This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2008 proceedings.

Opportunistic Spectrum Access in Cognitive Radio Networks

Senhua Huang
University of California, Davis
Davis, CA 95616
Email: senhua@ece.ucdavis.edu

Xin Liu
University of California, Davis
Davis, CA 95616
Email: liu@cs.ucdavis.edu

Zhi Ding
University of California, Davis
Davis, CA 95616
Email: zding@ece.ucdavis.edu

Abstract—**Driven by regulatory initiatives and radio technology advances, opportunistic spectrum access has the potential to mitigate spectrum scarcity and meet the increasing demand for spectrum. In this paper, we consider a scenario where secondary users can opportunistically access unused spectrum vacated by idle primaries. We introduce two metrics to protect primary performance, namely collision probability and overlapping time. We present three spectrum access schemes using different sensing, back-off, and transmission mechanisms. We show that they achieve indistinguishable secondary performance under given primary constraints. We provide closed form analysis on secondary user performance, present a tight capacity upper bound, and reveal the impact of various design options, such as sensing, packet length distribution, back-off time, packet overhead, and grouping. Our work sheds light on the fundamental properties and design criteria on opportunistic spectrum access.**

I. INTRODUCTION

The breakneck proliferation of wireless devices and rapid growth of wireless services continue to strain the limited spectral resource. In fact, most spectrum bands suitable for terrestrial wireless communication have already been allocated by the regulatory agencies to existing licensees. On the other hand, the current approach of static spectral allocation is highly wasteful. Measurement has shown that over 60% of the licensed spectrum below 6 GHz remains unused or underutilized [1], [2]. To exploit the reported "white space" in existing bands [1], cognitive radio networks have been considered as the viable technology to improve spectral efficiency. With primary licensee's consent, secondary users equipped with cognitive radios may be allowed to transmit on primary bands when the primary users are inactive [3], [4], [5].

Serious challenges must be resolved in order for the cognitive radios to be acceptable. First, secondary users must not be disruptive to primary user communications. Secondly, an access mechanism is required to reduce contention between secondary users to efficiently share spectrum opportunities. However, coordination and synchronization among secondary users may be limited due to the decentralized nature of secondary user access, particularly if secondary users of different networks must coexist. In addition, each secondary users may not be able to sense all channels due to the limitation on hardware or/and sensing capability, and thus algorithms to decide which channel to monitor are needed.

In this paper, we focus on non-intrusive spectrum access schemes that do not require primary users to alter their existing hardware or behavior. The activities of primary users on different frequency bands are modeled as independent M/G/1 queues, where secondary users can exploit a primary channel when it is idle. We introduce two protection metrics (collision probability and overlapping time), by which secondary users' access schemes must abide in order to protect the quality of service (QoS) of primary users. Such constraints can be imposed by primary licensees or spectral regulators.

Under the constraints, we investigate the capacity of secondary users under various sensing-based random access schemes. We propose three random access schemes for secondary users, namely, VX, VAC, and KS schemes, with different sensing, transmission, and back-off mechanisms. We study extensively the throughput performance of secondary users with the VX scheme. We investigate both the simple system setup with with one primary band and one secondary user, and the more general case in which there are multiple primary bands and multiple secondary users. Our results illustrate the fundamental properties and design criteria of opportunistic spectral access for cognitive radio networks.

II. RELATED WORKS

To facilitate spectrum sharing, researchers have considered the design of a common control channel to exchange spectrum access and sensing information and facilitate collaborative sensing and spectrum reservation/sharing, e.g., in [6], [7], [8]. Centralized and decentralized spectrum auction and brokerage have been proposed for efficient spectrum sharing, e.g., in [9], [10], [11]. Co-existence of cognitive users in unlicensed band has also been studied [12], [13], [14].

Researchers have also considered sensing-based decentralized cognitive medium access schemes [15], [16], [17]. In [15], the authors model the states of primary bands as a two-state Markovian process and maximize the transmission rate of secondary users in certain time slots. In [16], the authors design a CSMA/CA-based cognitive radio MAC protocol that uses channel statistics to determine the optimal access range and the number of channels to access. [17] developed a slotted transmission scheme of secondary user via periodic channel sensing based on a constrained Markov Decision Process. In comparison, our model is more general. We do not assume exponential busy period of primary users (required in Markovian models), nor do we require synchronization

978-1-4244-2026-1/08/$25.00 © 2008 IEEE

between multiple users or feedback from receivers. Our work introduces explicit guarantee on the performance of primary users and we provide closed form analysis on the capacity limit of secondary users under the primary constraints.

III. SYSTEM MODEL

Consider a system with N homogeneous primary bands (channels) and M secondary users (SUs). Primary users (PUs) are the legacy users of these bands and thus have higher priority over SUs. A primary band is busy if it is used by one or more PUs, and idle otherwise.

We assume the arrival process of a PU to be Poisson while the service time distribution can be arbitrary. This assumption holds in many situations. For example, in a data network, packet arrival process is Poisson while packet length distribution can be arbitrary. Similarly, the arrival process of a data session is often Poisson while the length of the data file can be heavy tail. In addition, the call arrival process for voice traffic can also be modeled as a Poisson process. When there are multiple PUs in a band, the busy and idle periods of the band can be modeled as the busy and idle periods of an M/G/1 queue (with multiple inputs). In this case, the idle time distribution is exponential while the busy time distribution is general. From the viewpoint of an SU, because the objective is to utilize the idle period, one can treat the aggregated primaries as one PU. Therefore, we assume, without loss of generality, there is one PU per band and the activities in different bands are independent and identically distributed (i.i.d.). We also assume that the system is stationary and ergodic.

Let V_p and L_p be random variables denoting the idle period and busy period of a primary band, respectively. Let $v_p = E[V_p]$ and $l_p = E[L_p]$. Let $\alpha = \frac{v_p}{v_p + l_p}$, i.e., α is the probability (or the percentage of time) that the PU is idle. In other words, a primary band is idle α fraction of time, which could be exploited by SUs.

The focus here is to develop non-intrusive spectrum access schemes of cognitive radio devices. The non-intrusiveness has two-folded meanings. On one hand, PUs are not required to change their existing transmission strategies and algorithms to coordinate with SUs. On the other hand, the access activities of SUs should guarantee that the impact on the transmissions of PUs is insignificant. We also assume that the PU's channel access is not affected by SU's behavior. For example, PUs does not sense the channel before its transmission. If a PU and a SU transmit simultaneously, the PU does not retransmit or back-off. In other words, the idle and busy periods of a primary band are not affected by SUs.

An SU performs sensing before transmission and only transmits if the primary band is idle. We assume that SUs perform perfect sensing, i.e., the false alarm and missing probability of the sensing is zero. Additionally, we assume that the sensing of the channel takes an infinitely small amount of time to finish. Furthermore, due to the limitation of the radio's front-end, we assume that the SU cannot sense the channel when it is transmitting. We do not assume synchronization between primary and secondary users or control channel among SUs.

A. Primary Protection

In our system, collision only happens in the following scenario: a secondary user senses the channel idle and starts transmission, and the primary user returns to the band before the SU finishes its transmission. To capture the impact of collision on PUs and thus to protect PUs, we introduce the following metrics. The first constraint metric is the probability of the collision observed by the PU. As shown in Fig. 1, collision is defined as the event that the PU starts transmission when the SU is transmitting a packet on the channel. Let P^c_{pb} and P^c_s be the collision probabilities observed by a PU and an SU, respectively. Under the assumption of stationary and ergodicity, we have:

$$P^c_{pb} = \lim_{T\to\infty} \frac{\text{No. of collisions in [0,T]}}{\text{No. of busy periods of PU in [0,T]}}$$

$$P^c_s = \lim_{T\to\infty} \frac{\text{No. of collisions in [0,T]}}{\text{No. of packets transmitted for SU in [0,T]}}.$$

This constraint is suitable for the situation where the average packet length of SUs, is close to that of PUs.

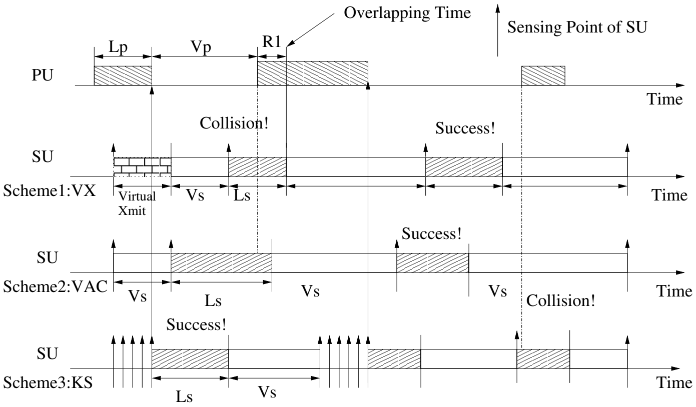

Fig. 1. Random Access Schemes of A Cognitive Radio User.

On the other hand, if a busy period of a PU consists of one or more service sessions (e.g., the whole duration of a telephone call or the total time to transfer a large file), then only a very small portion of the primary's traffic session is affected by collision. For example, suppose that the PU is hosting voice communications, the busy period (call session) may be thousands times longer than the duration of an IP packet transmitted by SUs. In this case, from the perspective of the PU, the collision probability metric is not appropriate, and the duration of the interruption caused by SUs is more important. Therefore, the second constraint metric is the percentage of overlapping time. One example of the overlapping time is illustrated in Fig. 1 with length $R1$. Specifically, we have the following definition of the percentage of overlapping time observed by the PU:

$$P^r_p = \lim_{T->\infty} \frac{\text{Length of overlapping time in [0,T]}}{T}.$$

To protect the transmission of PUs, the system can set the following constraints:

$$\begin{aligned} &P_{pb}^{c} \leq \eta, \\ \text{or,} \quad &P_{p}^{r} \leq r_0, \end{aligned} \tag{1}$$

where η and r_0 are performance thresholds predetermined by the network operator of the primary bands and/or the spectrum regulators. In fact, we will show that the above two constraints are closely related in the following sections.

B. Objective Function

We assume that there are always packets waiting for transmission for the SU. So the results obtained can be regarded as an upper bound on the capacity of SUs. We also assume that SUs have the knowledge of the statistics of the channels. In particular, each SU knows the average of busy period l_p and idle period v_p , i.e., the average channel occupancy behavior of the PUs. This knowledge can be obtained by the SU from historic information, measurement results, and/or database. SUs also have the information of the access constraints posed by the PUs or government regulators, i.e., η or r_0, which are predefined when they are admitted into the network.

The objective of the spectrum access is to maximize the achievable capacity (throughput) of the SU, denoted by C_s. More specifically, C_s is defined as the time proportion that an SU transmits on the channel without collision. Given that α is the fraction of time the PU is idle, we have $C_s \leq \alpha$. The resulting optimization problem is formulated as below:

$$\begin{aligned} &\max\{C_s\} \\ \text{such that} \quad &P_{pb}^{c} \leq \eta, \\ \text{or,} \quad &P_{p}^{r} \leq r_0. \end{aligned} \tag{2}$$

In other words, an SU can decide its sensing scheme, access scheme, packet length and distribution, and back-off duration and distribution to maximize its capacity. For the rest of the paper, let L_s be a random variable denoting the secondary transmission time, which is refereed to as packet length. Let V_s be a random variable denoting the back-off time, which is also referred to as vacation. Let $l_s = E[L_s]$ and $v_s = E[V_s]$.

IV. ONE PRIMARY BAND, ONE SU

In this section, we first consider the case where there is only one SU and one primary band. We analyze the throughput performance of the SU under the constraints posed by the PU.

A. Random Access Schemes

The media access schemes (or protocols) we consider in this paper are illustrated in Fig. 1 and described as below:

- VX Scheme (Virtual-Xmit-if-Busy): The SU senses the channel. If the channel is idle, the SU transmits a packet of length L_s. Then, the SU starts a vacation of length V_s. If the channel is busy, the SU starts a so-called virtual transmission stage and then enters into the vacation stage afterward. Here, virtual transmission means that the SU does not actually transmit the packet but waits for a time interval which is equal to the packet length. After vacation, the SU senses the channel again.
- VAC Scheme (Vacation-if-Busy): The only difference of the VAC scheme with respect to the VX scheme is that when the SU senses the channel busy, it discards the packet immediately and starts vacation.
- KS Scheme (Keep-Sensing-if-Busy): After a vacation, the SU senses the channel. If the channel is idle, the SU transmits a packet and then starts vacation. If the SU senses the channel busy, it keeps sensing until the channel is idle. Then, the SU transmits a packet and starts a random vacation of length V_s.

Since the sensing is perfect, given $L_s = \tau$, the collision probability observed by the SU in the above random access schemes is

$$Pr\{Collision|L_s = \tau\} = 1 - \int_{\tau}^{\infty} \frac{1}{v_p} e^{-\frac{t}{v_p}} dt = 1 - e^{-\frac{\tau}{v_p}}.$$

Note here we ignore the probability that one secondary packet collide with multiple PU's busy periods. This is reasonable when $l_s \ll l_p + v_p$. However, in all simulations, such events are not ignored. We show that the analysis and simulation match well for a wide range of values.

In the VX scheme, the transmission activity (including virtual transmission) of the SU is independent of the PU's occupancy of the channel, thus its analysis is simplified. In this paper, we obtain closed-form analysis on the collision probability, the overlapping time, the capacity of the SU. The closed-form solutions provide insights into the system performance and facilitate the implementation of the MAC protocol. On the other hand, the analysis is more difficult in the KS scheme, since the transmission of the SU is somewhat dependent of the activities of the PU. Interesting enough, simulation results show that the throughput performance of the KS scheme is indistinguishable from that of the VX scheme under the same collision probability constraint.

B. Performance Analysis of VX Scheme Under Collision Probability Constraint

In the VX scheme, the probability that the SU actually transmits a packet is equal to the probability that the SU senses the channel idle. Due to the independence between the sensing activities of the SU and the activities of the PU, this probability is $\alpha = v_p/(v_p + l_p)$. Let $f_{L_s}(\tau)$ be the probability density function of l_s. The average collision probability observed by the SU can be expressed as below:

$$P_s^c = \int_0^{\infty} (1 - e^{-\frac{\tau}{v_p}}) f_{L_s}(\tau) d\tau. \tag{3}$$

The virtual transmission stage in the VX scheme has the same statistic characteristics as the actual transmission. Thus, the number of "collision" events in time interval $[0, T]$ can be calculated as $\alpha \cdot P_s^c \cdot \frac{T}{l_s + v_s}$. Consequently, the collision probability for the PU can be calculated as

$$P_{pb}^c = \lim_{T \to \infty} \frac{\alpha \cdot P_s^c \cdot \frac{T}{l_s + v_s}}{\frac{T}{l_p + v_p}} = P_s^c \frac{v_p}{l_s + v_s}. \tag{4}$$

Let $\hat{L}_s$ denote the contribution of each transmission to the throughput of the SU. In particular, if a SU's packet of length L_s is successfully transmitted, $\hat{L}_s = L_s$, otherwise $\hat{L}_s = 0$. Let $\hat{l}_s = E[\hat{L}_s]$. We can regard $\hat{l}_s$ as the average length of effective packets. Then we have

$$\begin{aligned} \hat{l}_s &= \int_0^\infty \frac{1}{v_p} e^{-\frac{t}{v_p}} \int_0^t \tau f_{L_s}(\tau) d\tau dt \\ &= \int_0^\infty \tau e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau. \end{aligned} \tag{5}$$

The achievable capacity of the SU is obtained as the time proportion that the SU actually transmits packets (excluding the time when the SU performs virtual transmission) without collision, i.e.,

$$C_s = \alpha \frac{\hat{l}_s}{v_s + l_s} = \alpha \frac{\int_0^\infty \tau e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau}{v_s + l_s}. \tag{6}$$

Our objective is to find the optimum l_s, f_{L_s}, and v_s to maximize C_s in (6) under the collision probability constraint $P_{pb}^c \leq \eta$.

According to (4), to satisfy the collision probability constraint $P_{pb}^c \leq \eta$, we have

$$l_s + v_s \geq \frac{v_p \int_0^\infty (1 - e^{-\frac{\tau}{v_p}}) f_{L_s}(\tau) d\tau}{\eta}. \tag{7}$$

Note that in (7), when $l_s \ll v_p + l_p$, we have $v_s > 0$ since the right hand side in (7) can be approximated as l_s/η. The maximum throughput of the SU is achieved when equality holds in the above inequality. Thus, we have

$$C_s^{max} = \eta\alpha \frac{\int_0^\infty \frac{\tau}{v_p} e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau}{\int_0^\infty (1 - e^{-\frac{\tau}{v_p}}) f_{L_s}(\tau) d\tau}. \tag{8}$$

For the VX scheme, we have the following result.

Proposition 1. *For a primary channel of which the idle period V_p obeys exponential distribution, suppose that the probability of channel being idle is α, under the constraint that the collision probability of the primary user is less than η, there exists an upper bound on the achievable capacity of the SU:*

$$C_s \leq \eta \cdot \alpha. \tag{9}$$

Proof: We have

$$\begin{aligned} &\int_0^\infty \frac{\tau}{v_p} e^{-\frac{\tau}{v_p}} f(\tau) d\tau + \int_0^\infty e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau \\ &= \int_0^\infty (\frac{\tau}{v_p} + 1) e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau \\ &\leq \int_0^\infty (\frac{\tau}{v_p} + 1) \frac{1}{1 + \frac{\tau}{v_p}} f_{l_s}(\tau) d\tau \\ &= 1, \end{aligned}$$

where the inequality holds because $e^{-x} < \frac{1}{x+1}$ for $x > 0$. Thus,

$$\int_0^\infty \frac{\tau}{v_p} e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau \leq 1 - \int_0^\infty e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau.$$

Therefore, we have: $\frac{\int_0^\infty \frac{\tau}{v_p} e^{-\frac{\tau}{v_p}} f_{L_s}(\tau) d\tau}{\int_0^\infty (1 - e^{-\frac{\tau}{v_p}}) f_{L_s}(\tau) d\tau} \leq 1.$

As a result, we have

$$C_s \leq \eta\alpha.$$

■

Note that, through the derivation, we only require that the primary idle period be exponential and do not pose any requirement on the distribution of L_p, L_s, and V_s.

The result is somewhat surprising: C_s cannot be larger than $\eta\alpha$, i.e., the product of the collision probability constraint and the time fraction of spectrum vacancy. The intuition is as follows. Since the idle period is exponentially distributed, whenever an SU starts to transmit on an idle channel, it generates the same collision probability to the PU. Therefore, the aggressiveness of an SU's transmission should be proportional to the collision probability allowed by PUs. When $\eta = 1$, the SU can fully utilize the spectrum opportunity, and thus reach the capacity limit α when l_s goes to zero. On the other hand, if $\eta = 0$, the SU can never transmit and thus the capacity is zero. We note that, the aggressiveness of the SU's access is controlled by l_s and v_s.

The other side of the story is more straightforward. If the idle period is deterministic, the capacity of SU, C_s, can reach α regardless of the value of η. An SU can simply track the beginning of the idle period and transmit until the end of the idle period. Deterministic and exponential distribution are the two extremes in terms of predictability/entropy. While one can maximize C_s in the case of deterministic idle period, our conjecture is that the exponential idle period results in the worst opportunistic spectrum access capacity among all idle period distributions (for a given l_p and v_p).

In the following, we consider two special cases, i.e., exponentially distributed L_s and fixed L_s.

- Exponentially distributed L_s:
  When the packet length of the SU, L_s, is exponentially distributed, we have

  $$P_s^c = \frac{l_s}{l_s + v_p}, \tag{10}$$

  $$P_{pb}^c = \frac{v_p}{v_s + l_s} \cdot \frac{l_s}{l_s + v_p}. \tag{11}$$

  Following (5), we have

  $$\hat{l}_s = l_s \frac{v_p^2}{(l_s + v_p)^2}. \tag{12}$$

  From (7), in order to satisfy the collision probability, for a given l_s, the optimal v_s should be chosen such that

  $$v_s = \max\{0, \frac{v_p l_s}{\eta(v_p + l_s)} - l_s\}. \tag{13}$$

  Therefore, for given l_s and η, C_s^{max} is given as:

  $$C_s^{max} = \eta\alpha \frac{v_p}{l_s + v_p}. \tag{14}$$

We can observe that smaller l_s leads to larger C_s. This is intuitive. With smaller l_s, the collision probability is smaller, and the amount of transmission wasted is smaller when a collision happens. Therefore, more packets can be transmitted successfully with the collision constraint satisfied. We note that $C_s^{max} \to \eta\alpha$ when $l_s \to 0$.

- Fixed Packet Length of SU:
  If the SU uses fixed packet length, i.e., $L_s = l_s$, we have

$$P_{pb}^c = (1 - e^{-\frac{l_s}{v_p}})\frac{v_p}{v_s + l_s}, \tag{15}$$

From (7), in order to satisfy the collision probability, for a given l_s, v_s should be chosen such that

$$v_s = \max\{0, \frac{v_p(1 - e^{-l_s/v_p})}{\eta} - l_s\}. \tag{16}$$

Following similar approach in (6), we have

$$C_s^{max} = \eta\alpha\frac{l_s e^{-l_s/v_p}}{v_p(1 - e^{-l_s/v_p})}. \tag{17}$$

Again, we can observe that $C_s^{max} \to \eta\alpha$ when $l_s \to 0$.

It is interesting to compare the capacity performance of the SU with random and fixed length packet (with the same mean). We have the following *Proposition* for the VX scheme.

Proposition 2. *For VX, let the largest packet length be* $l_{max} = (2 - \frac{l_s/v_p}{e^{l_s/v_p}-1})v_p$. *Under the constraint* $P_{pb}^c \le \eta$ *and* $E[L_s] = l_s$, *the SU achieves the maximum throughput when it transmits fixed length packets, i.e.,* $L_s = l_s$. *In other words,*

$$C_s(l_s) = \eta\alpha\frac{\int_0^\infty \frac{\tau}{v_p}e^{-\frac{\tau}{v_p}}f_{l_s}(\tau)d\tau}{\int_0^\infty (1 - e^{-\frac{\tau}{v_p}})f_{L_s}(\tau)d\tau} \le \eta\alpha\frac{\frac{l_s}{v_p}e^{-l_s/v_p}}{1 - e^{-l_s/v_p}},$$

where $f_{L_s}(\tau)$ *is the probability density function of* l_s *with finite support* $0 \le \tau \le l_{max}$.

Proof: Define $X = \frac{L_s}{v_p}$ and its expectation $m_X = \frac{l_s}{v_p}$. From our assumption of $f_{L_s}(\tau)$, we know that the probability density function of X, $f_X(x) = 0$ if $x \notin [0,\ 2 - \frac{l_s/v_p}{e^{l_s/v_p}-1}]$. Next, we define an auxiliary function:

$$g(x) = (e^{m_X} - 1)xe^{-x} + m_X e^{-x},\ 0 < x < 2 - \frac{m_X}{e^{m_X} - 1},$$

whose second order derivative is

$$g''(x) = (x(e^{m_X} - 1) - (2e^{m_X} - 2 - m_X))e^{-x}.$$

For $0 < x < 2 - \frac{m_X}{e^{m_X}-1}$, $g''(x) \le 0$; thus, $g(x)$ is concave over the support of $f_X(x)$. Therefore, by Jensen's inequality, we have:

$$\begin{aligned} E[g(X)] &= \int_0^\infty g(x)f_X(x)dx \\ &\le g(E[X]) = g(m_X) = m_X, \end{aligned}$$

or,

$$\int_0^\infty [xe^{-x}(1 - e^{m_X}) + e^{-m_X}m_X e^{-x}]f_X(x)dx \le e^{-m_X}m_X.$$

Simple manipulations of this equation leads to the result of the proposition:

$$\frac{\int_0^\infty xe^{-x}f_X(x)dx}{\int_0^\infty (1 - e^{-x})f_X(x)dx} \le \frac{m_X e^{-m_X}}{1 - e^{-m_X}}.$$

■

Proposition 2 specifies a maximum packet length. First, packet size is limited in practice. Second, we note that, if $L_s > v_p$, the collision probability will be very high and undesirable for both the PU and SU. Since $l_{max} = (2 - \frac{l_s/v_p}{e^{l_s/v_p}-1})v_p > v_p$, l_{max} is a reasonable length constraint.

Simulations

In our simulations, we set $v_p = 1$, $\eta = 0.1$, and $l_p = 0.5$, leading to the channel idling probability $\alpha = 0.667$ which approximately equals the proportion of white space according to measurement.

In Fig. 2, we present results for the VX scheme when the SU adopts exponentially distributed packets and fixed length packets, respectively. For 10^6 busy PU periods, we vary the average SU packet length l_s from 0.1 to 1.0. Parameter v_s of the SU is obtained according to (13) and (16) to satisfy the collision probability constraint. With no assumption about the distribution of L_p and V_s in the analysis, different distributions are tested to verify the analytical results. Here, we include the cases L_p being exponentially distributed (denoted by E) and fixed (denoted by F). The distributions of V_s are exponential (denoted by E) and uniform over $[0, 2v_s]$ (denoted by U). In the legend, X/Y denotes that L_p follows X distribution and V_s follows Y distribution. Simulation result matches our analysis very well, both in terms of collision probability and capacity for different distributions of L_p and V_s.

In Fig. 3, we compare the throughput of VX and KS. Without a selection criterion of v_s for KS, for given l_s, we set v_s for KS scheme using the value obtained for the VX scheme when $\eta = 0.1$. We then adjust the value of v_s in the VX scheme using the actual collision probability obtained from the simulation results of KS scheme. The comparison is fair because KS achieves the same collision probabilities as in the VX scheme. For fixed v_s, we observe higher collision probabilities for the more aggressive KS scheme. Similar results can be obtained for the VAC scheme. Also notice that, the KS scheme is the most aggressive access scheme among the proposed three schemes. For all access schemes, the SU with fixed length packet always achieves larger throughput.

Observations

- VX, VAC, and KS schemes have indistinguishable throughput for the SU under the same collision probability constraint. Therefore, quick back-off in the VAC scheme and insistent sensing in the KS scheme do not help. The main reason is that the idle period of channel, V_p, is exponentially distributed and the SU has to guarantee the collision probability observed by the PU.
- For VX scheme, an upper-bound of the throughput of the SU is $C \le \eta\alpha$. We conjecture that this upper-bound is valid for any access schemes that exploit the idle time of

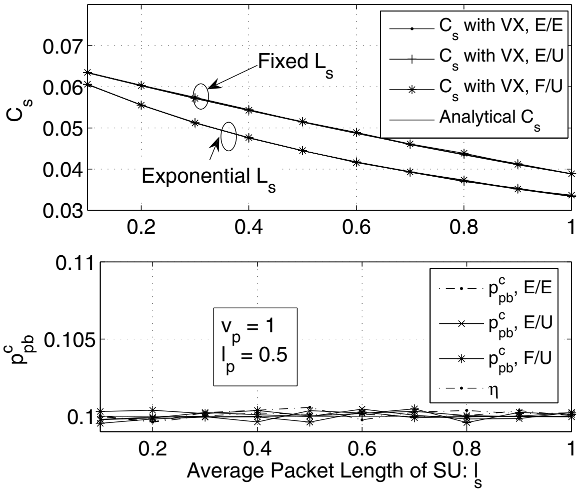

Fig. 2. Throughput of SU in VX scheme.

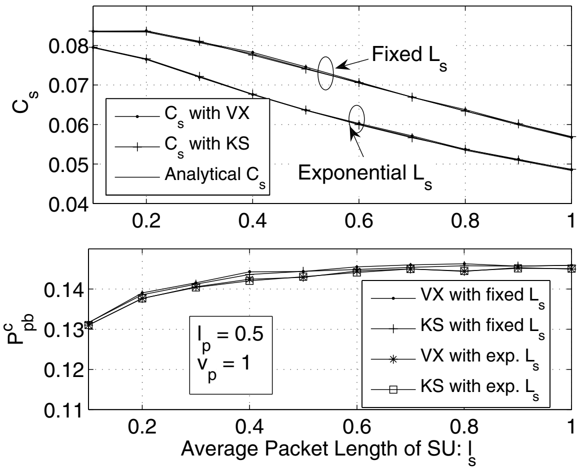

Fig. 3. Compare VX and KS schemes.

a memoryless channel without coordination from the PU under the collision probability constraint.

- For a large range of packet length l_s, fixed length packet achieves the best capacity over other packet length distributions.

C. Overlapping Time Constraint

The upper bound of the SU throughput $C_s \leq \eta\alpha$ may appear pessimistic, but not surprising. Consider applications with $l_s \ll l_p$. For example, when the primary has many packets in each busy session (e.g., a large file transfer) or the primary has voice traffic on the channel, one secondary packet may overlap with a very small proportion of PU transmission in a busy period. Most PU packets will be successful while only a small portion suffers from collision. Thus, the collision probability constraint on P^c_{pb} is overly pessimistic because it counts the whole busy period as a collision. To be more practical, we introduce the overlapping time constraint as defined in (III-A), and study the corresponding throughput performance of the SU.

Our objective is to relate the overlapping time constraint to the previously discussed collision probability constraint. The problem is to calculate P^r_p in (III-A). Denote L_v as the random length of overlapping time given that there is a collision. We have:

$$P^r_p = P^c_{pb}\frac{E[L_v]}{l_p + v_p}. \tag{18}$$

Under the assumption that $l_s \ll l_p + v_p$, the probability that the packet from the SU collides with more than one busy period of the PU is negligible. Therefore, $L_v = \min\{L_p, L^{residual}_s\}$, where $L^{residual}_s$ is the remaining transmission time of the SU's packet when the PU returns to the channel. For brevity, we present two special cases here.

First, when L_s is exponentially distributed, $L^{residual}_s$ follows the same distribution as L_s. If L_p is exponential with mean l_p, then L_v is also exponential with mean $\frac{l_p l_s}{l_p + l_s}$. Consequently, we have

$$P^r_p = P^c_{pb}\frac{l_p l_s}{(l_p + l_s)(l_p + v_p)}. \tag{19}$$

From (19), we can observe that P^r_p increases linearly with respect to P^c_{pb} and the increasing factor is much smaller than 1. This is reasonable since the overlapping part is only a very small proportion of the whole busy period if $l_s \ll l_p$.

If $L_s = l_s$ (fixed length packet) and L_p is exponentially distributed, we have

$$E[L_v] = l_p - \frac{l_p^2(e^{-l_s/v_p} - e^{-l_s/l_p})}{(v_p - l_p)(1 - e^{-l_s/v_p})}. \tag{20}$$

Consequently, the proportion of the overlapping time is

$$P^r_p = P^c_{pb}\frac{l_p[v_p(1 - e^{-l_s/v_p}) - l_p(1 - e^{-l_s/l_p})]}{(l_p + v_p)(v_p - l_p)(1 - e^{-l_s/v_p})}. \tag{21}$$

Based on (18), (19), and (21), we can easily convert the optimization problem under the overlapping time constraint to a problem under the corresponding collision probability constraint. Therefore, most of previous results in solving the collision probability constraint apply to this case.

In our simulations, we set $l_p = 0.5$, $v_p = 1$, and $l_s = 0.05 = 0.1 l_p$. The average vacation time v_s is set to satisfy the collision probability constraint $P^c_{pb} \leq \eta$. Fig. 4 shows the resulting P^r_p with respect to η for exponentially distributed l_s and fixed l_s. We can observe that, in both cases, P^r_p increases linearly with η and the analytical results agree with simulations. Also observe that for fixed L_s, the fraction of overlapping time is smaller than that with exponentially distributed L_s for any given η. This again shows the advantage of fixed SU packet length. Our results are much more optimistic by using overlapping time constraint. In particular, the achievable throughput for the SU is much closer to the limit of the available spectrum white space, α. For example, for exponentially distributed L_s, if the constraint is posed as $P^r_p \leq 0.018$, the throughput of the SU is $C_s = 0.4$. When the constraint is $P^r_p \leq 0.015$, the throughput of using fixed length packet is $C_s = 0.65$, very close to the maximum $\alpha = 0.67$.

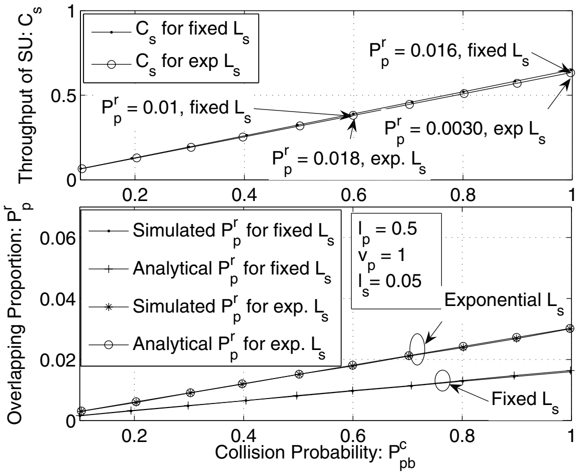

Fig. 4. Throughput of SU under different constraints.

D. Packet Overhead

Our results have thus far favored smaller l_s for better throughput C_s. This requires quick actions by SU. However, packet overhead must be considered in practical transmission by the SU. Therefore, we consider the effect of packet overhead here by assuming that a fixed length overhead is added to a payload of length L_s. Let l_0 be the length of the overhead. The collision probability observed by the SU is:

$$P_s^c = Pr\{l_0 + L_s > v_p\} = \int_0^\infty (1 - e^{-\frac{l_0+t}{v_p}}) f_{L_s}(t)dt. \quad (22)$$

Correspondingly, the PU collision probability constraint is

$$P_{pb}^c = P_s^c \cdot \frac{v_p}{l_s + v_s + l_0} \le \eta. \quad (23)$$

The average length of successfully transmitted payload of the SU is

$$\hat{l}_s = \int_0^\infty t f_{L_s}(t) \int_{t+l_0}^\infty \frac{1}{v_p} e^{-\frac{\tau}{v_p}} d\tau dt. \quad (24)$$

The effective throughput of the SU(excluding the overhead) is:

$$C_s = \alpha \frac{\hat{l}_s}{l_s + v_s + l_0} \le \alpha\eta \frac{\int_0^\infty \frac{t}{v_p} e^{-\frac{l_0+t}{v_p}} f_{L_s}(t)dt}{\int_0^\infty (1 - e^{-\frac{l_0+t}{v_p}}) f_{L_s}(t)dt}. \quad (25)$$

Using a similar approach as in the proof of *Proposition 1*, we can show that $C_s^{max} < \alpha\eta$. Next, we present the optimal packet size when L_s is exponential and fixed, respectively.

- Exponentially distributed L_s:
  We have the following results for exponentially distributed packet length:

$$P_{pb}^c = \alpha \left(1 - \frac{v_p}{l_s + v_p} e^{-\frac{l_0}{v_p}}\right) \frac{l_p + v_p}{l_s + v_s}, \quad (26)$$

$$C_s^{max} = \alpha\eta \frac{l_s v_p}{(l_s + v_p)^2} \frac{e^{-l_0/v_p}}{1 - \frac{v_p}{l_s+v_p} e^{-l_0/v_p}}. \quad (27)$$

The optimal average packet length l_s is:

$$l_s^* = v_p \sqrt{1 - e^{-\frac{l_0}{v_p}}}. \quad (28)$$

- Fixed Length L_s:
  For fixed payload length of the SU,

$$P_{pb}^c = (1 - e^{-\frac{l_0 + l_s}{v_p}}) \frac{v_p}{v_s + l_s + l_0}, \quad (29)$$

$$C_s^{max} = \eta\alpha \frac{l_s}{v_p} \frac{e^{-(l_0+l_s)/v_p}}{1 - e^{-(l_s+l_0)/v_p}}. \quad (30)$$

The optimal l_s is the solution of

$$1 - \frac{l_s^*}{v_p} - e^{-\frac{l_s^* + l_0}{v_p}} = 0, \quad (31)$$

Clearly, the throughput of the SU with overhead is smaller than that without overhead. The optimal l_s for both exponentially distributed L_s and fixed L_s demonstrate the trade-off between overhead cost and packet collision.

V. RANDOM ACCESS SCHEME FOR MULTI-BAND COMPETITIVE SYSTEM

Now consider the system with multiple primary channels and multiple SUs. Denote the number of channels N, and the number of SUs M. Let only one PU own each channel. Each SU can only transmit in one channel at a time. Multiple SUs compete for available spectrum in N channels. We consider the VX scheme only in this section. All SUs adopt the same access parameters, and thus they can be viewed as homogeneous. They do not communicate with one other. Additionally, as perfect sensing is assumed, an SU can detect transmissions by both the PU and other SU's in a channel. Assuming instantaneous sensing, as in Section IV, collision in channels can only happen between the returning incumbent PU and a transmitting SU. The aggregated SU throughput is defined as the sum throughput of all SUs in one particular channel. Similarly, the aggregated collision probability is the collision probability observed by the PU.

A. One Primary Band, Multiple SUs

We first consider the case when there are multiple SUs (with VX scheme) contending for one primary band, and obtain the relationship between the number of SUs and the aggregated collision probability observed by the PU. For each SU, since there are other SUs transmitting on the channel, the probability of virtual transmission become a function of the activities of both SU and PU. Denote the probability of virtual transmission of each SU as p_d. Then the transmission probability of each SU is $p = 1 - p_d$. Denote the event that a SU is transmitting as Φ. Then the aggregated collision probability observed by the PU can be expressed as:

$$P_{pb}^c = MPr[Collision|\Phi]p = M(1 - p_d)P_s^c \frac{l_p + v_p}{l_s + v_s} \quad (32)$$

When $M = 1$, we have $p_d = 1 - \alpha$, and the above formula becomes the same as (4).

Whenever the SU senses the channel being busy and performs virtual transmission, the channel is occupied by the PU and other SUs. Notice that there is probably transmission overlapping between the PU and other SUs. We have the following expression for the probability of virtual transmission of any SU:

$$p_d = 1 - \alpha + (M-1)(1-p_d) - (M-1)P_p^r, \qquad (33)$$

where P_p^r is the overlapping proportion observed by the PU caused by one SU with transmission probability p. As long as we can obtain the relationship between the overlapping probability and the collision probability (which is caused by each SU) observed by the PU, i.e., P_{pb}^c/M, as in section IV-C, we can solve equations (32) and (33) to obtain the expression of P_{pb}^c.

As an example, when the SU uses exponentially distributed packet length, and the PU's busy period follows exponential distribution, we have the following result on the probability of virtual transmission:

$$p_d = \frac{1-\alpha+\frac{(M-1)l_s}{l_s+v_s}-\frac{(M-1)l_p l_s^2}{(l_s+v_s)(l_p+l_s)(l_s+v_p)}}{\frac{(M-1)l_s}{l_s+v_s}+1-\frac{(M-1)l_p l_s^2}{(l_s+v_s)(l_p+l_s)(l_s+v_p)}}. \qquad (34)$$

Relying on (32), we can obtain a closed-form expression for the aggregated collision probability observed by the PU. Therefore, if SUs can estimate the number of SUs in the system and the distribution of the busy period of the PU, they can adjust their access parameters using (32), such that the collision probability/overlapping time constraint imposed by the PU is satisfied.

For given values of v_s and l_s, we can derive the achieved throughput of each SU as in (6) by replacing α with $1-p_d$. As a result, we have the following expression for the aggregated throughput of SUs in one primary band:

$$C_s = P_{pb}^c \alpha \frac{v_p}{l_s + v_p}, \qquad (35)$$

which is in accordance to (14) for the One-Band-One-Secondary system (OBOS).

In Fig. 5, we plot the aggregated throughput of SUs and the aggregated collision probability observed by the PU when there are M SUs in the system. We can observe that, the analytical results match very well with the simulations. For given l_s and under the same collision probability, the aggregated throughput of M SUs is the same as the throughput of the SU in an OBOS system. This is reasonable, because there is no collision between SUs under perfect sensing. Since SUs are homogeneous, the collision caused by all SUs can be treated as from a super-node. Each SU achieves an equal fraction $(\frac{1}{M})$ of the total throughput. We can also observe that collision probability caused by one individual SU with $M > 1$ is less than the collision probability with $M = 1$. This is due to the lower transmission probability of each SU.

B. Multiple Primary Bands, Multiple SUs

When there are multiple primary bands, we assume that each SU can have access to at most one band during each access period. First, the SU determines which band to sense (sensing strategy). After that, it uses the VX scheme(or any other access scheme) to exploit the spectrum opportunities in the band. Two sensing strategies are considered:

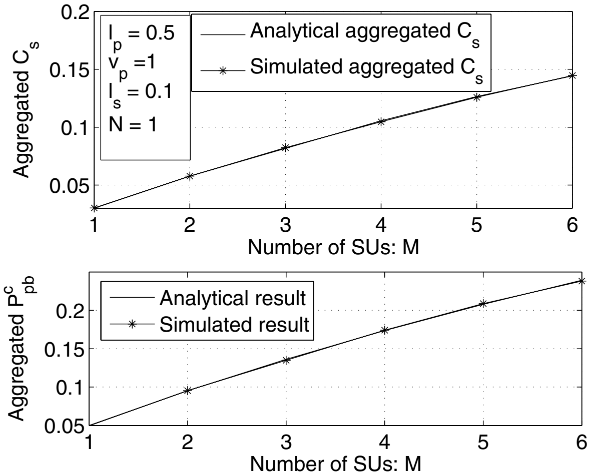

Fig. 5. Aggregated Throughput of SUs in VX scheme.

- Random-Sensing: After a random vacation time v_s, each SU randomly selects a channel, and then detects whether the channel is busy. If it is, then SU enters the Virtual Transmission stage. If the channel is idle, the SU transmits its packet before taking a vacation.
- All-Channel-Sensing: After a vacation, each SU senses all channels. If there is no idle channel, the SU enters the Virtual Transmission stage. Otherwise, the SU randomly selects an idle channel for packet transmission.

With the Random-Sensing strategy, the SU only needs to monitor one band at each instant. By comparison, the All-Channel-Sensing strategy requires that each SU monitor all channels. Thus, the former is much easier to implement than the latter.

We present Monte-Carlo simulations on the performance of the two strategies. We set $l_p = 0.5$, $v_p = 1$, and $l_s = 0.1$. Due to limited paper length, we only present results for exponentially distributed l_s here. We set $M = 3N$. Note that the primaries' activities are i.i.d., and all SUs behave in the same way, thus the performance is the same for all channels. Therefore, we only show the results for one of N channels here. The aggregated throughput of SUs and collision probability in each channel for Random-Sensing and All-Channel-Sensing strategies are shown in Fig. 6. Based on the simulation results, we have the following observations.

- For the same collision probability constraint in each primary band as in the corresponding OBOS system, the system with multiple SUs has no loss/gain in terms of total throughput.
- Under the same collision probability constraint, sensing all the frequency bands does not improve the total throughput of SUs, though it has higher complexity. If we adjust the values of l_s and v_s, such that the

aggregated collision probabilities observed by the PU for the Random-Sensing and All-Channel-Sensing strategies are the same, then they will have the same throughput. This is mainly due to the memoryless characteristics of the idle time and the collision probability constraint, rather than the limitation that each SU can access one channel each time.

- The simpler autonomous random access performs the same as the coordinated method of organizing SUs into separate groups, each assigned the same amount of spectral resource. From the perspective of the SU, the total throughput it can achieve is $\frac{N}{M}$ times the aggregated throughput in each channel.

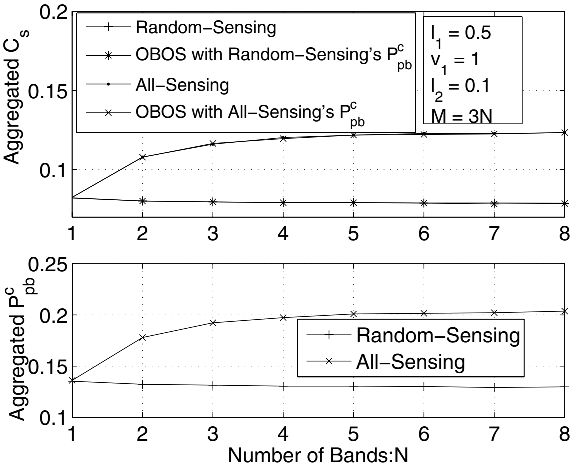

Fig. 6. Aggregated Throughput of SUs with Multiple Primary Bands.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper, we study the data capacity of cognitive radio users in opportunistic spectral access under stringent intrusion constraints on collision probability and the overlapping collision time. Three random access schemes are proposed for cognitive radio devices to exploit the spectral opportunities in primary bands. We obtain closed-form expressions for the collision probability of the PU and the capacity of the SU. We show that the proposed random access schemes have similar capacity performance. Furthermore, we find that the collision probability and the overlapping time constraints are closely related. SUs can utilize a significant amount of spectral vacancy in the primary band under overlapping time constraint when appropriate. In addition, we consider the overhead cost in the SUs' packets and demonstrate that an optimal trade-off can be obtained for exponential and fixed packet length. Finally, we investigate the aggregated throughput performance and collision probability in a multi-band multi-secondary system.

Our work provides a better understanding on the fundamental properties and performance limit of opportunistic spectrum access. Future works in this direction may involve the extension to systems with inaccurate sensing devices. Imperfect sensing will induce collisions between different SUs and hidden nodes problems in networks. Advanced signal processing techniques in physical layers can also be integrated in the design of media access schemes.

VII. ACKNOWLEDGMENT

This work is supported in part by the National Science Foundation Grant CNS-0520126.

REFERENCES

[1] "Spectrum policy task force report," Federal Communications Commision, Report, Et docket No. 02-135, November 2002.

[2] J. Yang, "Spatial channel characterization for cognitive radios," MS Thesis, UC Berkeley, 2004.

[3] "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order," Federal Communications Commision, Report, Et docket No. 03-322, December 2003.

[4] M. Marcus, "Real time spectrum markets and interruptible spectrum: new concepts of spectrum use enabled by cognitive radio," in *First IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2005, pp. 512– 517.

[5] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005, 0733-8716.

[6] D. Raychaudhuri and X. Jing, "A spectrum etiquette protocol for efficient coordination of radio devices in unlicensed bands," in *14th IEEE Proceedings on Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 1, 2003, pp. 172–176.

[7] D. Carbric, S. M. Mishra, D. Willkomm, R. Brodersen, and A. Wolisz, "A cognitive radio approach for usage of virtual unlicensed spectrum," in *14th IST Mobile and Wireless Communications Summit*, 2005.

[8] Y. Yuan, P. Bahl, R. Chandra, P. A. Chou, J. I. Ferrell, T. Moscibroda, S. Narlanka, and Y. Wu, "Knows: Cognitive radio networks over white spaces," in *Second IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2007, pp. 416–427.

[9] M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "Dimsumnet: New directions in wireless networking using coordinated dynamic spectrum access," in *IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks (IEEE WoWMoM)*, aromina/Giardini Naxos, Italy, 2005.

[10] J. Huang, R. Berry, and M. Honig, "Auction-based spectrum sharing," *ACM Mobile Networks and Applications Journal*, pp. 405–418, 2006.

[11] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A general framework for wireless spectrum auctions," in *Second IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2007, pp. 22–33.

[12] X. Jing, S.-C. Mau, D. Raychaudhuri, and R. Matyas, "Reactive cognitive radio algorithms for co-existence between ieee 802.11b and 802.16a networks," in *IEEE Globecom*, St. Louis, MO, 2005.

[13] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," in *First IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, Baltimore MD, 2005, pp. 251– 258.

[14] J. Huang, R. A. Berry, and M. L. Honig, "Spectrum sharing with distributed interference compensation," in *First IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2005, pp. 88–93.

[15] Q. Zhao, L. Tong, and A. Swami, "Decentralized cognitive mac for dynamic spectrum access," in *First IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2005, pp. 224–232.

[16] A. C.-C. Hsu, D. S. L. Wei, and C. C. J. Kuo, "A cognitive mac protocol using statistical channel allocation for wireless ad-hoc networks," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007, pp. 105–110.

[17] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Optimal dynamic spectrum access via periodic channel sensing," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007.