Unit 5: Karnaugh Maps

EEC180A

5.1 Minimum Forms of Switching Functions

Find a minimum sum-of products expression for:

$$F(a,b,c)=\sum m(0,1,2,5,6,7)$$

Note: Use XY' + XY = X

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

F = a'b' + b'c + bc' + ab

None of these terms can be eliminated

However, if we combine in a different way.

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

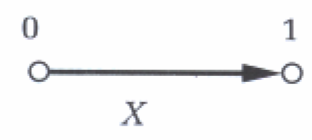
F = a'b' + bc' + ac

5.2 Karnaugh Maps

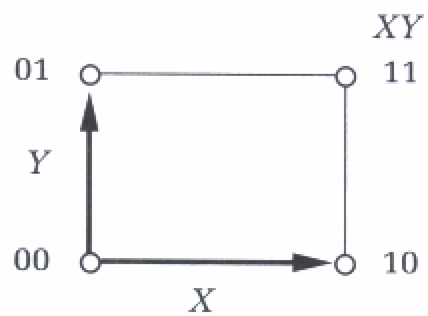
We can represent a 1 and 2-input truth table as 1-D and 2-D cube

X	F
0	
1	

XY	F
00	
01	
10	
11	



1-cube



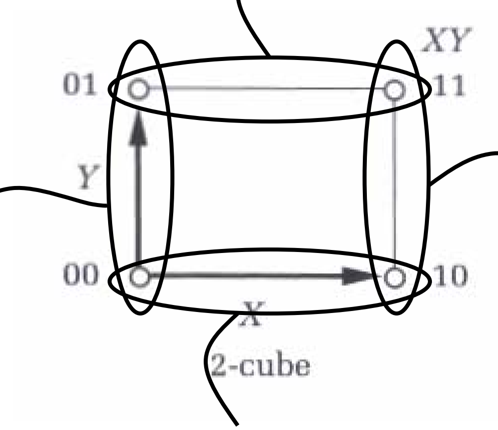
2-cube

5.2 Karnaugh Maps

Allows for easy application of XY + XY' = X

X'Y+XY = Y

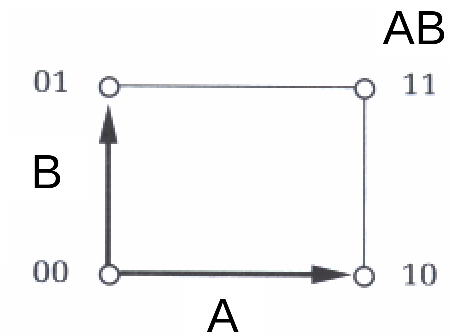
X'Y + X'Y' = X'

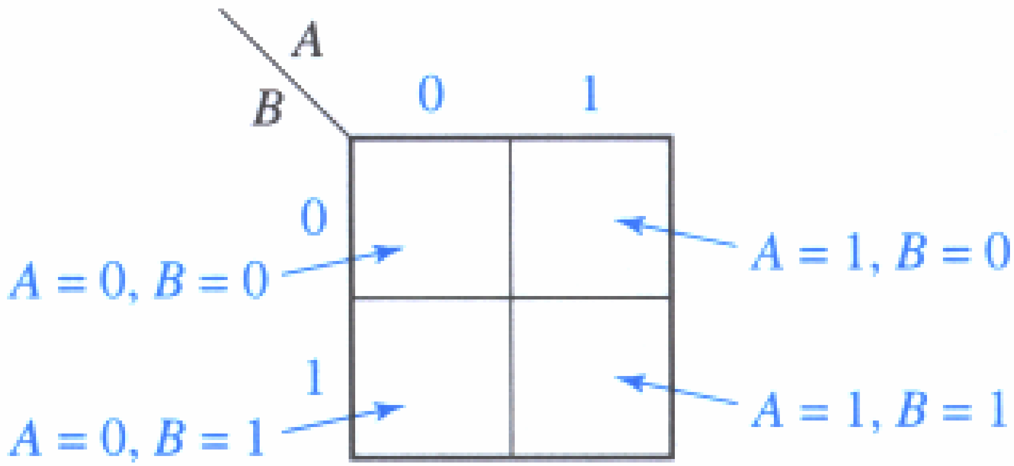


XY + XY' = X

X'Y'+XY' = Y'

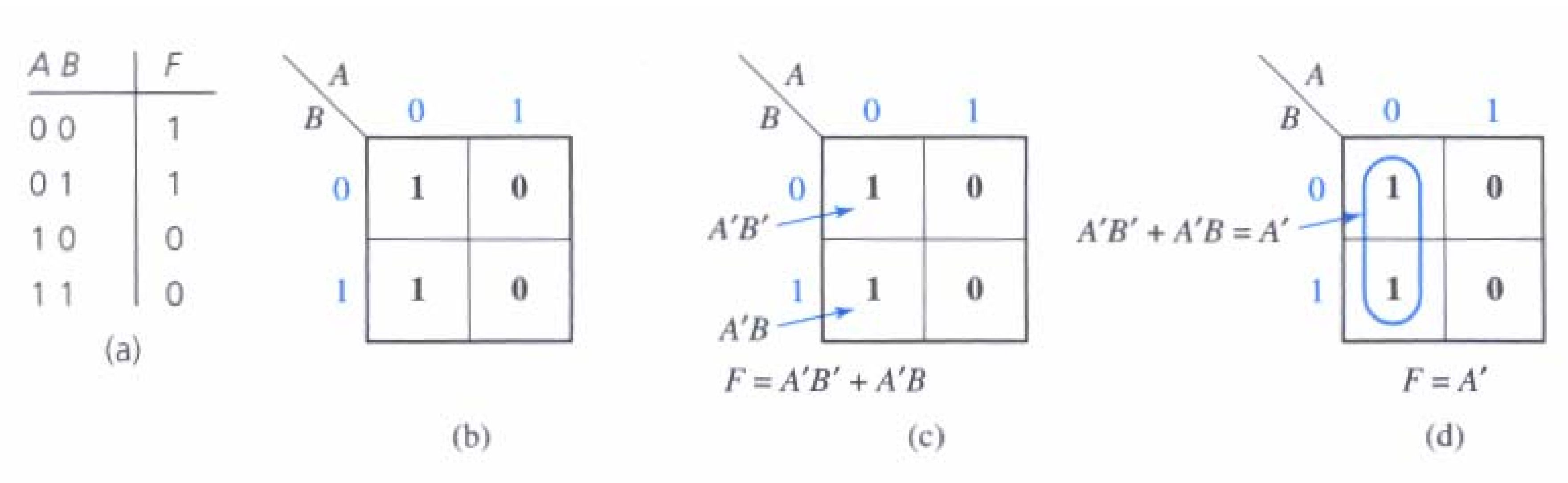
5.2 Two-Variable Karnaugh Maps





5.2 Two-Variable Karnaugh Maps

Two Variable Karnaugh Map Example:

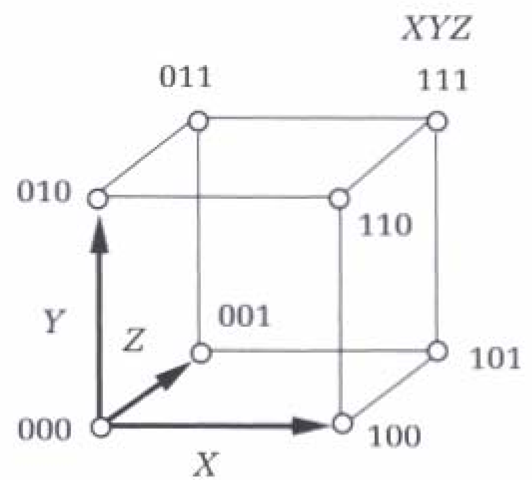


- **Minterms in adjacent squares on the map can be combined since they differ in only one variable (i.e. XY' + XY = X)**

5.2 Three-Variable Karnaugh Maps

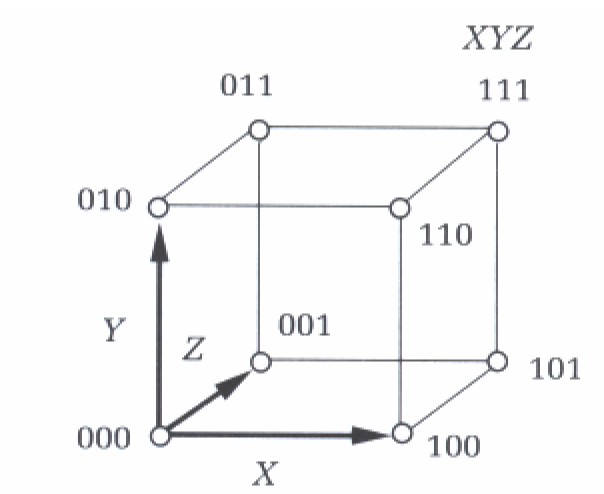
We can represent a 3-input truth table as a 3-D cube

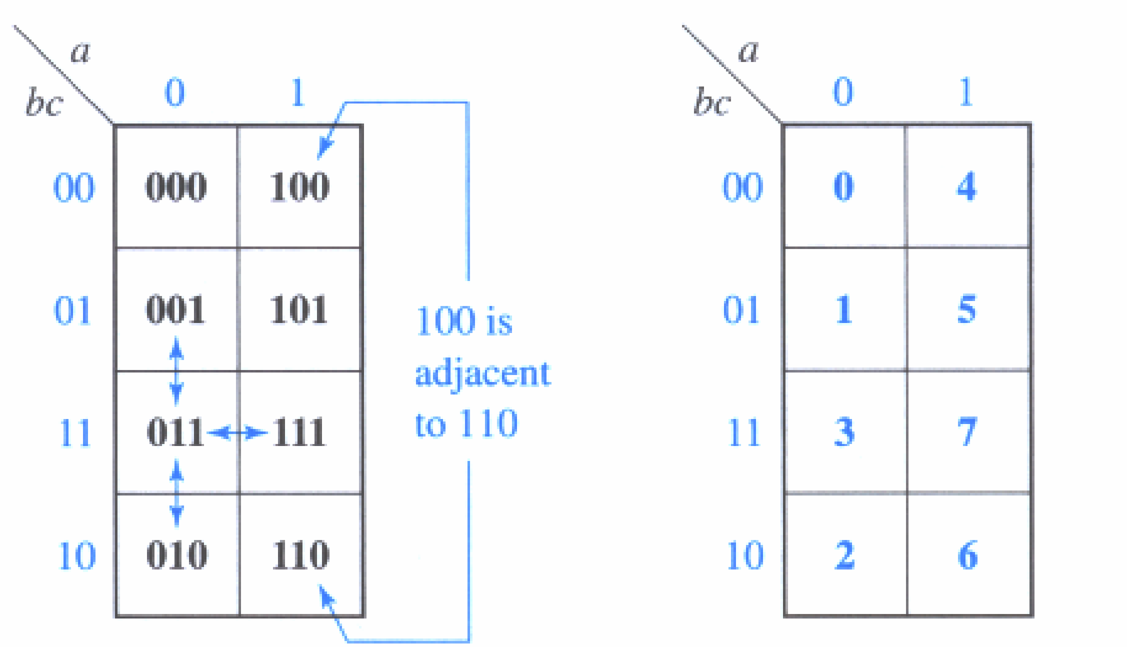
X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



5.2 Three-Variable Karnaugh Maps

Location of Minterms on a Three Variable Karnaugh Map





(a) Binary notation

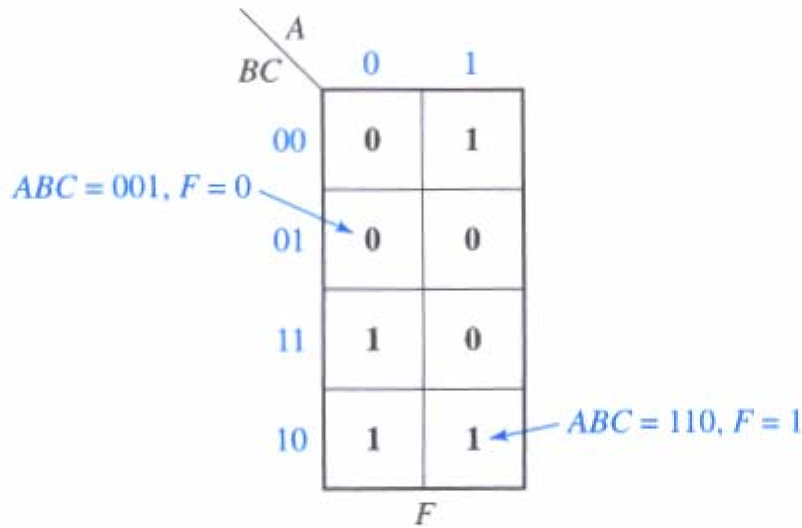
(b) Decimal notation

5.2 Three-Variable Karnaugh Maps

Truth Table and resulting Karnaugh Map for Three-Variable Function

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

(a)

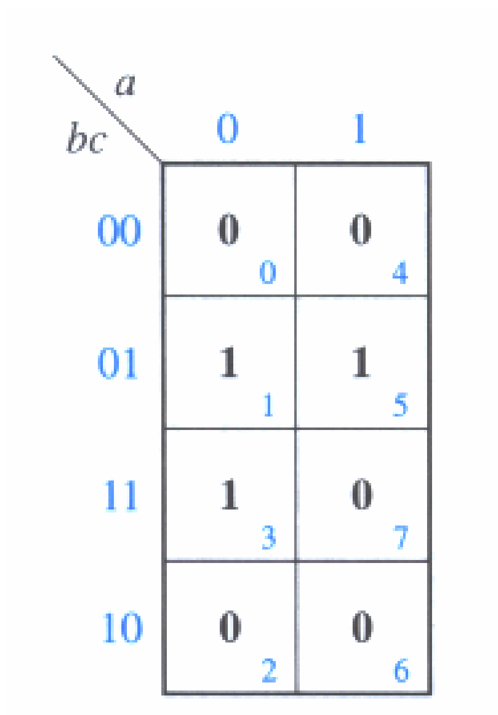


(b)

5.2 Three-Variable Karnaugh Maps

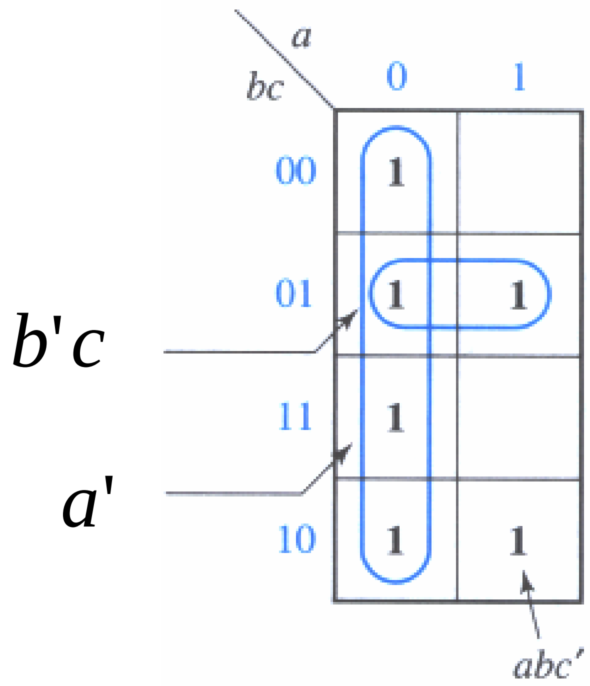
Location of Minterms on a Three Variable Karnaugh Map

$$F(a,b,c) = \sum m(1,3,5) = \prod (0,2,4,6,7)$$



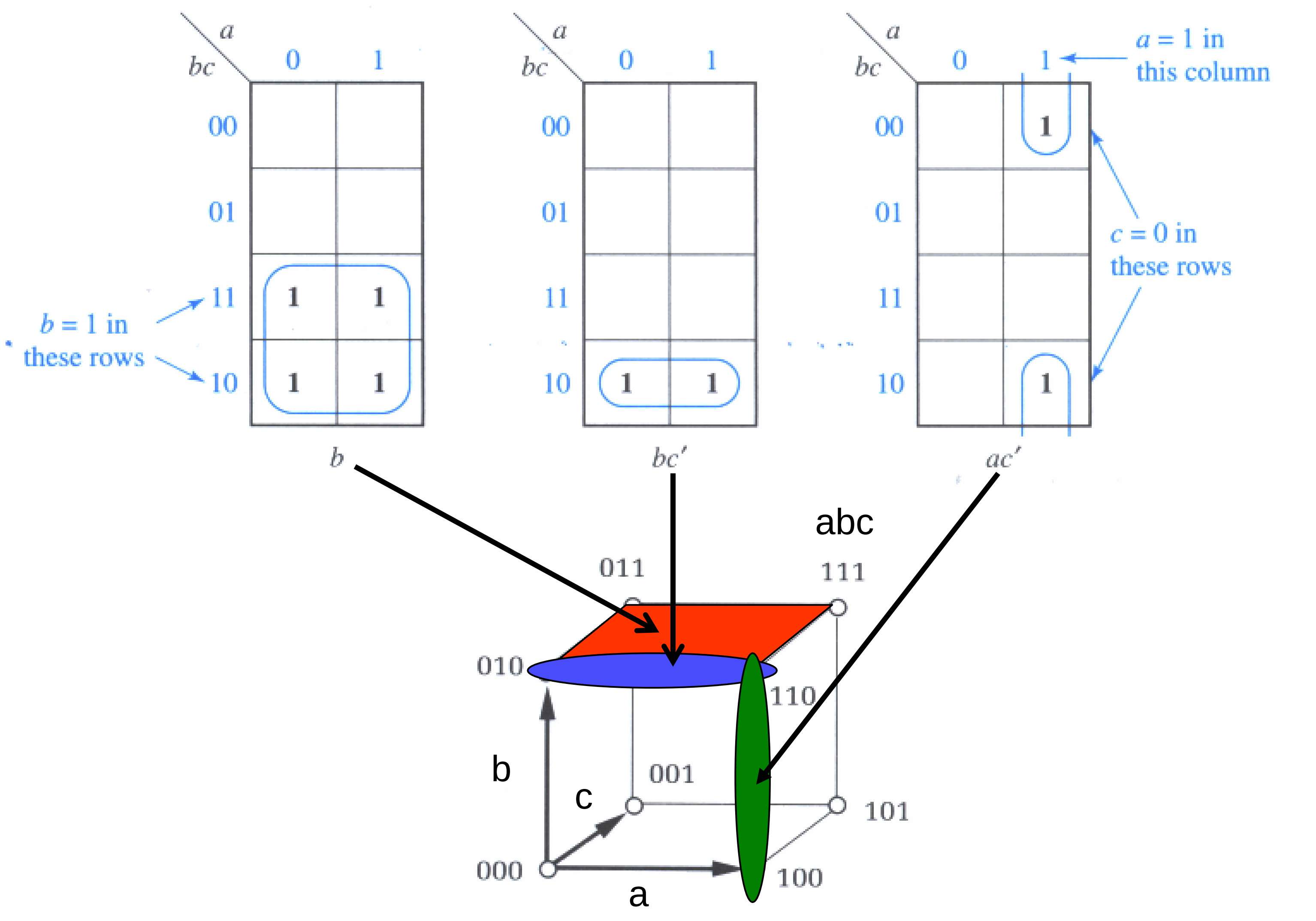
5.2 Three-Variable Karnaugh Maps

Karnaugh Map for $F = abc' + b'c + a'$



5.2 Three-Variable Karnaugh Maps

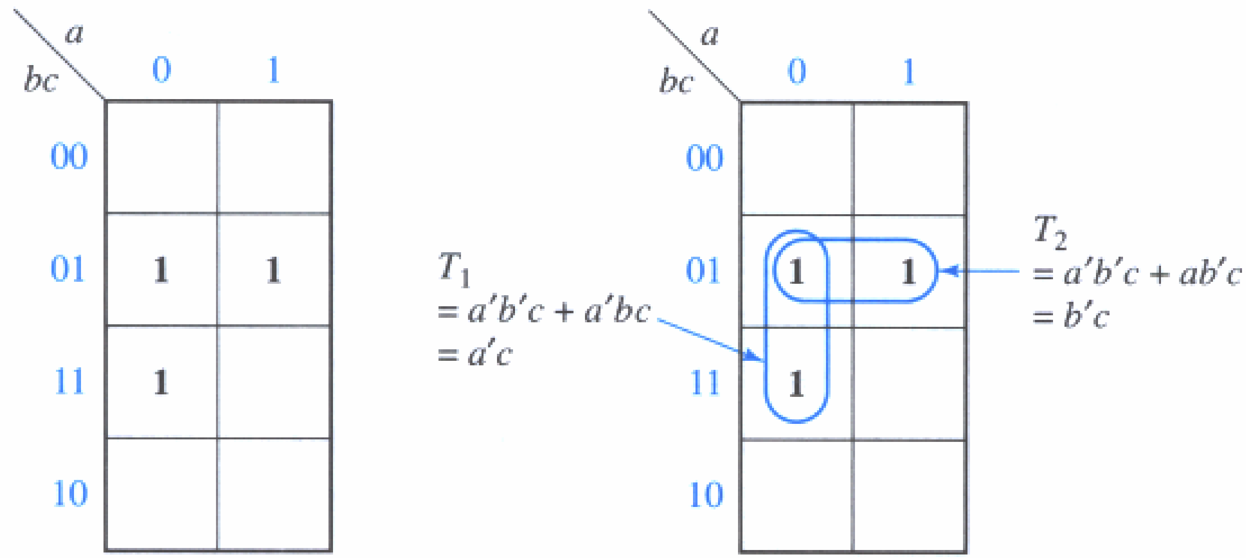
Karnaugh Maps for Product Terms



5.2 Three-Variable Karnaugh Maps

Simplification of a Three-Variable Function

$$F = \sum m(1,3,5)$$



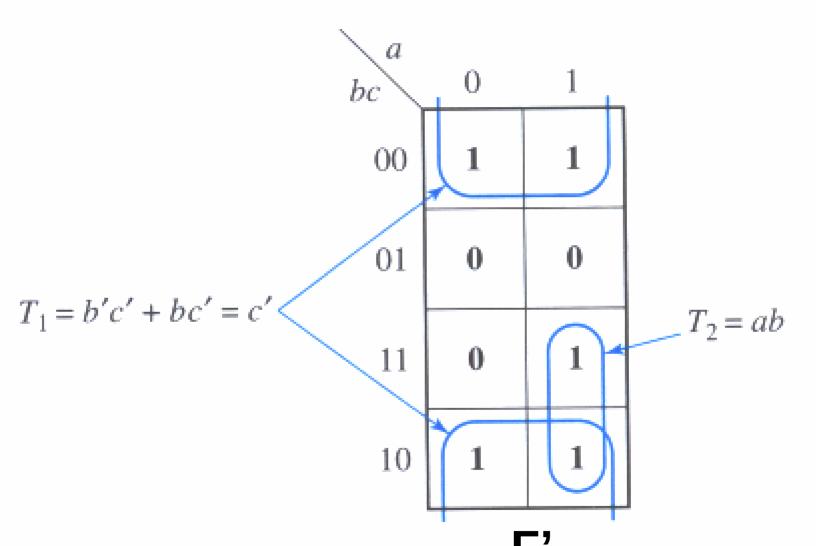
$$F = a'c + b'c$$

5.2 Three-Variable Karnaugh Maps

Simplification of F’

$$F = \sum m(1,3,5)$$

$$F' = \sum m(0,2,4,6,7)$$

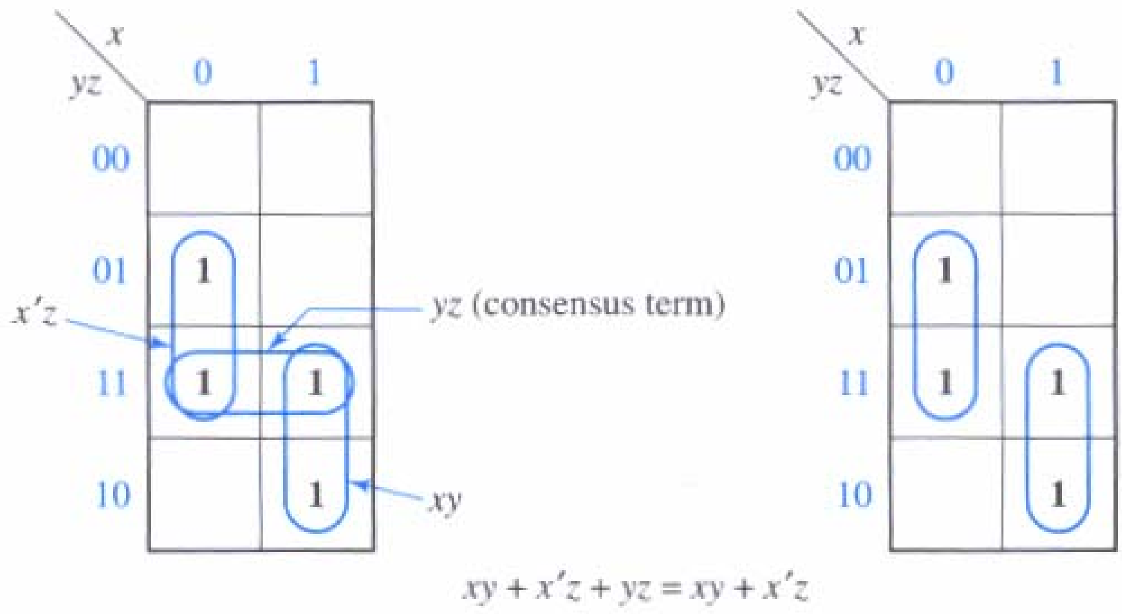


F’

$$F' = c' + ab$$

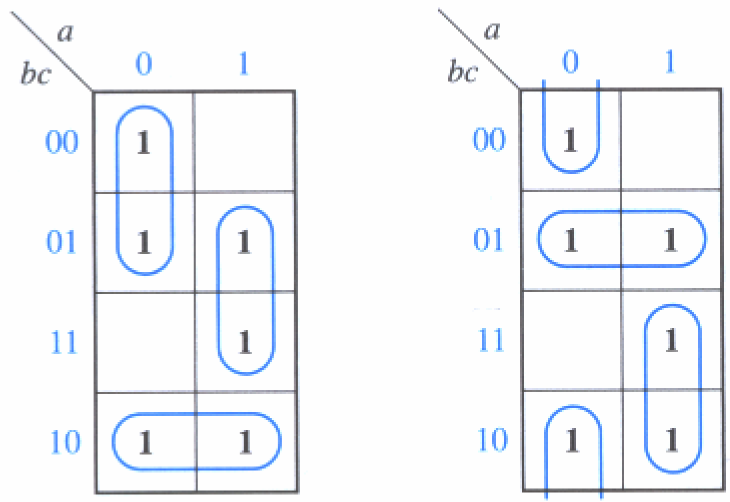
5.2 Three-Variable Karnaugh Maps

Karnaugh Maps which illustrate the Consensus Theorem



5.2 Three-Variable Karnaugh Maps

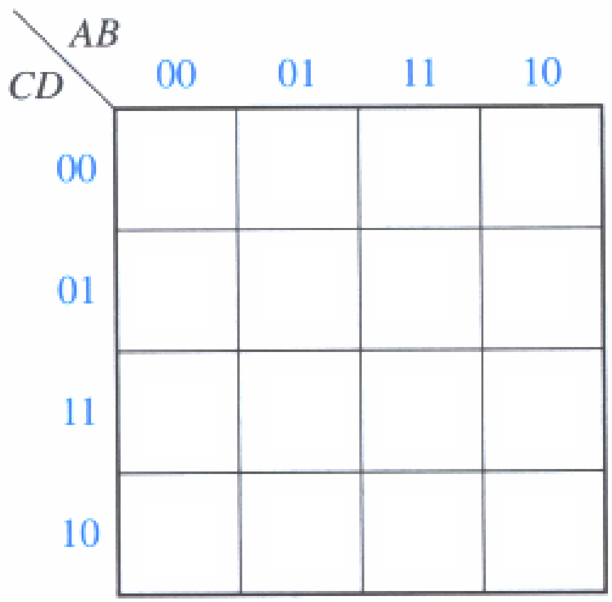
Function with Two Minimum Forms



$F = a'b' + bc' + ac$ $\qquad$ $F = a'c' + b'c + ab$

5.3 Four-Variable Karnaugh Maps

Adjacent squares should differ by only one variable



5.3 Four-Variable Karnaugh Maps

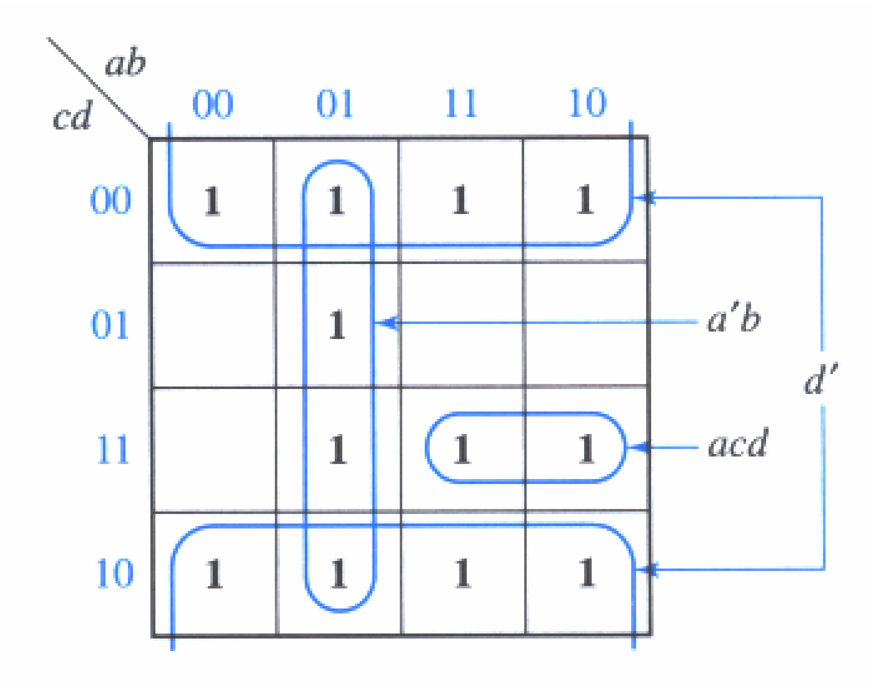
Location of Minterms on a Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	**0**	**4**	**12**	**8**
01	**1**	**5**	**13**	**9**
11	**3**	**7**	**15**	**11**
10	**2**	**6**	**14**	**10**

5.3 Four-Variable Karnaugh Maps

Sample 4-variable Karnaugh Map

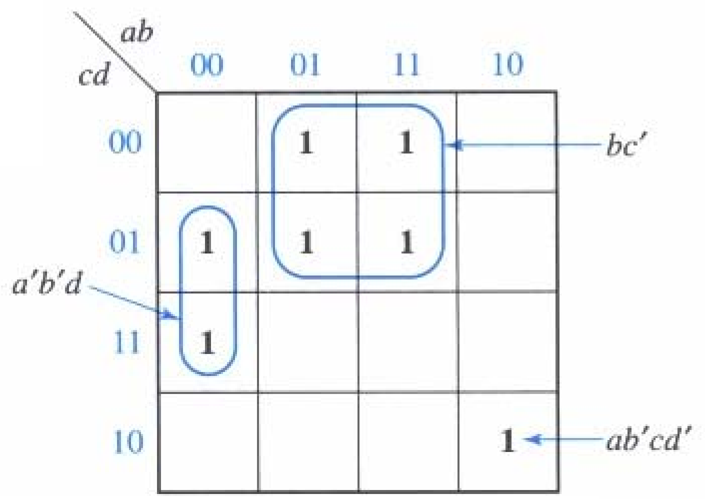
$$F = acd + a'b + d'$$



5.3 Four-Variable Karnaugh Maps

Simplification of Four-Variable Functions

$$F = \sum m(1,3,4,5,10,12,13)$$

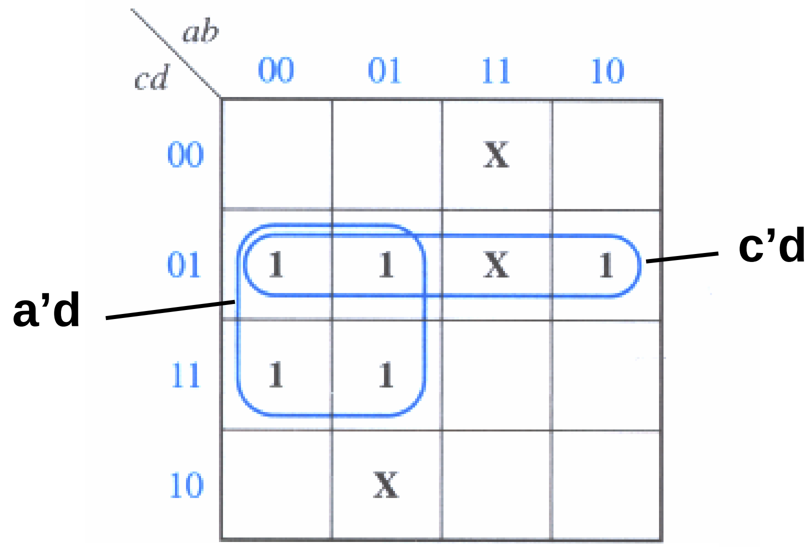


$$F = bc' + a'b'd + ab'cd'$$

5.3 Four-Variable Karnaugh Maps

Simplification of Incompletely Specified Function

$$F = \sum m(1,3,5,7,9) + \sum d(6,12,13)$$



$$F = a'd + c'd$$

5.3 Four-Variable Karnaugh Maps

Finding Minimum Product of Sums from Karnaugh Maps

$$F = x'z' + wyz + w'y'z' + x'y$$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

F

wxz'

yz \ wx	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	1	0	0
10	0	1	1	0

y'z

w'xy

F'

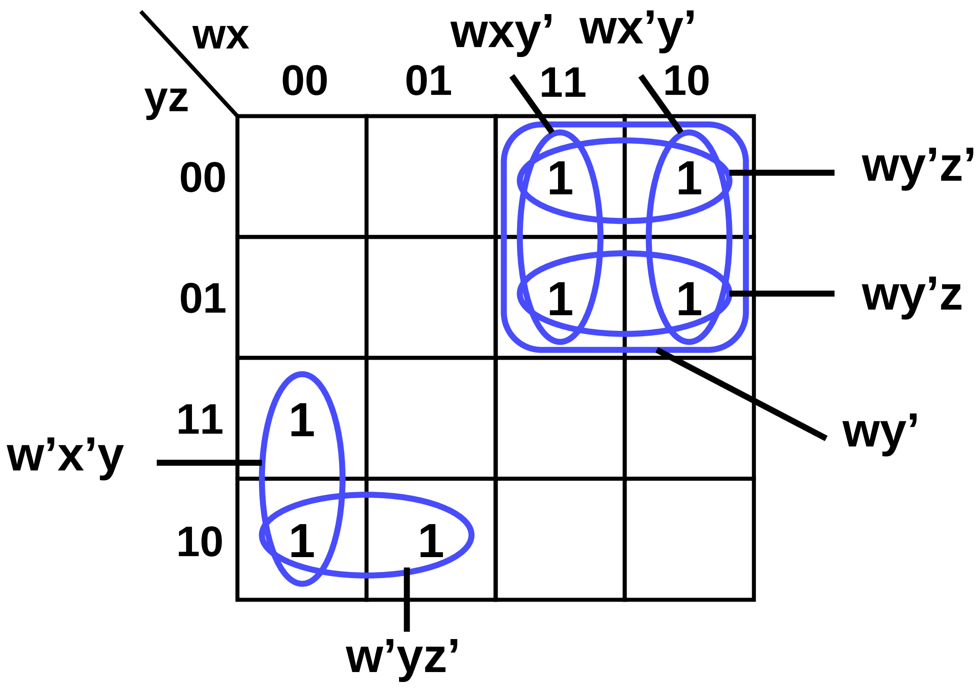
$$F' = y'z + wxz' + w'xy$$

Using DeMorgan's

$$F = (y + z')(w' + x' + z)(w + x' + y')$$

5.4 Determination of Minimum Expressions

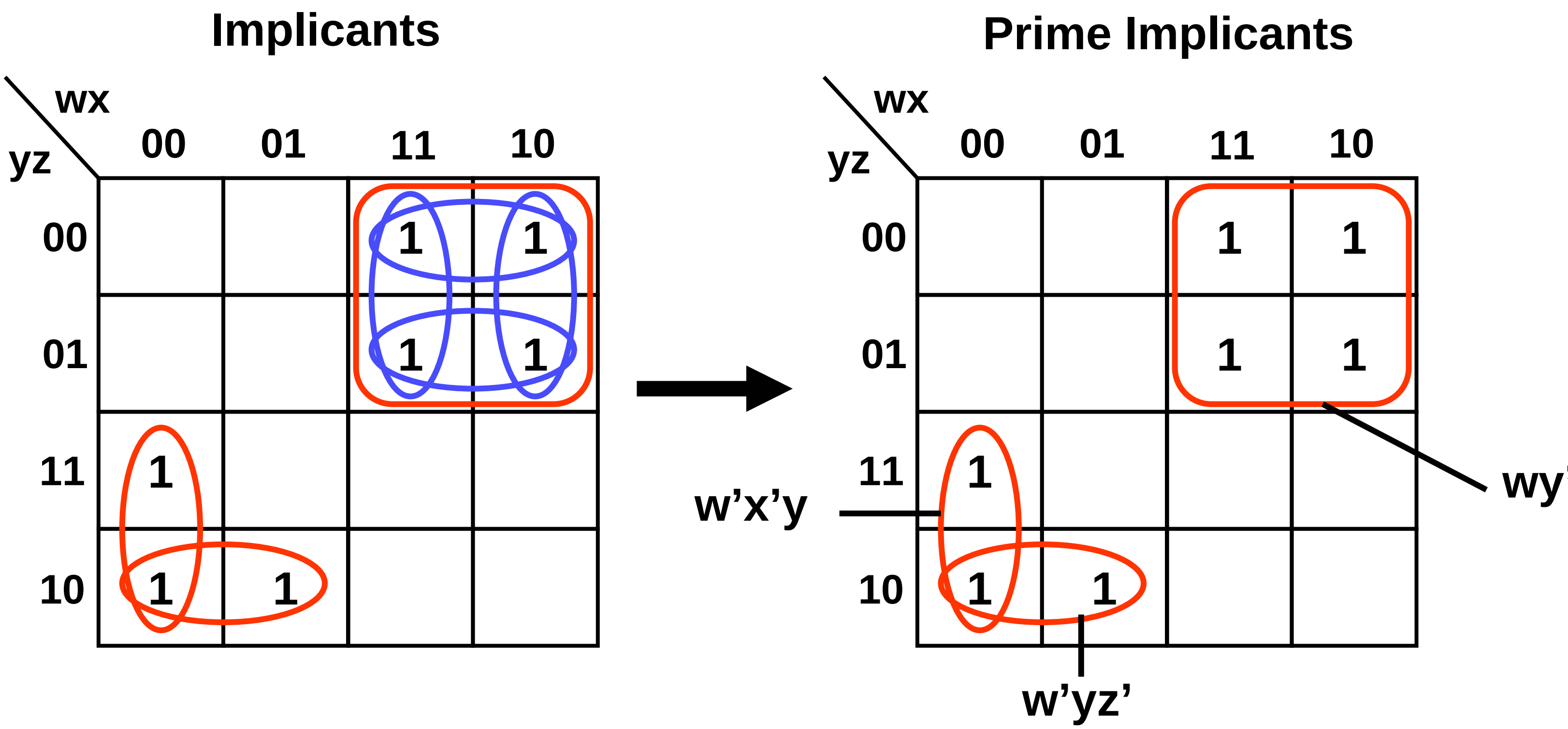
Implicant – any single 1 or any group of 1's which can be combined together on a map of the function F



List of Implicants – wxy', wx'y', wy'z', wy'z, wy', w'x'y, w'yz' and all single 1's

5.4 Determination of Minimum Expressions

Prime Implicant – an implicant which can not be combined with another term to eliminate a variable.



List of Prime Implicants: w'x'y, w'yz', wy'

5.4 Determination of Minimum Expressions

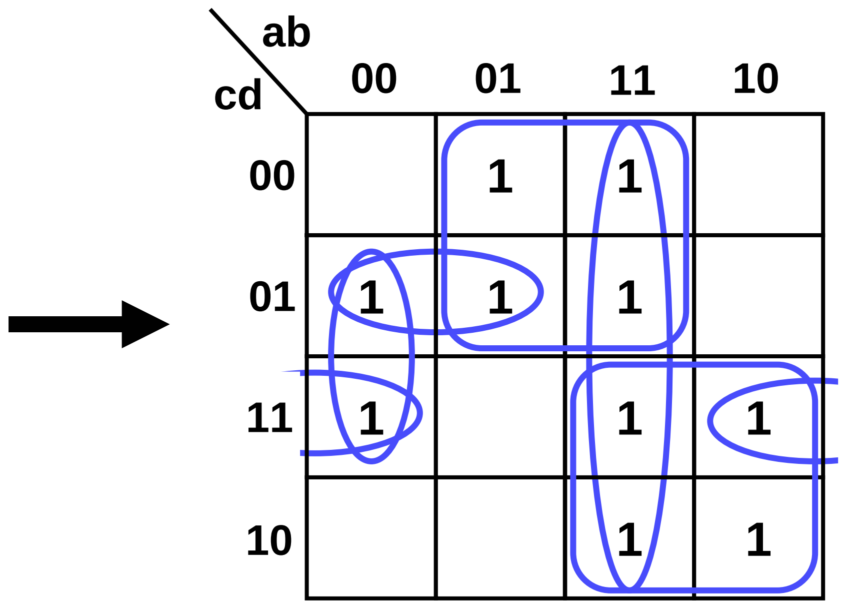
Find the prime implicants:

cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1

5.4 Determination of Minimum Expressions

Find the prime implicants:

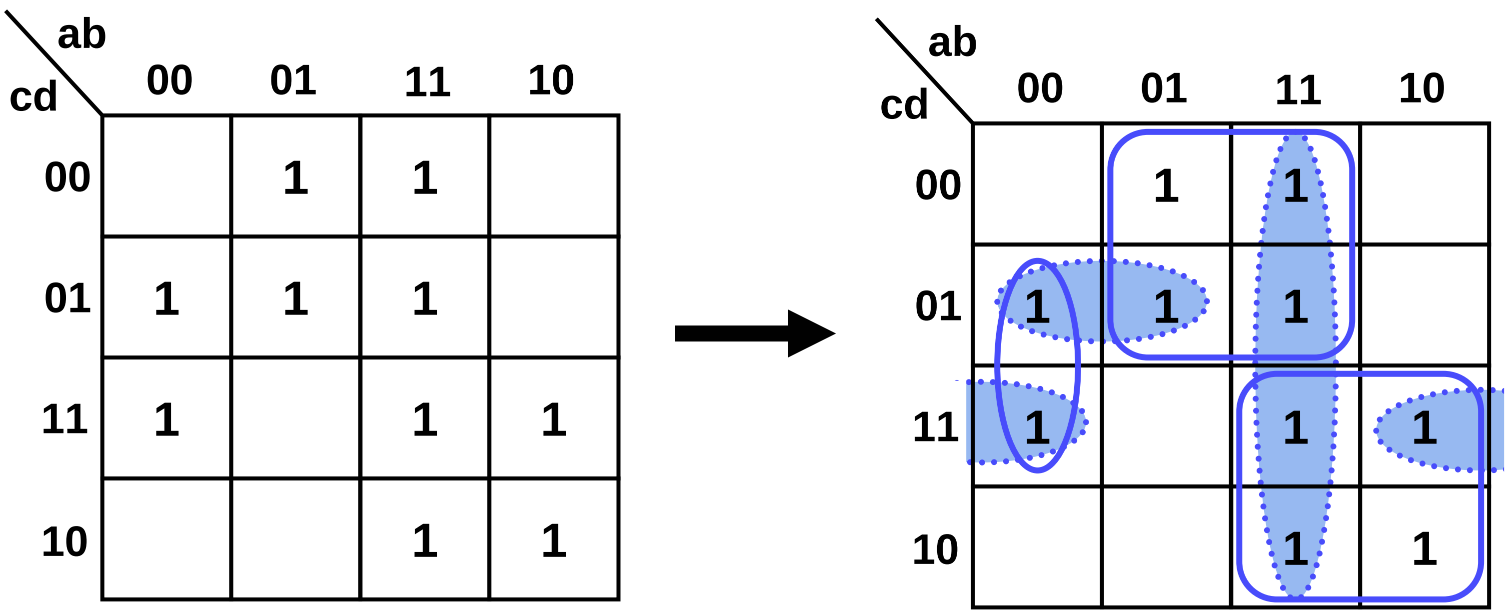
cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1



All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

5.4 Determination of Minimum Expressions

Minimum Solution might not utilize all prime implicants



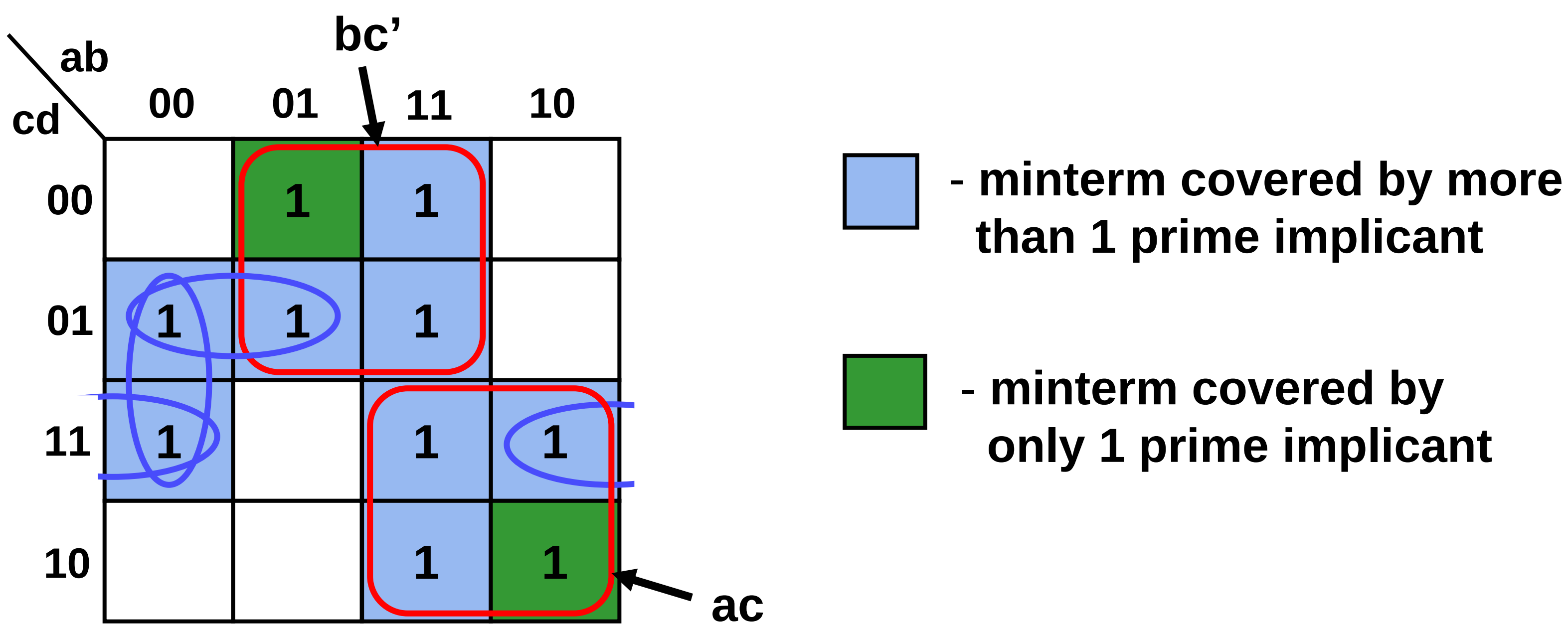
Minimum solution: $F = a'b'd + bc' + ac$

All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

5.4 Determination of Minimum Expressions

Essential Prime Implicant –

A prime implicant that contains a minterm that is covered by only one prime implicant

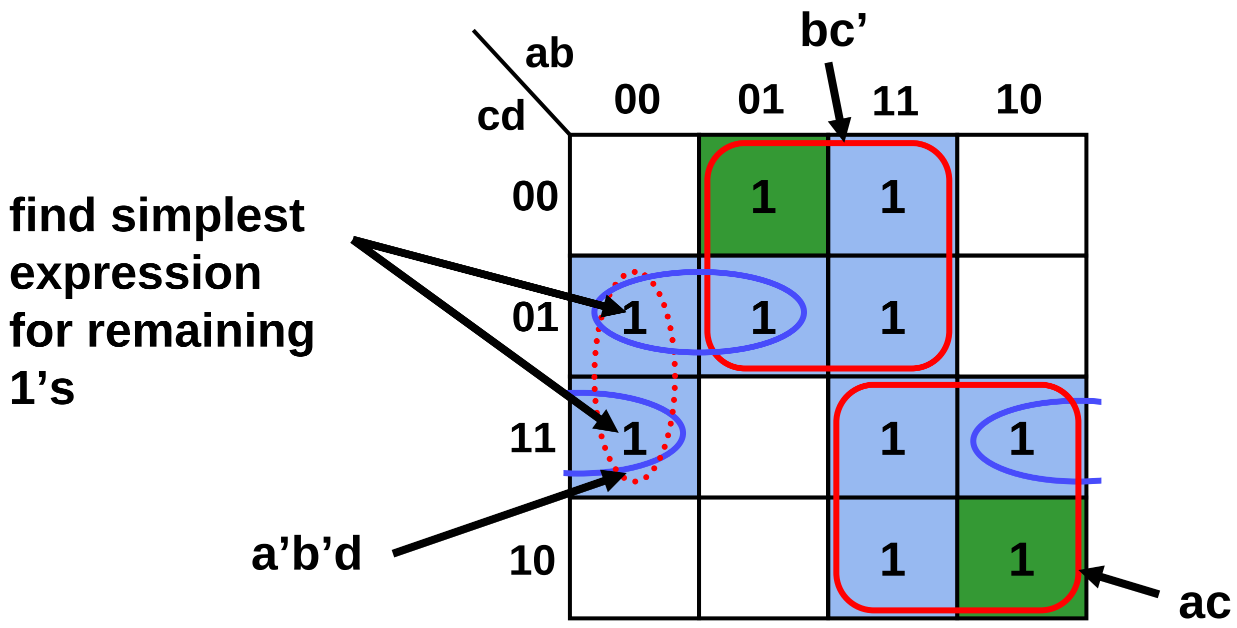


List of Essential Prime Implicants: bc', ac

5.4 Determination of Minimum Expressions

To find minimum expression:

- *Find all Prime Implicants*
- *Determine Essential Prime Implicants*
- *Find Simplest Expression for remaining uncovered 1's*



F = a'b'd + bc' + ac

5.4 Determination of Minimum Expressions

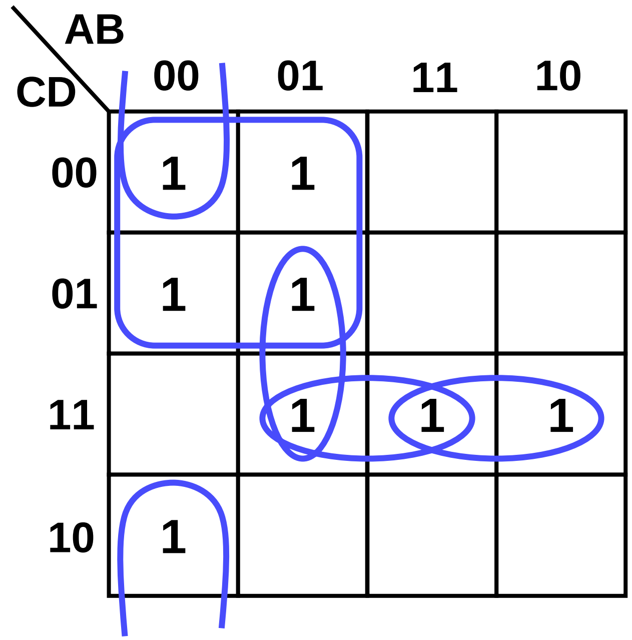
Find Minimum Sum-of-Products Expression

CD \ AB	00	01	11	10
00	1	1		
01	1	1		
11		1	1	1
10	1			

5.4 Determination of Minimum Expressions

First find all Prime Implicants

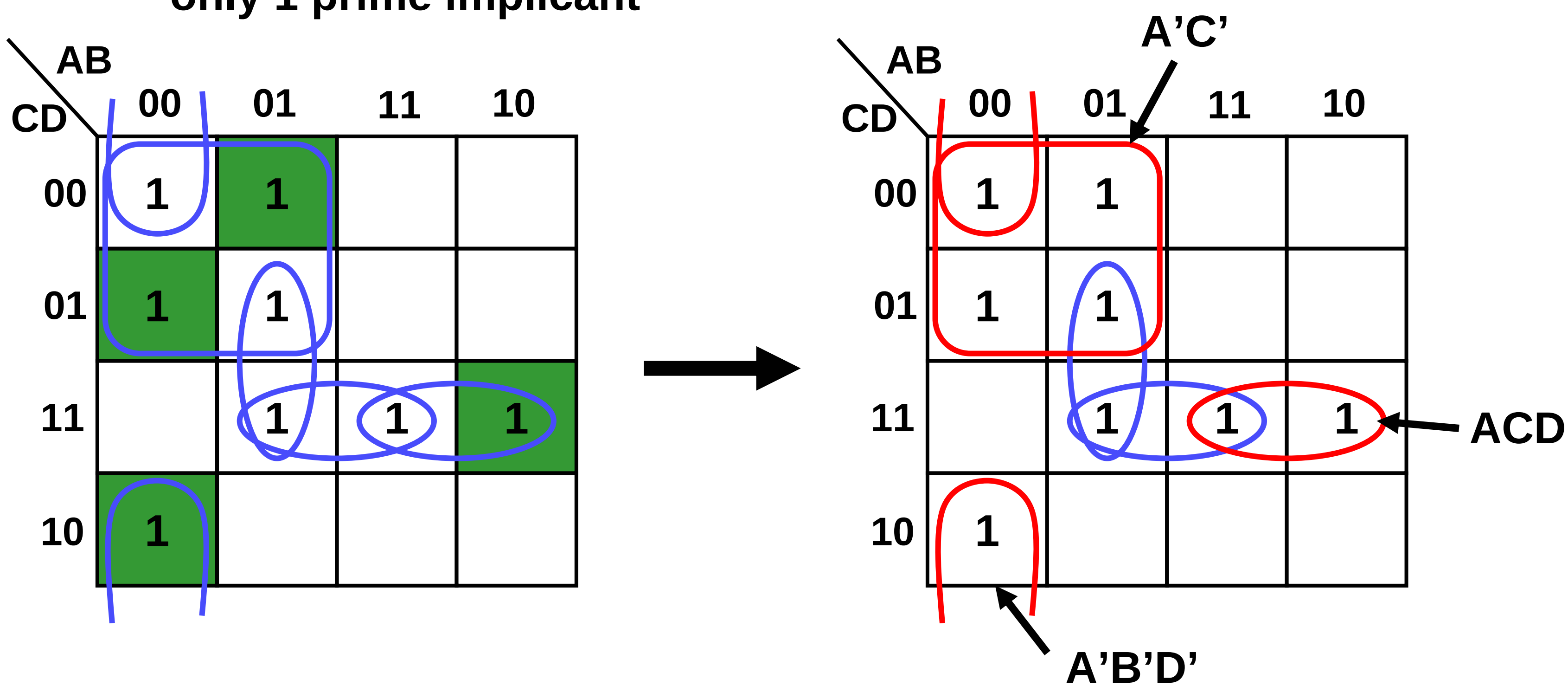
CD \ AB	00	01	11	10
00	1	1		
01	1	1		
11		1	1	1
10	1			



5.4 Determination of Minimum Expressions

Next find all essential Prime Implicants

- mintern covered by only 1 prime implicant



List of Essential Prime Implicants: A'C', ACD, A'B'D'

Minimum Solution: F = A'C' + ACD + A'B'D' + $\left\{ \begin{array}{c} \textbf{A'BD} \\ \textbf{or} \\ \textbf{BCD} \end{array} \right\}$

	0	1
00	1	1
01	1	1
11		
10		

	0	1
00		
01		
11		
10		

	0	1
00		
01	1	
11	1	1
10		1

	0	1
00		
01		
11		
10		

	0	1
00	1	
01		1
11		1
10	1	

	0	1
00		
01		
11		
10		

	00	01	11	10
00	0 1	4	12	8 1
01	1	5	13	9
11	3	7	15	11
10	2 1	6	14	10 1

CD \ AB	00	01	11	10
00	0	4 1	12 1	8
01	1	5 1	13 1	9
11	3	7	15	11
10	2	6	14	10

$F = B\bar{C}$

	00	01	11	10
00	0	4 1	12 1	8
01	1	5	13	9
11	3	7	15	11
10	2	6 1	14 1	10

	00	01	11	10
00	0	4 1	12	8
01	1	5 1	13 1	9
11	3	7	15 1	11
10	2 1	6	14	10

	00	01	11	10
00	0	4 1	12 1	8
01	1	5 1	13 1	9
11	3	7	15 1	11 1
10	2	6	14 1	10 1

CD \ AB	00	01	11	10
00	0	4 1	12	8
01	1 1	5 1	13	9 1
11	3 1	7 1	15	11 1
10	2	6 1	14	10

$F = \bar{A}B + \bar{B}D$

	00	01	11	10
00				
01				
11				
10				

	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00		1		
01		1	1	
11			1	1
10				

$F = \bar{C}D\bar{A} + CAB + \bar{A}BD$

	00	01	11	10
00				
01				
11				
10				

	00	01	11	10
00				
01				
11				
10				

	00	01	11	10
00				
01				
11				
10				

1) Minimum sum of products

$$f(a,b,c,d) = b'c'd' + bcd + acd' + a'b'c + a'bc'd$$

	a	*b*	*c*	*d*	*f*
0	0	0	0	0	1
	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
	0	1	0	0	0
5	0	1	0	1	1
	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
	1	0	0	1	0
10	1	0	1	0	1
	1	0	1	1	0
	1	1	0	0	0
	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1

$$f(a,b,c,d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 14, 15)$$

$f(a,b,c,d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 14, 15)$

$\diagdown$ ab / cd				
00	0 1	4	12	8 1
01	1	5 1	13	9
11	3 1	7 1	15 1	11
10	2 1	6	14 1	10 1

cd \ ab				
00	1			1
01		1		
11	1	1	1	
10	1		1	1

cd \ ab	00	01	11	10
00	1			1
01		1		
11	1	1	1	
10	1		1	1

$F = \bar{b}\bar{d} + \bar{a}bd$
$+ abc + \bar{a}cd$

cd \ ab				
00	1			1
01		1		
11	1	1	1	
10	1		1	1

2) Minimum product of sums

Ploting Karnaugh map for f':

$$f(a,b,c,d) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 14, 15)$$

$$f'(a,b,c,d) = \Sigma m(1, 4, 6, 9, 11, 12, 13)$$

cd \ ab				
00	0 0	4 1	12 1	8 0
01	1 1	5 0	13 1	9 1
11	3 0	7 0	15 0	11 1
10	2 0	6 1	14 0	10 0

cd \ ab	00	01	11	10
00	[0]	1 [4]	1 [12]	[8]
01	1 [1]	[5]	1 [13]	1 [9]
11	[3]	[7]	[15]	1 [11]
10	[2]	1 [6]	[14]	[10]

AB'

$A\overline{B} \leftarrow$

$\overline{AB}$ —

$(AB)'$

cd \ ab	00	01	11	10
00	[0]	1 [4]	1 [12]	[8]
01	1 [1]	[5]	1 [13]	1 [9]
11	[3]	[7]	[15]	1 [11]
10	[2]	1 [6]	[14]	[10]

$$f' = \bar{a}b\bar{d} + \bar{b}\bar{c}d + a\bar{b}d + ab\bar{c}$$

$$f = (a+\bar{b}+d)(b+c+\bar{d})(\bar{a}+b+\bar{d})(\bar{a}+\bar{b}+c)$$