BOOLEAN ALGEBRA

Define:

1° Set of Elements B $a, b \in B \quad a \neq b$

2° Set of operations $\cdot\,, +\,, \bar{\ }$ $\sim$

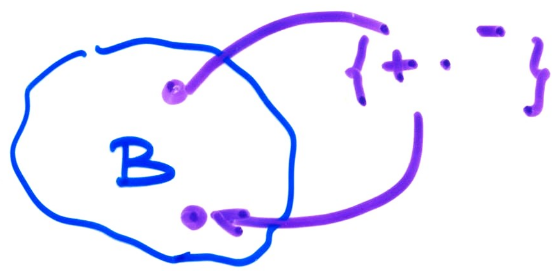
Properties:

1. **Closure:** $\forall\, a, b \in B$

$a + b \in B$

$a \cdot b \in B$

$\bar{a} \in B$



$\cdot = \wedge$

$+ = \vee$

$\oplus = \veebar$

2° **Commutative Law:**

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

3° **Associative Law:**

$$(a + b) + c = a + (b + c) = a + b + c$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b \cdot c$$

4. Identities

$\exists\, I$ with respect to σ

$\forall \sigma,\ \exists I$

(a) $a + Z = a \quad \forall a \in B$

(b) $a \cdot I = a \quad \forall a \in B$

also: $a \cdot Z = Z$; $a + I = I$; $Z \neq I$

since $a, b \in B \{0, 1\}$

$Z = 0 \qquad I = 1$

5. Distributive Law

(a) $a + (b \cdot c) = (a + b) \cdot (a + c)$

(b) $a \cdot (b + c) = a \cdot b + a \cdot c$

6. Complement

$\forall a \in B \ \exists\, \bar{a} \in B$ such that

$a + \bar{a} = I$; $a \cdot \bar{a} = Z$

$Z = 0$; $I = 1$

$a + \bar{a} = 1$; $a \cdot \bar{a} = 0$

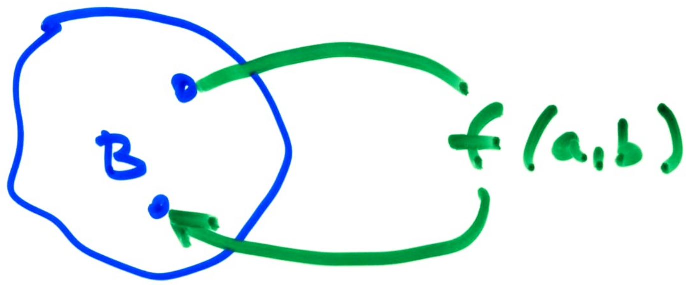
7. Idempotency

$a + a = a$

$a \cdot a = a \quad \forall a \in B$

Boolean Function f

uniquely mapps some number of inputs $a, b \in B$ into B



Boolean Expression is an algebraic statement containing Boolean variables and operands

Theorem: Any Boolean function can be expressed in terms of: $\cdot$, $+$, $\bar{\ }$ operations:

Example: $f = \bar{a} \cdot \bar{b} \cdot (c + d)$

$= \bar{a} \cdot (\bar{b} \cdot (c+d))$

7° Idempotency

$\forall a \in B$; $a \cdot a = a$, $a + a = a$

8° Involution Theorem

$\forall a \in B$; $\overline{(\bar{a})} = a$

Simplification Theorems:

1. $a \cdot b + a\bar{b} = a$
2. $a + a \cdot b = a$
3. $(a + \bar{b}) \cdot b = a \cdot b$
4. $(a+b)(a+\bar{b}) = a$
5. $a \cdot (a+b) = a$
6. $(a\bar{b}) + b = a + b$

Simplification Theorems (Cont.)

1. $a \cdot b + a \cdot \bar{b} = a \cdot (b + \bar{b}) = a \cdot 1 = a$

complement(6)

associativity(3°)

2. $a + a \cdot b = a \cdot 1 + a \cdot b = a \cdot (1 + b) = a \cdot 1 = a$

Identity(4) Assoc (3) Ident(4)

Identities

3. $(a + \bar{b}) \cdot b = a \cdot b + \bar{b} \cdot b = a \cdot b + 0 = a \cdot b$

Distribut (5) Complement (6)

4. $(a + b)(a + \bar{b}) = a \cdot a + a \cdot b + a \cdot \bar{b} + b \cdot \bar{b}$

Distribut (5)

$= a + a \cdot b + a \cdot \bar{b} + 0$

Idempot(7) Compl. (6)

$= a \cdot (1 + b) + a \cdot \bar{b} = a \cdot 1 + a \cdot \bar{b}$

Distrib(6) Ident (4)

$= a \cdot (1 + \bar{b}) = a \cdot 1 = a$

6.

Dist(5) Ident(4)

5. $a\cdot(a+b) = a\cdot a + a\cdot b = a + a\cdot b$

Ident

$= a\cdot 1 + a\cdot b = a\cdot(1+b) =$

Dist(5) Assoc

$= a\cdot 1 = a$

Ident

6. $\underline{b + a\bar{b} = a + b}$

(2) Compl(6)

$b + ab + a\bar{b} = b + a(b+\bar{b})$

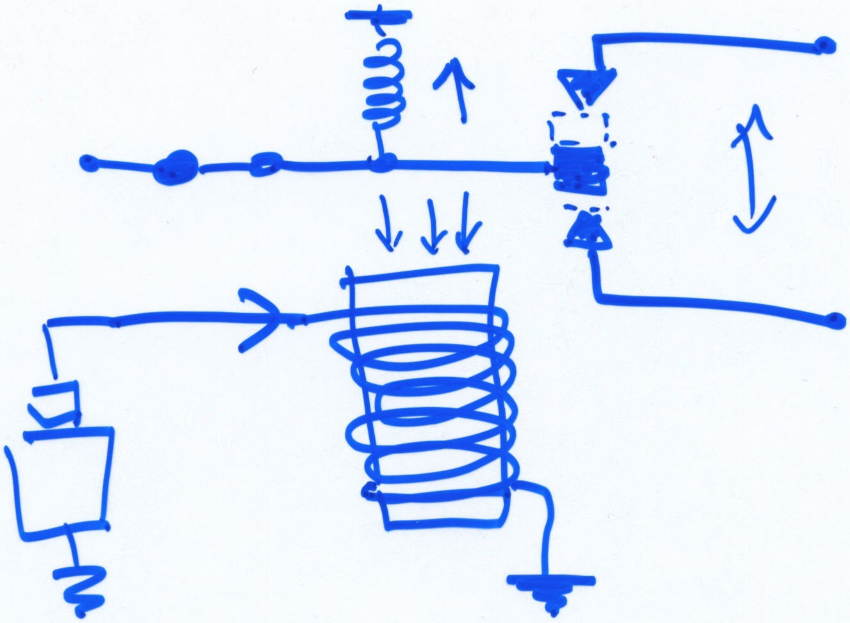
$= b + a\cdot 1$ ← Ident

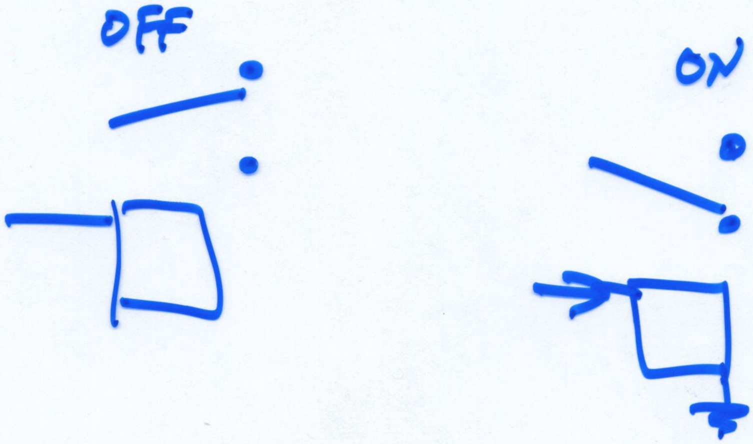
$= b + a$

Assoc(3)

$= a + b$

$$\boxed{a + \bar{a}b = a + b}$$

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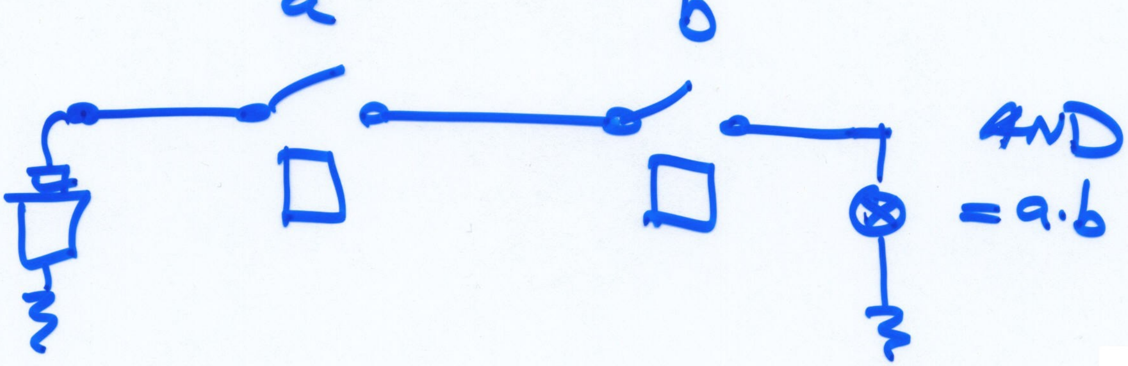
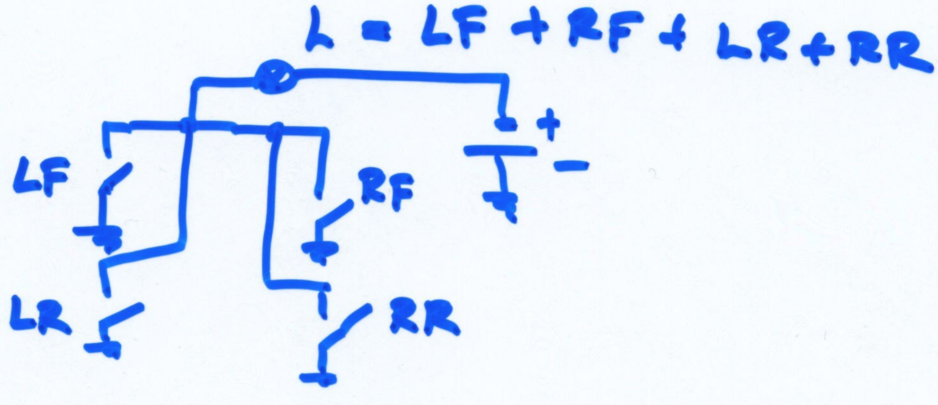
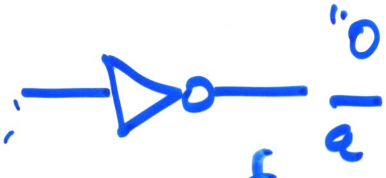

a
b
AND
= a·b


L = LF + RF + LR + RR
LF
RF
LR
RR
+
−

Implementation (Symbols)

Compl: $a \Rightarrow \bar{a}$



Truth Table:

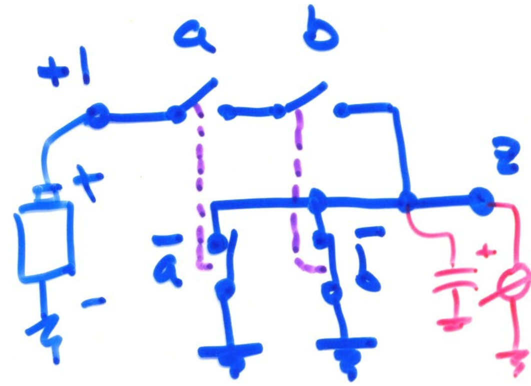
f

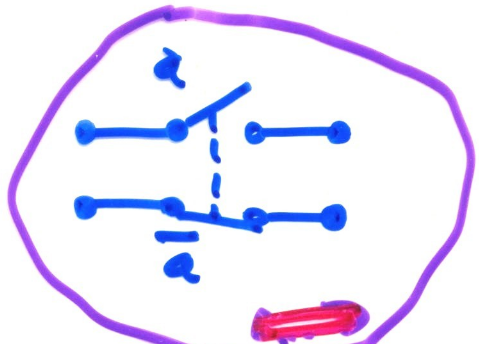
a	$\bar{a}$
0	1
1	0

AND "·"

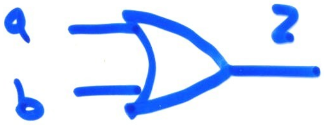
(a·b)

a	b	z
0	0	0
0	1	0
1	0	0
1	1	1



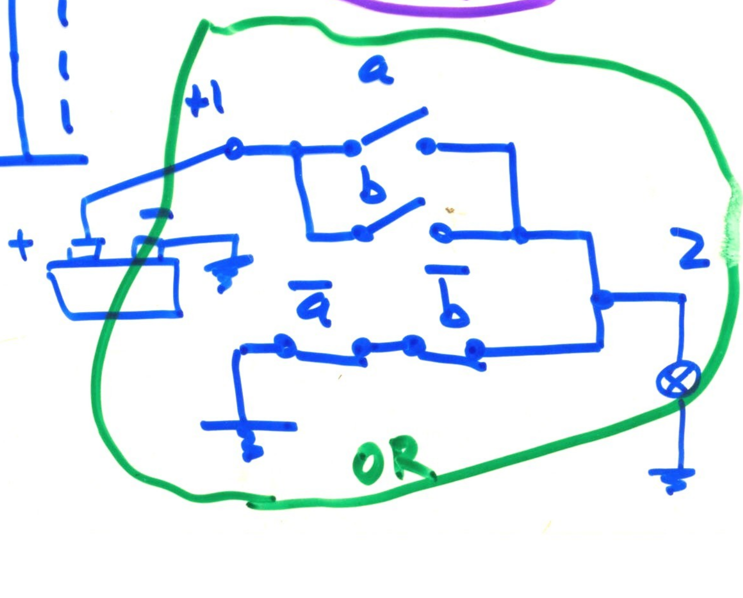


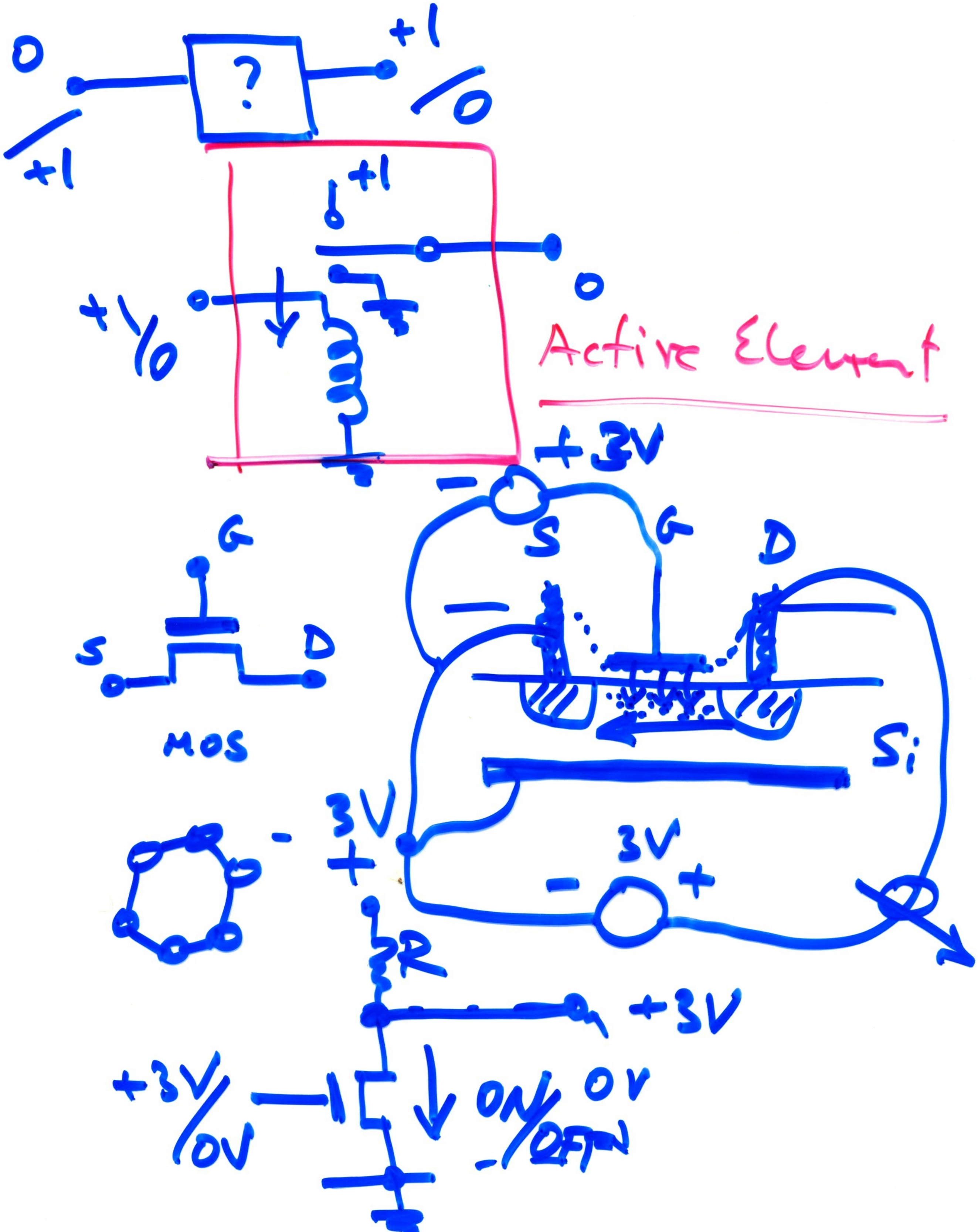
OR " + "



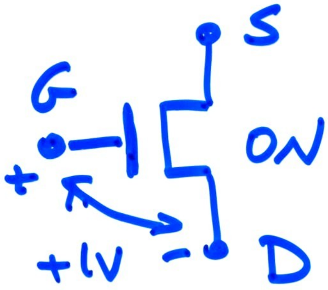
(a+b)

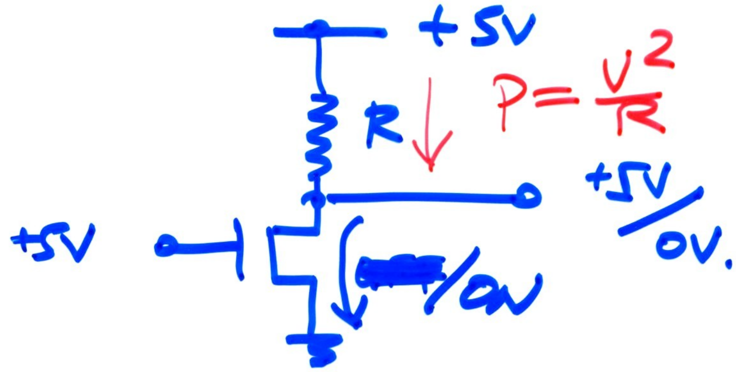
a	b	z
0	0	0
0	1	1
1	0	1
1	1	1

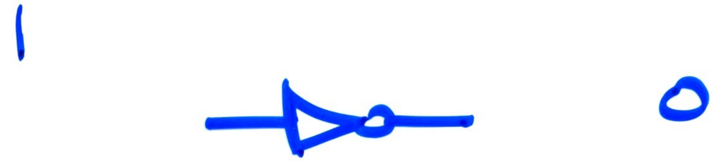



0
?
+1
/0
+1
+1
0
+1/0
Active Element
+3V
S
G
D
G
S
D
MOS
Si
3V
-
+
3V
- +
R
+3V
+3V/0V
ON/OFF
0V

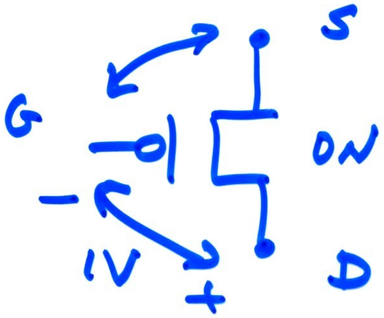
nMOS



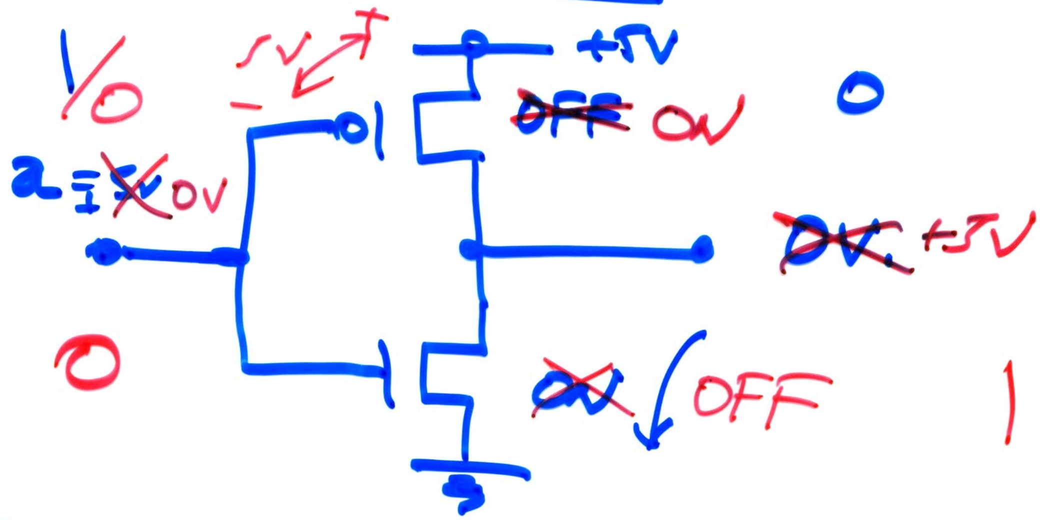


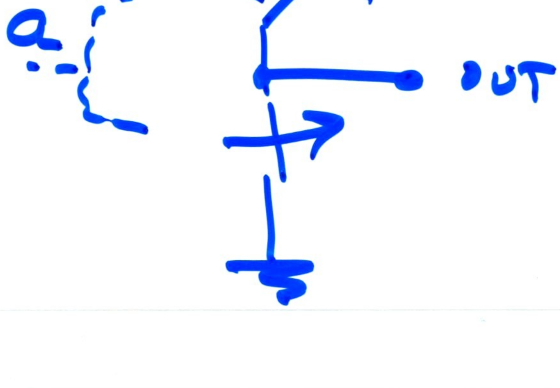


pMOS



CMOS





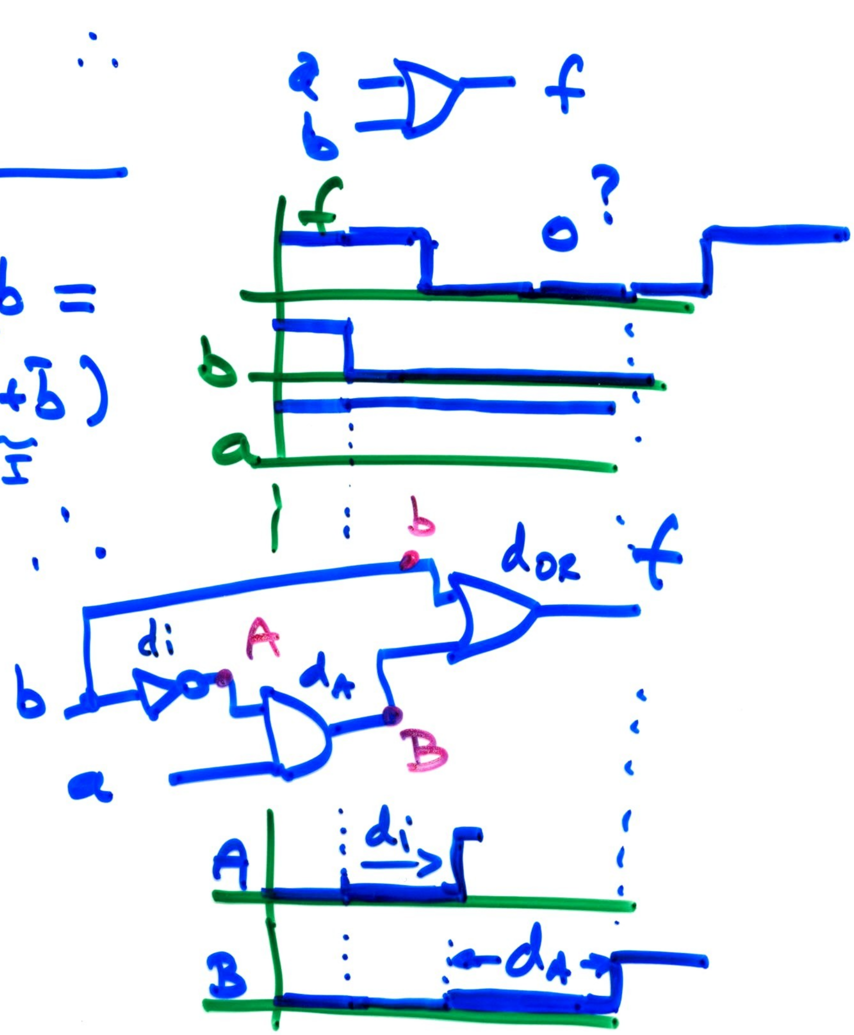
6. $\underline{b + a\cdot\bar{b}} = a + b$ $\therefore$

$b(a+\bar{a}) + a\cdot\bar{b} =$ ———

$\underbrace{ab + \bar{a}b} + \underbrace{a\bar{b} + ab} =$

$b\cdot(\underbrace{a+\bar{a}}_{I}) + a\cdot(\underbrace{b+\bar{b}}_{I})$

$a + b =$ $\therefore$



$a = 1$

$b = 1 \searrow 0$

$f = 1$