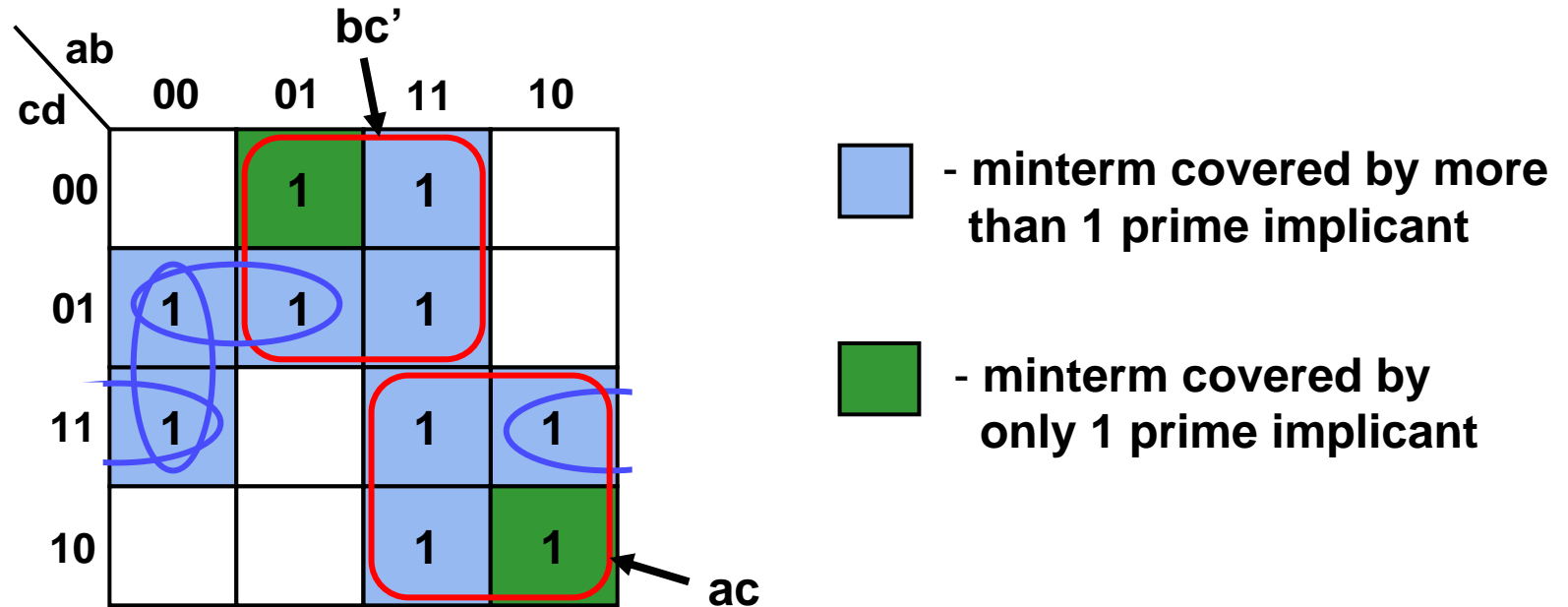
Minimum solution:  $F = a'b'd + bc' + ac$

All prime implicants:  $a'b'd, bc', ac, a'c'd, ab, b'cd$

## 5.4 Determination of Minimum Expressions

### *Essential Prime Implicant* –

A prime implicant that contains a minterm that is covered by only one prime implicant

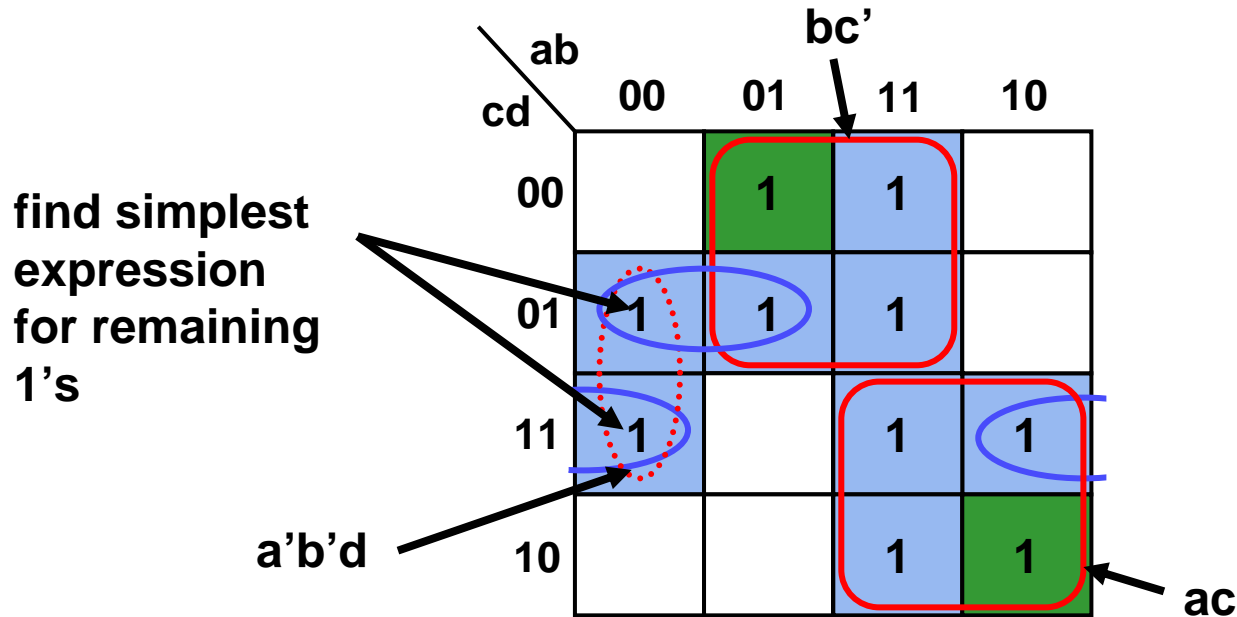


List of Essential Prime Implicants:  $bc'$ ,  $ac$

## 5.4 Determination of Minimum Expressions

To find minimum expression:

- Find all Prime Implicants
- Determine Essential Prime Implicants
- Find Simplest Expression for remaining uncovered 1's



$$F = a'b'd + bc' + ac$$

## 5.4 Determination of Minimum Expressions

*Find Minimum Sum-of-Products Expression*

		AB			
		00	01	11	10
CD	00	1	1		
	01	1	1		
	11		1	1	1
	10	1			

## 5.4 Determination of Minimum Expressions

First find all Prime Implicants

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11		1	1	1
10	1			



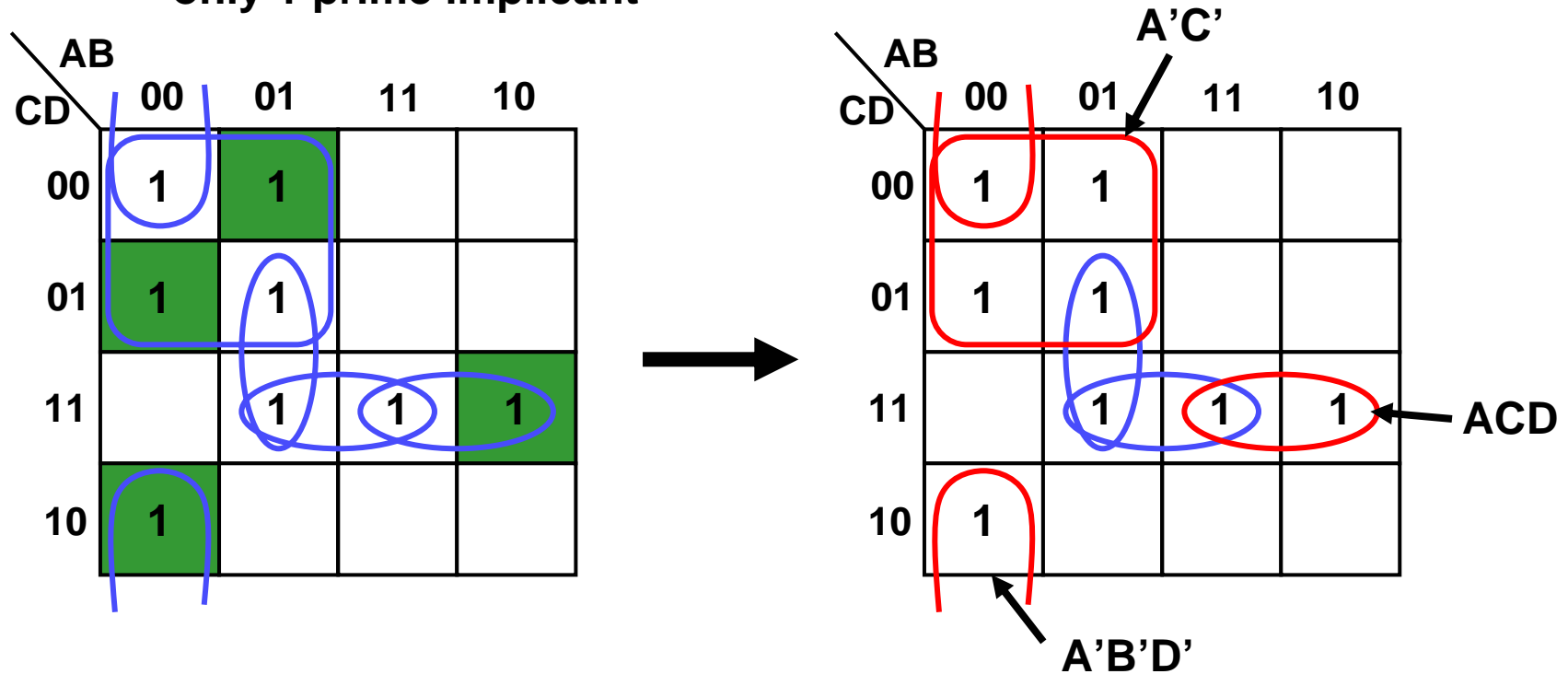
AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11		1	1	1
10	1			

The right Karnaugh map shows the same data as the left one, but with blue circles highlighting the prime implicants. There are four prime implicants circled: a vertical circle around the first column (CD=00), a vertical circle around the second column (CD=01), a horizontal circle around the first two rows (AB=00 and AB=01), and a horizontal circle around the last three cells of the third row (CD=11).

# 5.4 Determination of Minimum Expressions

Next find all essential Prime Implicants

 - minterm covered by only 1 prime implicant



List of Essential Prime Implicants:  $A'C'$ ,  $ACD$ ,  $A'B'D'$

Minimum Solution:  $F = A'C' + ACD + A'B'D' + \left\{ \begin{array}{l} A'BD \\ \text{or} \\ BCD \end{array} \right\}$



	0	1
00	1	1
01	1	1
11		
10		

	0	1
00		
01		
11		
10		

	0	1
00		
01	1	
11	1	1
10		1

	0	1
00		
01		
11		
10		

	0	1
00	1	
01		1
11		1
10	1	

	0	1
00		
01		
11		
10		

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

AB

CD

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$F = B\bar{C}$$

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

AB

CD

	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$F = \bar{A}B + \bar{B}D$$

	00	01	11	10
00				
01				
11				
10				

	00	01	11	10
00				
01				
11				
10				

**AB** **CD**

	00	01	11	10
00		1		
01		1	1	
11			1	
10				

	00	01	11	10
00				
01				
11				
10				

$$F = \bar{c}d\bar{a} + c\bar{a}b + \bar{a}bd$$

	00	01	11	10
00				
01				
11				
10				

	00	01	11	10
00				
01				
11				
10				

1) Minimum sum of products

$$f(a,b,c,d) = b'c'd' + bcd + acd' + a'b'c + a'bc'd$$

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>f</i>
0	0	0	0	0	1
	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
	0	1	0	0	0
5	0	1	0	1	1
	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
	1	0	0	1	0
10	1	0	1	0	1
	1	0	1	1	0
	1	1	0	0	0
	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

$$f(a,b,c,d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 14, 15)$$

$$f(a,b,c,d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 14, 15)$$

$cd \backslash ab$				
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

The image shows a 4x4 Karnaugh map for a 4-variable function f(a,b,c,d). The rows are labeled with cd (00, 01, 11, 10) and the columns are labeled with ab (0, 4, 12, 8). The cells contain the minterm numbers. Red checkmarks are placed in the cells corresponding to the minterms listed in the function definition: 0, 2, 3, 5, 7, 8, 10, 14, and 15.

$ab \backslash cd$				
00	1			1
01		1		
11	1	1	1	
10	1		1	1

$ab \backslash cd$				
00	1			1
01		1		
11	1	1	1	
10	1		1	1

$$F = \bar{b}\bar{d} + \bar{a}bd + abc + \bar{a}cd$$

$ab \backslash cd$				
00	1			1
01		1		
11	1	1	1	
10	1		1	1

## 2) Minimum product of sums

Plotting Karnaugh map for  $f'$ :

$$f(a,b,c,d) = \sum m(0, 2, 3, 5, 7, 8, 10, 14, 15)$$

$$f'(a,b,c,d) = \sum m(1, 4, 6, 9, 11, 12, 13)$$

$cd \backslash ab$	00	01	11	10
00	0 0	4 1	12 1	8 0
01	1 1	5 0	13 1	9 1
11	3 0	7 0	15 0	11 1
10	2 0	6 1	14 0	10 0

$ab \backslash cd$	00	01	11	10
00	0	1 <sup>4</sup>	1 <sup>12</sup>	8
01	1 <sup>1</sup>	5	1 <sup>13</sup>	1 <sup>9</sup>
11	3	7	15	1 <sup>11</sup>
10	2	1 <sup>6</sup>	14	10

$AB'$

$A\bar{B}$  ←

$\bar{A}\bar{B}$  —

$(AB)'$  ,

$ab \backslash cd$	00	01	11	10
00	0	1 <sup>4</sup>	1 <sup>12</sup>	8
01	1 <sup>1</sup>	5	1 <sup>13</sup>	1 <sup>9</sup>
11	3	7	15	1 <sup>11</sup>
10	2	1 <sup>6</sup>	14	10

$$f' = \bar{a}b\bar{c} + \bar{b}c\bar{d} + a\bar{b}d + abc$$

$$f = (a + \bar{b} + \bar{c})(b + c + \bar{d})(\bar{a} + b + \bar{c})(\bar{a} + \bar{b} + c)$$