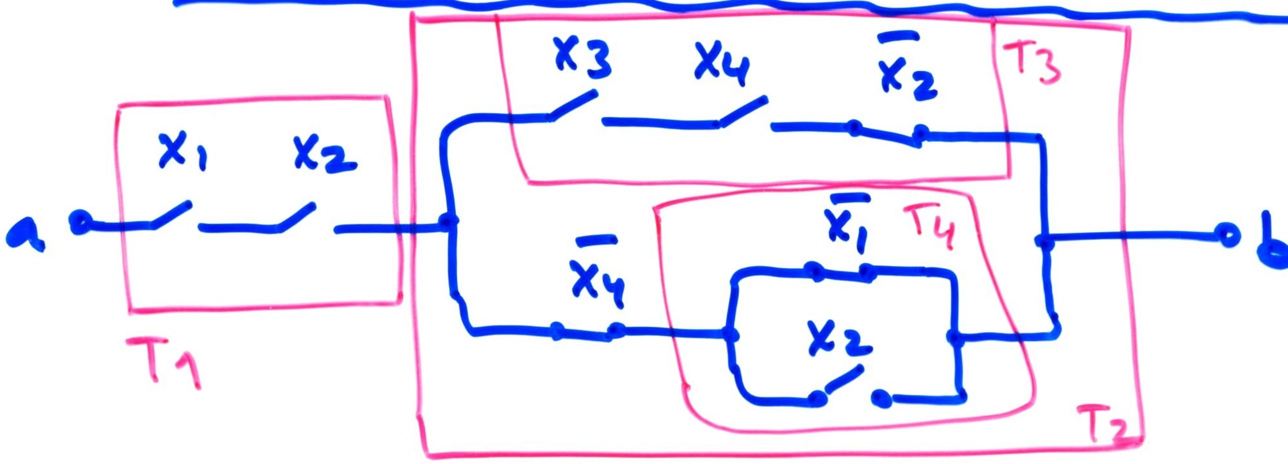
8.

Analysis of series-parallel networks:



$$f = T_1 \cdot T_2 = (x_1 \cdot x_2) \cdot T_2$$

$$T_2 = T_3 + \overline{x_4} T_4 = x_3 \cdot x_4 \cdot \overline{x_2} + \overline{x_4} \cdot (\overline{x_1} + x_2)$$

$$f = (x_1 \cdot x_2)\left[x_3 \cdot x_4 \cdot \overline{x_2} + \overline{x_4}(\overline{x_1} + x_2)\right]$$

$$f = (x_1 \cdot x_2)(x_3 \cdot \overline{x_2} \cdot x_4 + \overline{x_1}\,\overline{x_4} + x_2 \overline{x_4})$$

$$f = (x_1 \cdot x_2)\left[x_3 \cdot \overline{x_2} \cdot \overline{x_4} + (\overline{x_1} + x_2)\overline{x_4}\right]$$

$$f = (x_1 \cdot x_2)(\overline{x_2} \cdot x_3 \cdot \overline{x_4} + \overline{x_4}(\overline{x_1} + x_2)]$$

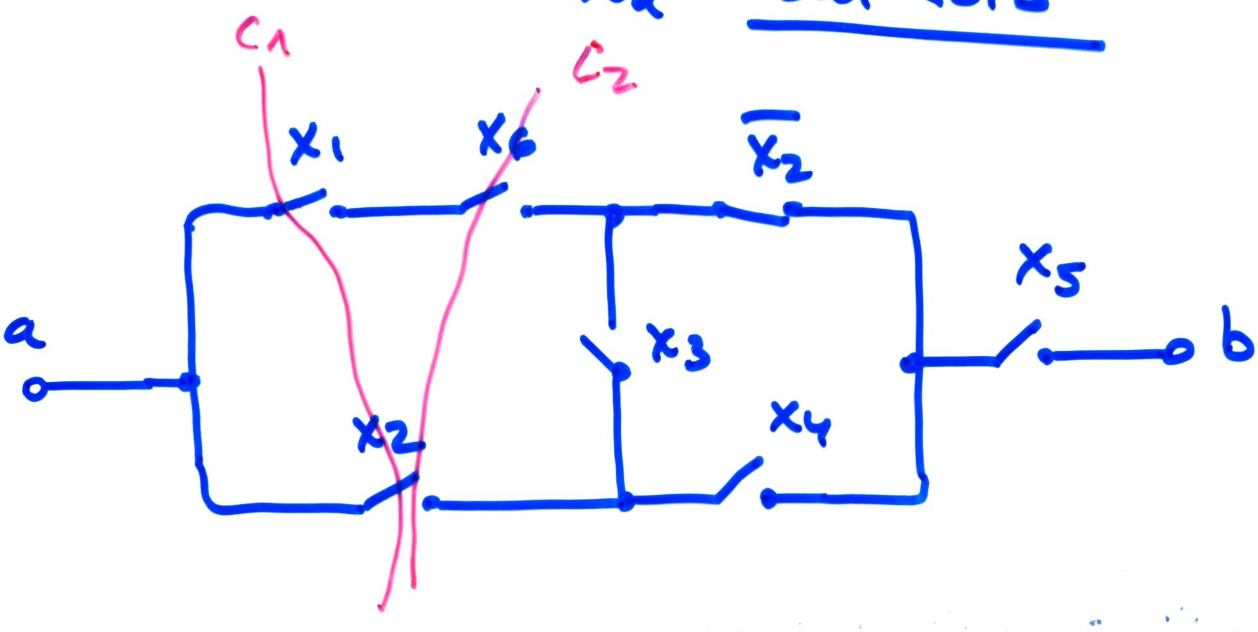
$$f = (x_1 \cdot x_2 \cdot \overline{x_4})(\overline{x_2} \cdot x_3 + \overline{x_1} + x_2) = x_1 x_2 \overline{x_4}$$

Analysis of Switching Networks

TIE SET : Connects a & b (of a network) and no parts form a loop.

CUT SET :

- All contacts open simultaneously
- if two contacts connect two parts of the net they are called cut sets



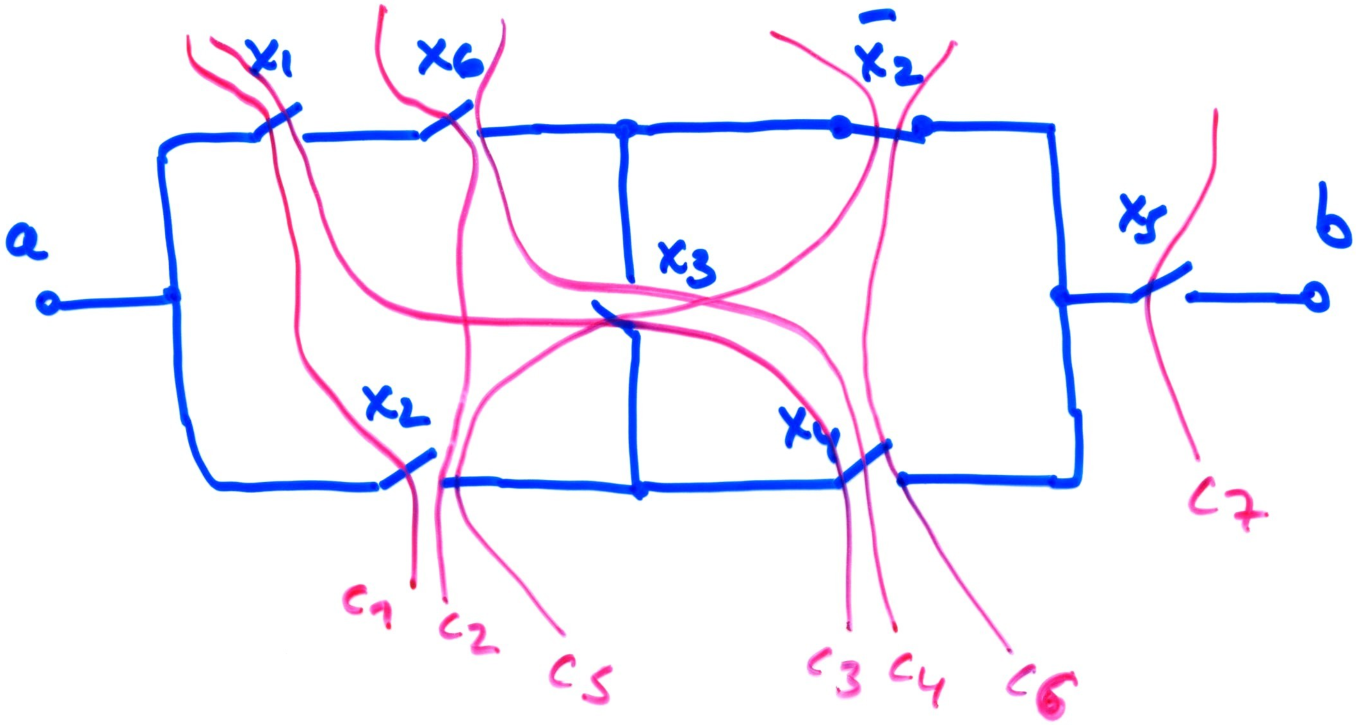
Procedure :

FIND ALL cut-sets : form OR of all literals in each cut-set

⇒ The product (AND) of all OR's yields the function f.

10.

Analysis of Switching Networks



$$f = c_1 \cdot c_2 \cdot c_3 \cdot c_4 \cdot c_5 \cdot c_6 \cdot c_7$$

$$f = (x_1 + x_2)(x_2 + x_6)(x_1 + \cancel{x_3} + x_4)(x_6 + x_3 + x_4) \cdot$$

$$\cdot (\bar{x}_2 + \cancel{x_3} + x_2)(\bar{x}_2 + x_4)(x_5)$$

$$f = \cancel{(x_1 + x_2)(x_2 + x_6)(x_6 + x_3 + x_4)(\bar{x}_2 + x_4) \cdot x_5}$$

Consensus Theorem

If we have: $\overbrace{xy + \bar{x}z}^{\text{other two}} + yz$

xy, $\bar{x}z$: complemented

CONSENSUS TERM IS REDUNDANT
CAN BE ELIMINATED!

Proof:

consensus

$$xy + \bar{x}z + yz = xy + \bar{x}z + yz(x + \bar{x})$$

$$= xy + xyz + \bar{x}z + \bar{x}zy$$

$$= xy(1 + z) + \bar{x}z(1 + y)$$

$$= xy + \bar{x}z \quad \therefore$$

De Morgan Law:

1° $\quad \overline{a+b} = \bar{a}\cdot\bar{b}$ ✓

2° $\quad \overline{a\cdot b} = \bar{a}+\bar{b}$ ✓

} Truth Table

a	b	$\overline{a+b}$	$\bar{a}\cdot\bar{b}$
0	0	$\vdots$	$= \vdots$
0	1		
1	0		
1	1		

$$\boxed{\overline{(x_1+x_2 \cdots + x_k)} = \bar{x}_1\cdot\bar{x}_2\cdots\bar{x}_k}$$

$$\overline{a+b+c} = \overline{\underbrace{(a+b)}_{①}+\underbrace{c}_{②}} = \overline{(a+b)}\cdot\bar{c}$$

$\uparrow 1^\circ$

$$= \bar{a}\cdot\bar{b}\cdot\bar{c}$$

N ✓ Valid

$N+1$

} Induction $\quad$ H.S 3^{rd} gr

$$\overline{(\underbrace{x_1+x_2 \cdots + x_k}_{①} + \underbrace{x_{k+1}}_{②})} = \underbrace{\bar{x}_1\cdot\bar{x}_2\cdots}_{①} \underbrace{\bar{x}_{k+1}}_{②}$$ ✓

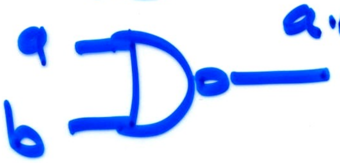
1°

$$\boxed{\overline{(x_1\cdot x_2\cdot \cdots x_k)} = \bar{x}_1+\bar{x}_2+\cdots+\bar{x}_k}$$

$$\overline{f(x_1,x_2\ldots x_k,0,1,+,\cdot)} =$$

$$= f(\bar{x}_1,\bar{x}_2\ldots\bar{x}_k,1,0,\cdot,+)$$

NAND $\qquad \overline{a\cdot b} = \bar{a} + \bar{b}$ $\qquad$ OR



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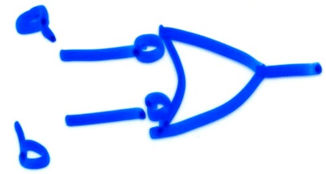
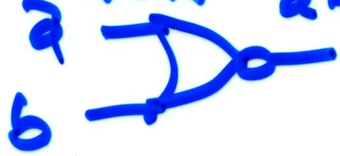


NOR $\qquad \overline{a+b} = \bar{a}\cdot\bar{b}$



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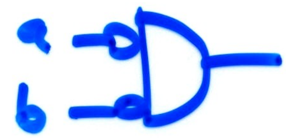
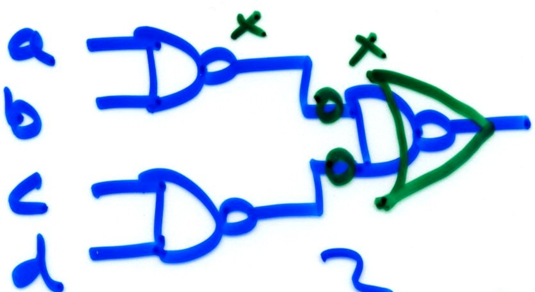
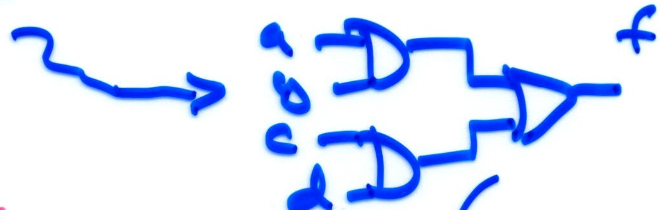
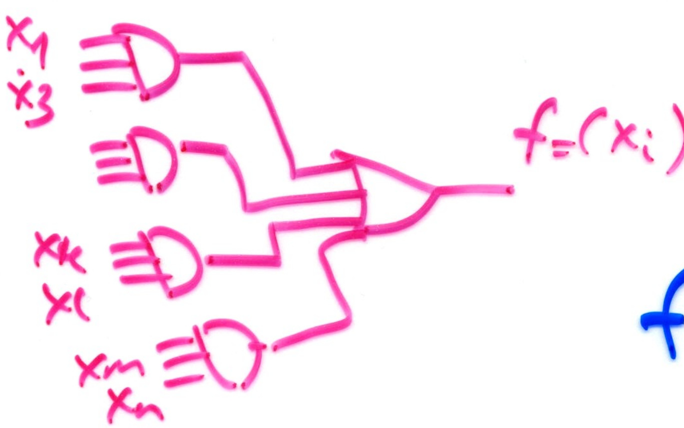




$f = ab + cd$





SOP:

$f = \_\_ + \_\_ + \_\_ + \_\_$

NAND-NAND

$[f(x_1, x_2 \dots x_n), 0, 1, +, \cdot] =$

$= [f(\bar{x}_1, \bar{x}_2 \dots \bar{x}_n), 1, 0, \cdot, +]$

Example (De Morgan):

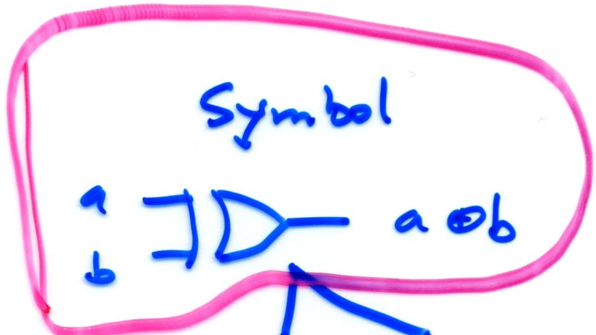
a	b	XOR	$\overline{}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0 (1)	1

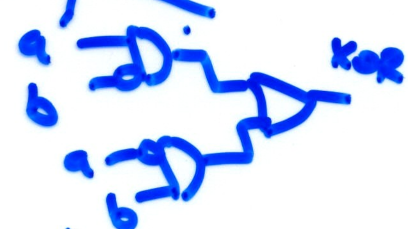
$$f = \bar{a}b + a\bar{b} = a \oplus b \quad : \text{XOR}$$

$$\bar{f} = \overline{(\bar{a}b + a\bar{b})} = \overline{(\bar{a}b)} \cdot \overline{(a\bar{b})} = (a+\bar{b})(\bar{a}+b)$$

$$= a\bar{a} + \bar{a}\bar{b} + ab + \bar{b}b = ab + \bar{a}\bar{b} : \text{XNOR}$$

XOR Properties:





(1) Commutative:

$$a \oplus b = b \oplus a$$

(2) Associative:

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

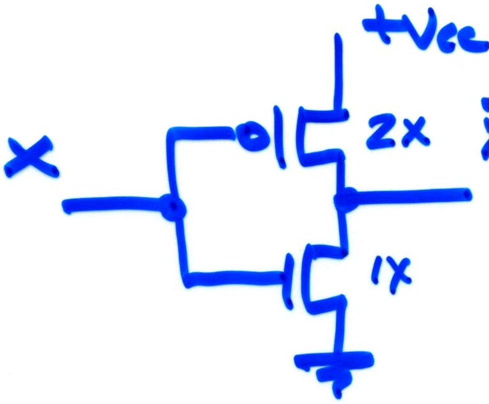
(3) Distributive:

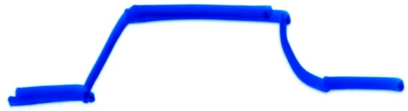
$$a \cdot (b \oplus c) = ab \oplus ac$$

(4) Identity Element:

$$a \oplus 0 = a \; ; \quad \boxed{a \oplus 1 = \bar{a}}$$

$$a \oplus a = 0 \; ; \quad a \oplus \bar{a} = 1$$





$$\boxed{F = \bar{a}b + a\bar{b} = a \oplus b}$$

XOR

a	b	$\oplus$	$\bar{F}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$$\bar{F} = \overline{\bar{a} \cdot b + a\bar{b}} =$$

$$= (a + \bar{b})(\bar{a} + b)$$

$$\boxed{\bar{F} = \bar{a}\bar{b} + ab}$$

XOR properties

$1°$ $a \oplus b = b \oplus a$

$$a \oplus \overset{0}{Z} = a$$

$$a \oplus 1 = \bar{a}$$

$$a \oplus a = 0$$

$$a \oplus \bar{a} = 1$$

2° Assoc:

$$(a \oplus b) \oplus c =$$

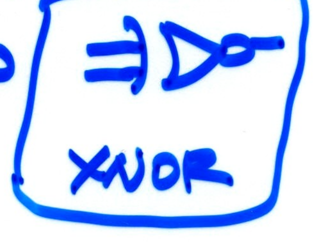
$$a \oplus (b \oplus c) =$$

$$= a \oplus b \oplus c$$

3° Distrib:

$$a \cdot (b \oplus c) = a \cdot b \oplus a \cdot c$$

Ident= $\overline{a \oplus b} = \bar{a} \oplus b = a \oplus \bar{b}$

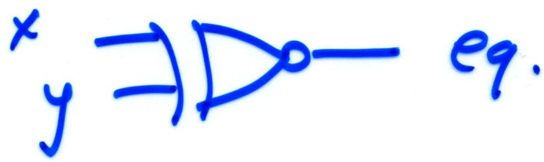


Equivalence

$x \equiv y$

x	y	$x \equiv y$
0	0	1
0	1	0
1	0	0
1	1	1

$Eq = \overline{x \oplus y}$



Proof of Distributive Property for XOR

$$x(y \oplus z) = xy \oplus xz = xy\,\overline{(xz)} + \overline{(xy)} \cdot xz$$

$$= xy(\bar{x} + \bar{z}) + xz(\bar{x} + \bar{y})$$

$$= xy\bar{z} + x\bar{y}z = x(y\bar{z} + \bar{y}z)$$

$$= x(y \oplus z)$$

Duality Theorem:

$$\boxed{[f(x_1 \dots x_k, 0, 1, +, \cdot)]^D = f(x_1 \dots x_k, 1, 0, \cdot, +)}$$

Prove:

$$\overline{F} = \overline{G}$$

$$\boxed{\begin{array}{l} \text{if } F = G \\ F^D = G^D \end{array}}$$

$$f(\overline{x}_1 \dots \overline{x}_k, 0, 1, +, \cdot) = g(\overline{x}_1 \dots \overline{x}_k, 0, 1 +, \cdot)$$

$$y_i = \overline{x}_i$$

$$f(y_1, y_2 \dots y_k, 0, 1, +, \cdot) = g(y_1 \dots y_k, 0, 1, + \cdot)$$

$$\overline{\qquad\qquad}$$

$$(x + \overline{y}) \cdot y = xy$$

dual

$$\boxed{x\overline{y} + y = x + y}$$

$$F = a\overline{b} + c + 0 \cdot \overline{d}(1 + e) = a \cdot \overline{b} + c$$

$$\Big\downarrow d \qquad\qquad\qquad\qquad\qquad\qquad \Big\downarrow d$$

$$(a + \overline{b}) \cdot c \cdot (1 + \overline{d} + 0 \cdot e)$$

$$(a + \overline{b})c \qquad = (a + \overline{b})c$$

Example Dual:

$$f = a\bar{b} + c + 0\cdot\bar{d}(1+e) = a\bar{b} + c$$

$$f^D = (a+\bar{b})c\cdot(1+\bar{d}+0\cdot e) = (a+\bar{b})\cdot c$$

Proof by duality:

$$(x+\bar{y})\cdot y = xy$$

$$\downarrow$$

$$\boxed{x\bar{y}+y = x+y}$$

$$x\bar{y}+y = x+y$$

$$\downarrow \text{dual}$$

$$(x+\bar{y})\cdot y = xy$$

Multiplying Out & Factoring

POS: $x\cdot(y+z) = xy + xz$: SOP

POS: $(x+y)(x+z) = x + yz$: SOP

$$\boxed{(x+y)(\bar{x}+z) = xz + \bar{x}y}$$

$$(x+y)(\bar{x}+z) = x\bar{x} + y\bar{x} + xz + yz$$

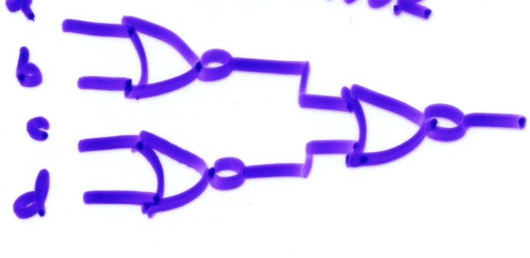
$$= xz + \bar{x}y + yz(x+\bar{x})$$

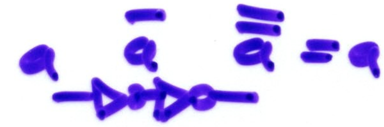
$$= xz + xyz + \bar{x}y + \bar{x}yz$$

$$= xz(1+y) + \bar{x}y(1+z)$$

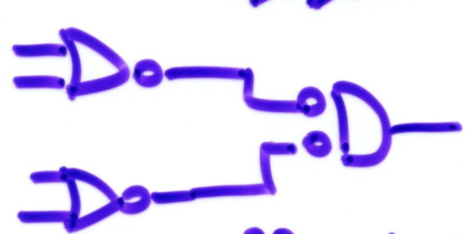
$$= xz + \bar{x}y$$

NOR – NOR





$\Rightarrow$

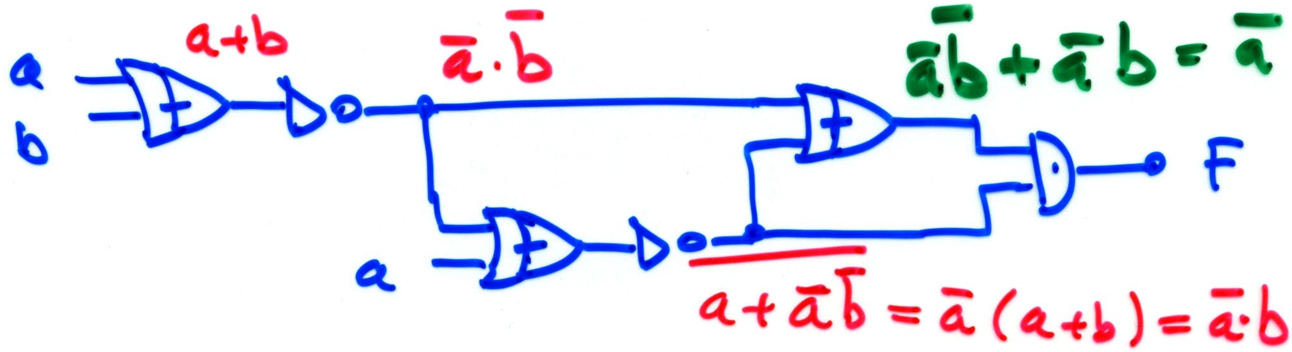


OR – AND

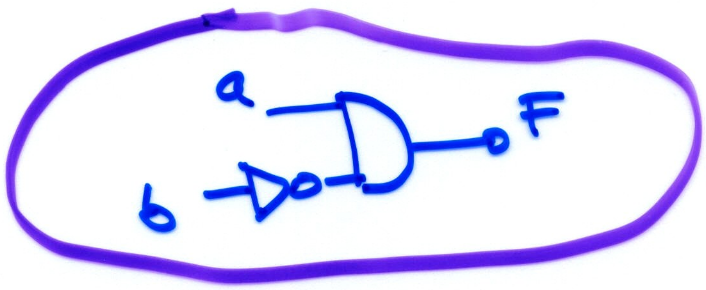
$f = (x_1 + \ )(\ + \ )(\ + \ )$

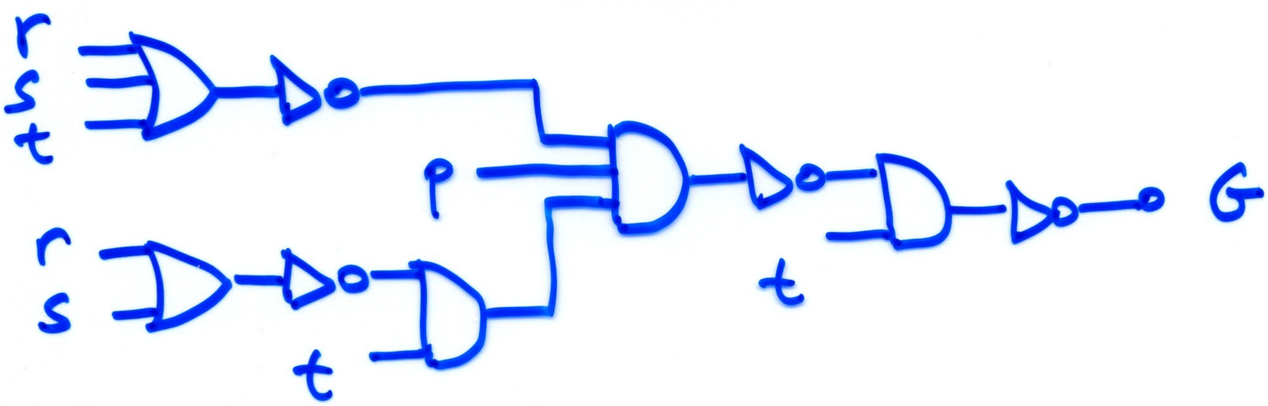
Problem 3.15

Find F & G & Simplify:

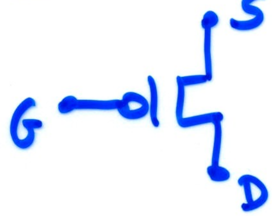


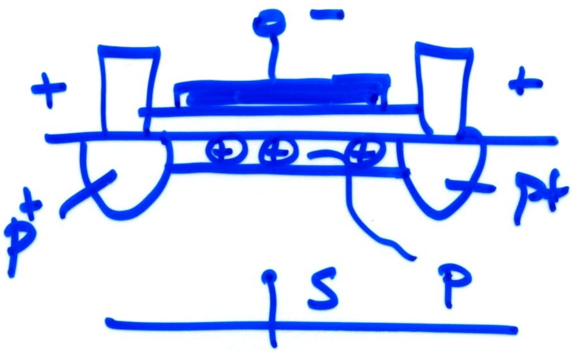
$F = \bar{a}\cdot\bar{a}b = \bar{a}b$

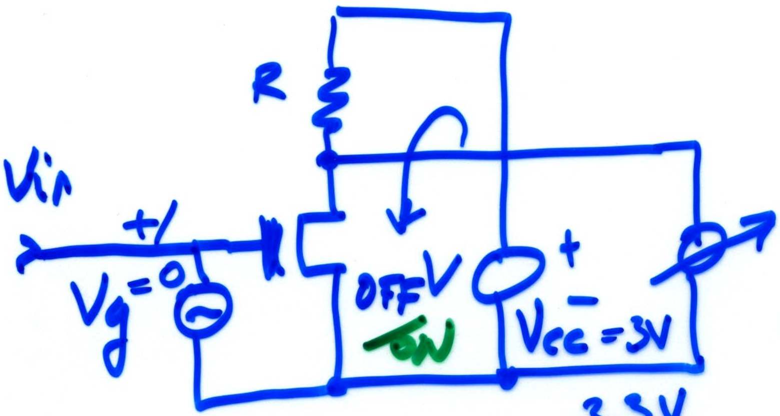


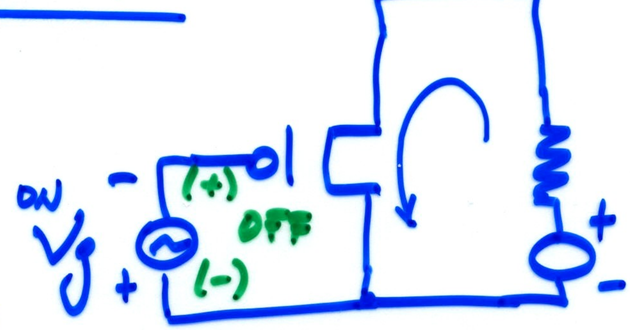


PMOS:









$V_i = V_{cc}$; $V_g = 0$; 1

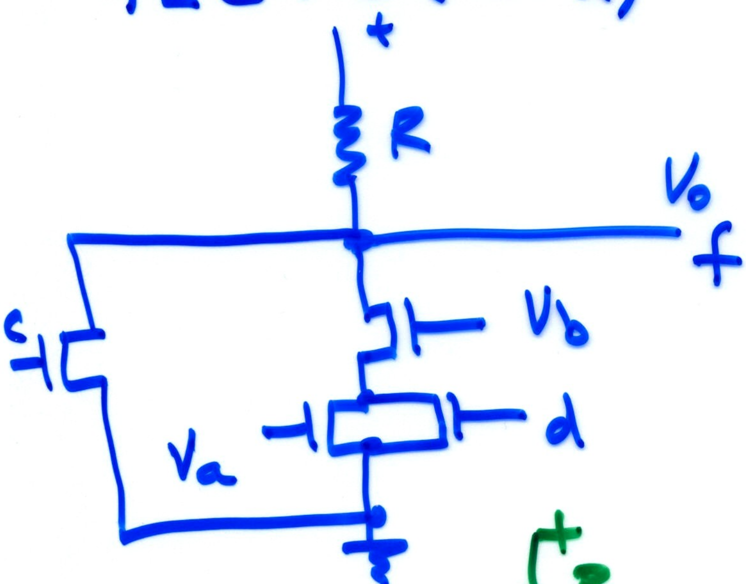
$V_i = 0$ $V_g > V_t$; 0

$r_c << R$

$\bar{f} = c + b\cdot(a+d)$

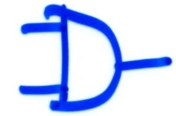
Vin	Vout
1	0
0	1

inversion

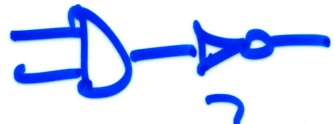


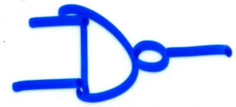
$V_a=1, V_b=1$ $V_o = 0$

$f = \overline{a\cdot b}$



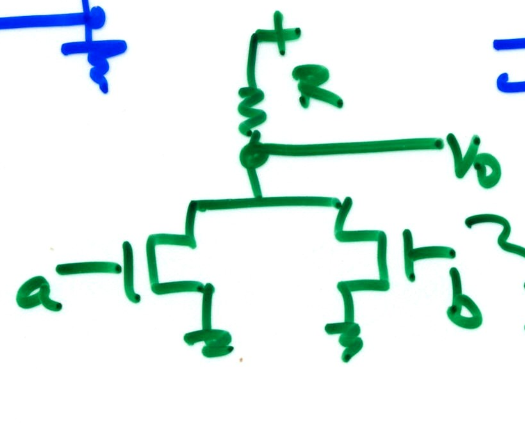
AND

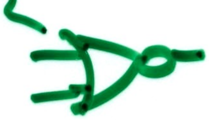




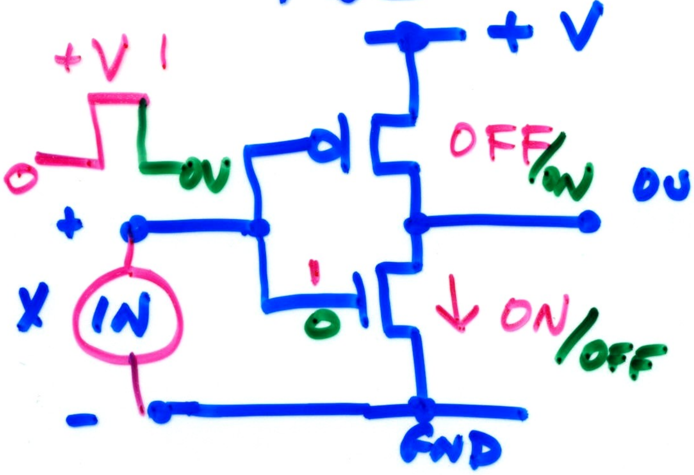
NAND

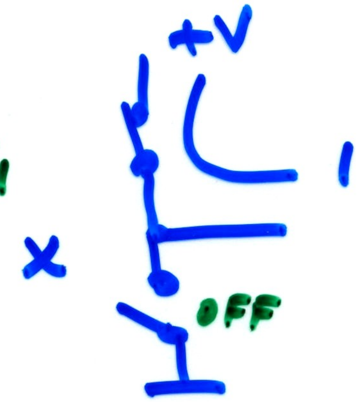
OR

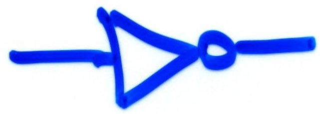




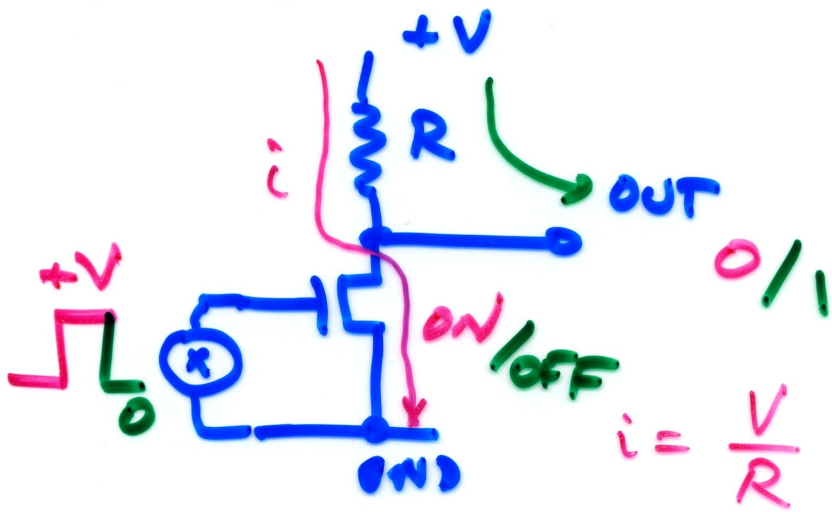
CMOS





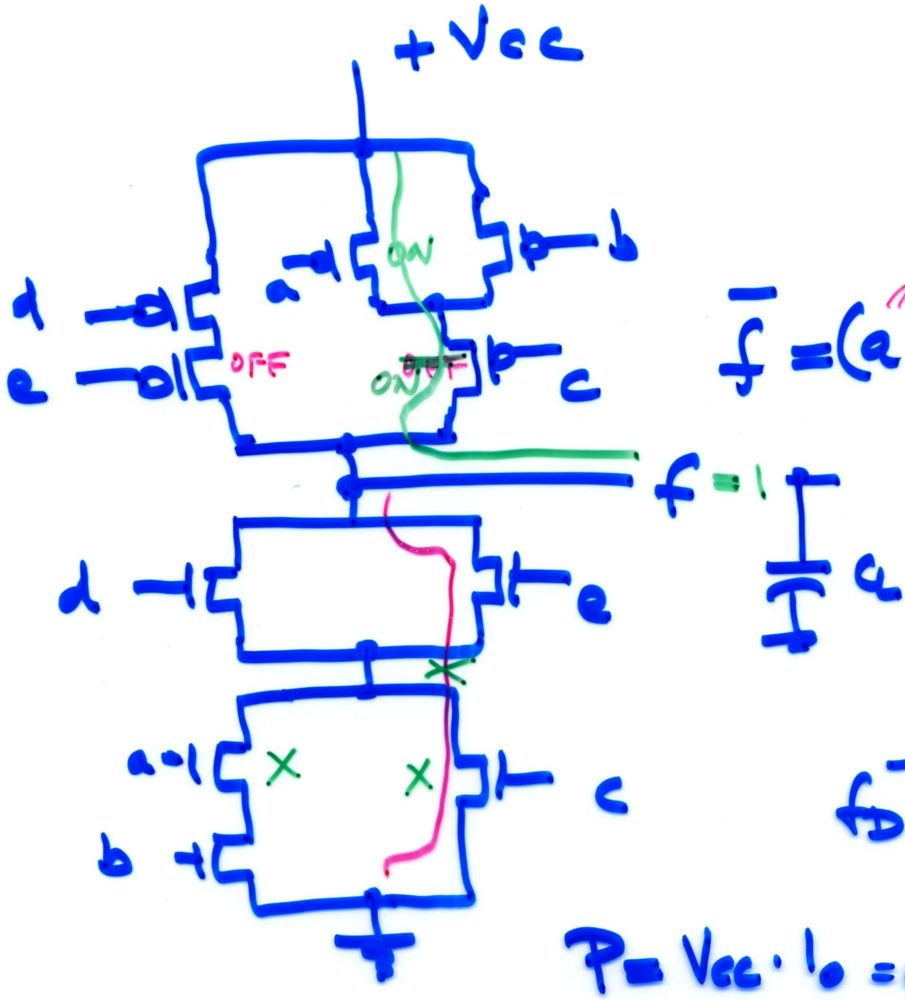
X  $\bar{X}$: INVERT CMOS

nMOS :



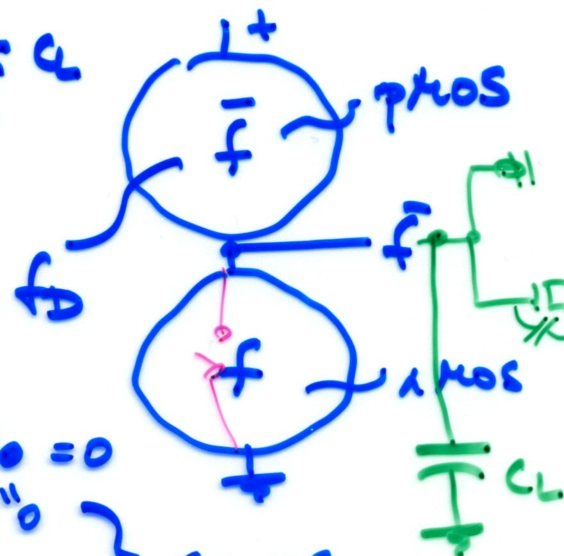
$i = \frac{V}{R}$ - $P = i \cdot V$

$P = \frac{V^2}{R}$



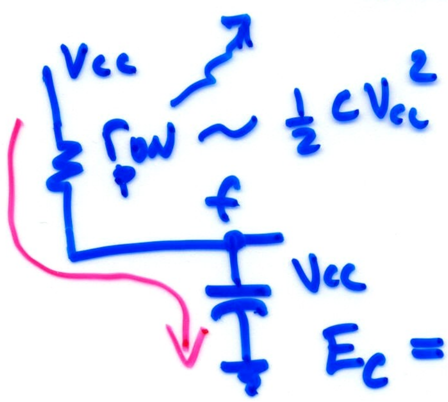
$$\bar{f} = (a \cdot b + c)\cdot(d + e)$$

CMOS



$$P = V_{cc} \cdot I_0 = 0$$

CMOS



$$E_C = \frac{1}{2} C V_{cc}^2$$