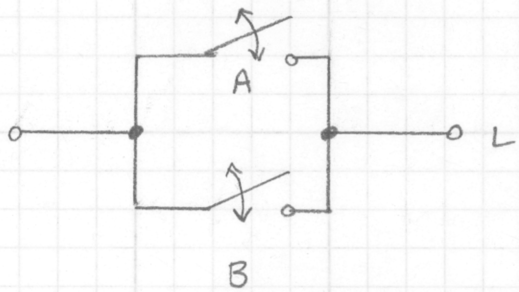
I. Digital Logic

Consider circuit elements in digital logic circuits as ideal switches (typically implemented using transistors.)

Circuits

+5V or logic "1"



Parallel Connection

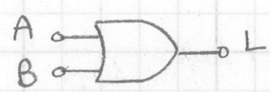
- When A, B are "1", then switches are closed.
- When A, B are "0", then switches are open.

Truth Table (mapping from input signals to output signal)

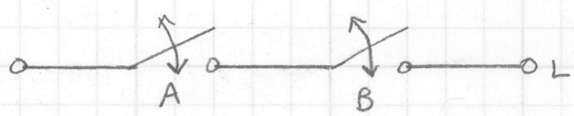
A	B	L
0	0	0
0	1	1
1	0	1
1	1	1

Mathematical (Logical) "OR" Operation

$L = A + B$



+5V or logic "1"

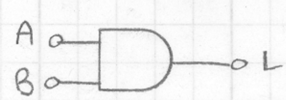


Series Connection

Truth Table

A	B	L
0	0	0
0	1	0
1	0	0
1	1	1

"AND" Operation

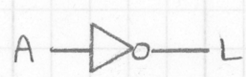


Mathematical Form

$A \cdot B = L$ or $AB = L$

"NOT" Operation (Inversion)

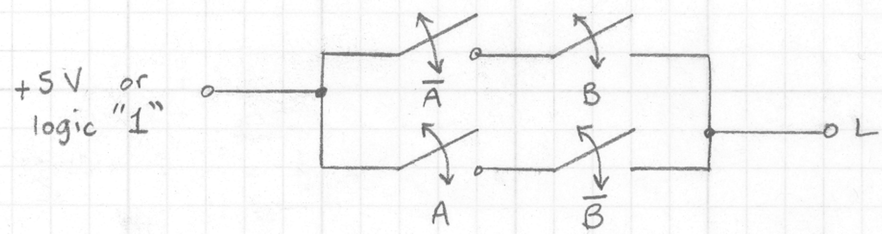
A	L
0	1
1	0

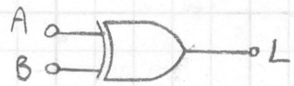


Mathematical Form

$L = \overline{A}$

"XOR" (Exclusive OR) Operation





Mathematical Form

$$L = (\bar{A} + \bar{B})(A + B) = \bar{A} \cdot B + A \cdot \bar{B}$$

$$L = A \oplus B$$

II. $\boxed{\text{Boolean Algebra}}$

Mathematical manipulations of equations involving single-bit binary numbers.

Basic Operations

"OR" $+$

"AND" $\cdot$

"NOT" $\bar{A}$

Commutative Law $A + B = B + A$ $AB = BA$

Associative Law $(A+B)+C = A + (B+C)$ $(AB)C = A(BC)$

Distributive Law $A(B+C) = AB + AC$

$A + BC = (A+B)(A+C)$

Identities: $A \cdot 1 = A$ $A + 0 = A$ $\bar{\bar{A}} = A$

$A \cdot 0 = 0$ $A + A = A$

$A \cdot A = A$ $A + 1 = 1$

$A \cdot \bar{A} = 0$ $A + \bar{A} = 1$

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Theorems of Boolean Algebra
(pp. 772–778 in book)

$$A + AB = A$$

$$A(A+B) = A$$

$$A + \bar{A}B = A + B$$

$$A(\bar{A} + B) = AB$$

DeMorgan's theorem

$$\overline{A+B} = \bar{A}\,\bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

Sum of Products

Express Boolean function as OR of ANDs

$$\underbrace{F}_{\text{Think about where } F=1} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \underbrace{ABC}_{\text{minterm}}$$

Truth Table for F

A	B	C	F	
0	0	0	0	
0	0	1	1	$\rightarrow \bar{A}\bar{B}C \equiv m_1$ (ABC = 001 = binary #1)
0	1	0	1	$\rightarrow \bar{A}B\bar{C} \equiv m_2$
0	1	1	0	
1	0	0	1	$\rightarrow A\bar{B}\bar{C} \equiv m_4$
1	0	1	0	
1	1	0	0	
1	1	1	1	$\rightarrow ABC \equiv m_7$

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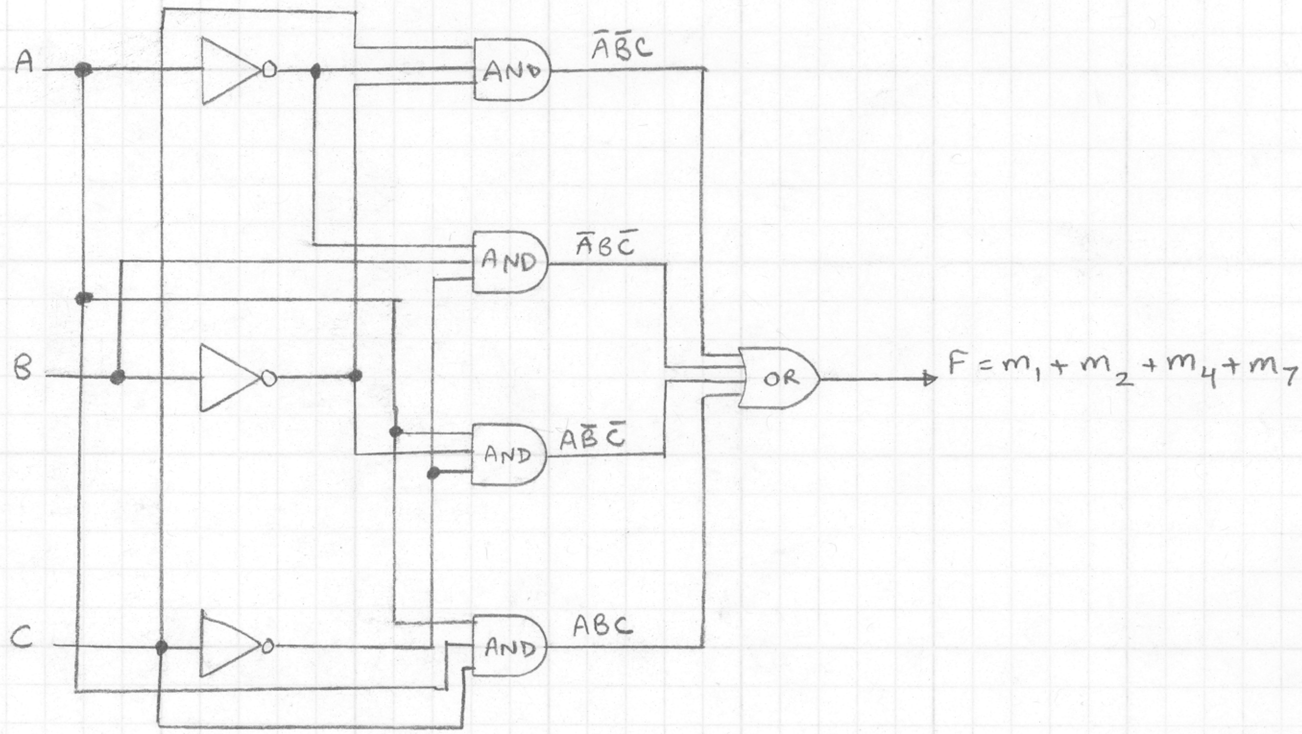
Implement F using Logic Gates



Product of Sums Express Boolean function as AND of ORs

$$\underbrace{\bar{F}} = \underbrace{AB\bar{C}}_{m_6} + \underbrace{A\bar{B}C}_{m_5} + \underbrace{\bar{A}BC}_{m_3} + \underbrace{\bar{A}\bar{B}\bar{C}}_{m_0}$$

Think about where $\bar{F} = 0$

$$F = \bar{\bar{F}} = \overline{AB\bar{C} + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}}$$

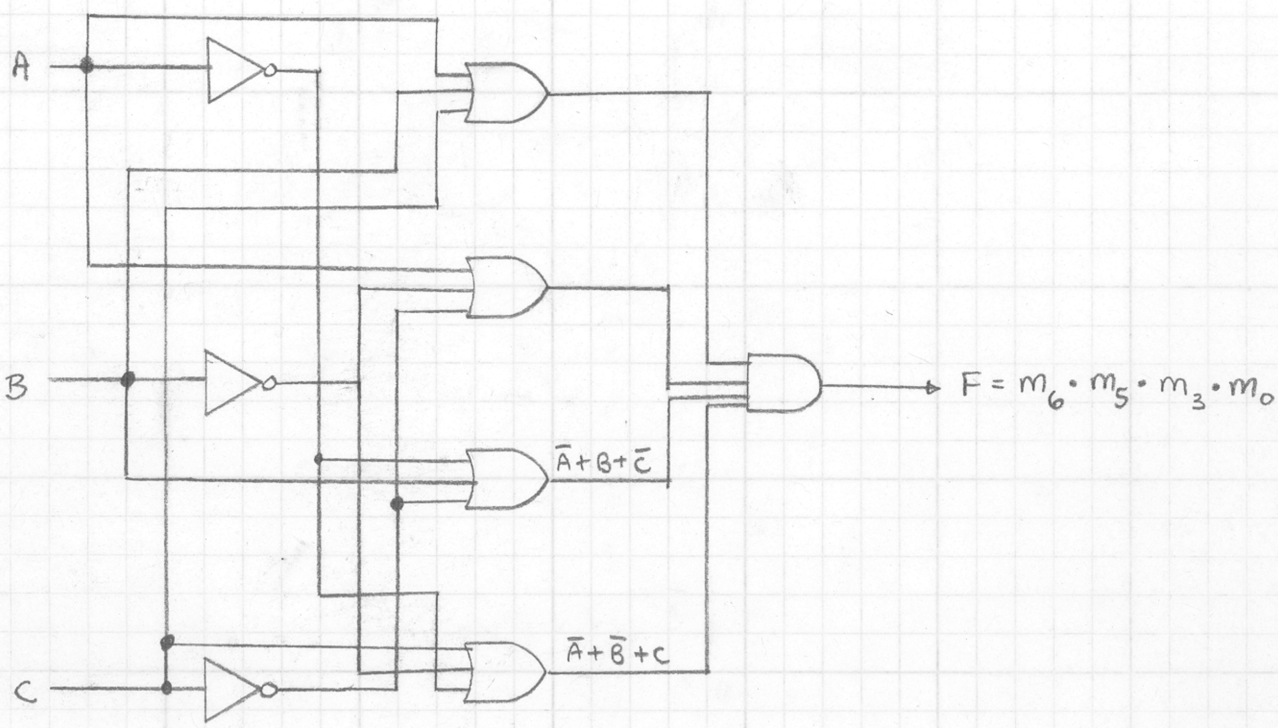
Using De Morgan's Theorem:

$$F = \overline{(AB\bar{C})}\,\overline{(A\bar{B}C)}\,\overline{(\bar{A}BC)}\,\overline{(\bar{A}\bar{B}\bar{C})} = \underbrace{(\bar{A}+\bar{B}+C)}_{\text{maxterm}}(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+C)$$

Truth Table for F Same as Above

A	B	C	F	
0	0	0	0	$\rightarrow \bar{A}\bar{B}\bar{C} \equiv m_0$
0	0	1	1	
0	1	0	1	
0	1	1	0	$\rightarrow \bar{A}BC \equiv m_3$
1	0	0	1	
1	0	1	0	$\rightarrow A\bar{B}C \equiv m_5$
1	1	0	0	$\rightarrow AB\bar{C} \equiv m_6$
1	1	1	1	

Logic Gate Implementation of F using Product of Sums



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III. Karnaugh Maps

Karnaugh maps are a graphical method of simplifying Boolean expressions.

Ex: $F = \underbrace{\bar{A}\bar{B}\bar{C}}_{m_0} + \underbrace{\bar{A}\bar{B}C}_{m_1} + \underbrace{\bar{A}B\bar{C}}_{m_2} + \underbrace{AB\bar{C}}_{m_6}$

First, try to simplify using Boolean algebra:

$$F = \bar{A}\bar{B}\underbrace{(\cancel{\bar{C}+C})}_{1} + B\bar{C}\underbrace{(\cancel{\bar{A}+A})}_{1} = \bar{A}\bar{B} + B\bar{C}$$

Karnaugh Maps of 3 Variables

F is sum of products (minterms)

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Ex: $F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C}$

A \ BC	00	01	11	10
0	1	1		1
1				1

$\bar{A}\bar{C}$

$\bar{A}\bar{B}$

$B\bar{C}$

Only need enough terms to "cover" all 1's:

$F = \bar{A}\bar{B} + B\bar{C}$

For minterms, think where is $F = \underline{1}$?

Karnaugh Maps of 4 Variables

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_3	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

<u>Ex:</u> Find expression corresponding to following Karnaugh map.

AB \ CD	00	01	11	10
00	1			1
01	1	1		
11	1	1		
10	1			1

$$F = \overline{C}\,\overline{D} + B\overline{C} + \overline{B}C\overline{D}$$

<u>Ex:</u> Find expression for following Karnaugh map.

AB \ CD	00	01	11	10
00				
01	1	1		
11			1	
10	1	1		

$$F = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + ABCD$$

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Karnaugh Maps for Maxterms (Product of Sums)

Group "0"s in Karnaugh Map.

Ex:

$$F_1 = \underbrace{(A+B+C+D)}_{m_0}\underbrace{(A+B+C+\bar{D})}_{m_1}\underbrace{(A+\bar{B}+C+D)}_{m_4}\cdot$$

$$\underbrace{(A+\bar{B}+\bar{C}+D)}_{m_6}\underbrace{(A+\bar{B}+\bar{C}+\bar{D})}_{m_7}\underbrace{(\bar{A}+B+C+\bar{D})}_{m_9}\cdot$$

$$\underbrace{(\bar{A}+\bar{B}+C+D)}_{m_{12}}\underbrace{(\bar{A}+\bar{B}+\bar{C}+D)}_{m_{14}}\underbrace{(\bar{A}+\bar{B}+\bar{C}+\bar{D})}_{m_{15}}$$

Truth Table for F_1

A	B	C	D	F_1		
0	0	0	0	0	m_0	$A+B+C+D$
0	0	0	1	0	m_1	$A+B+C+\bar{D}$
0	0	1	0	1	m_2	
0	0	1	1	1	m_3	
0	1	0	0	0	m_4	$A+\bar{B}+C+D$
0	1	0	1	1	m_5	
0	1	1	0	0	m_6	$A+\bar{B}+\bar{C}+D$
0	1	1	1	0	m_7	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	1	m_8	
1	0	0	1	0	m_9	$\bar{A}+B+C+\bar{D}$
1	0	1	0	1	m_{10}	
1	0	1	1	1	m_{11}	
1	1	0	0	0	m_{12}	$\bar{A}+\bar{B}+C+D$
1	1	0	1	1	m_{13}	
1	1	1	0	0	m_{14}	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	0	m_{15}	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$

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AB \ CD	00	01	11	10
00	0	0		
01	D		0	0
11	D		0	0
10		0		

$$F1 = (\overline{B} + D)(\overline{B} + \overline{C})(B + C + \overline{D})(A + B + C)$$

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I. Last Word on Karnaugh Maps

What does it mean to simplify an expression?

Ex: Find simplest expression for following Karnaugh map:

A \ BC	00	01	11	10
0	1	1		1
1		1	1	

One grouping: $\bar{A}\bar{C} + AC + \bar{A}\bar{B}$

3 terms, 6 "literals" (signals such as $A, B, \bar{B}$, etc.)

A \ BC	00	01	11	10
0	1	1		1
1		1	1	

Alternative grouping: $AC + \bar{A}\bar{C} + \bar{B}C$

3 terms, 6 literals

A \ BC	00	01	11	10
0	1	1		1
1		1	1	

$\bar{A} \cdot (\bar{B} + \bar{C}) + AC$ 5 literals, but no longer sum of products

A \ BC	00	01	11	10
0			0	
1	0			0

$(A + \bar{B} + \bar{C}) \cdot (\bar{A} + C)$ 5 literals

Consider circuit implementation: (starting with A, B, C)

$\bar{A}\bar{C} + AC + \bar{A}\bar{B}$ → 3 inverters, 3 2-input ANDs, 1 3-input OR

$AC + \bar{A}\bar{C} + \bar{B}C$ → 3 inverters, 3 2-input ANDs, 1 3-input OR

$\bar{A} \cdot (\bar{B} + \bar{C}) + AC$ → 3 inverters, 2 2-input ANDs, 2 2-input ORs

$(A + \bar{B} + \bar{C}) \cdot (\bar{A} + C)$ → 3 inverters, 1 2-input AND, 1 2-input OR, 1 3-input OR

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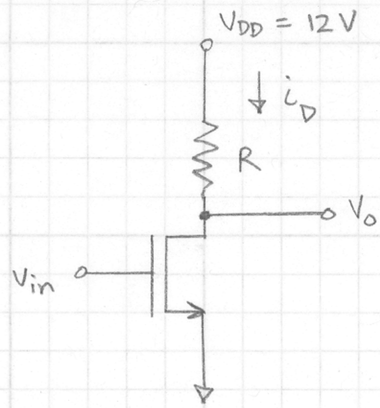
Need more information to decide which expression to choose! Must specify either sum of products, product of sums, smallest number of literals (signals), or simplest circuit implementation.

II. MOSFET Logic Gates

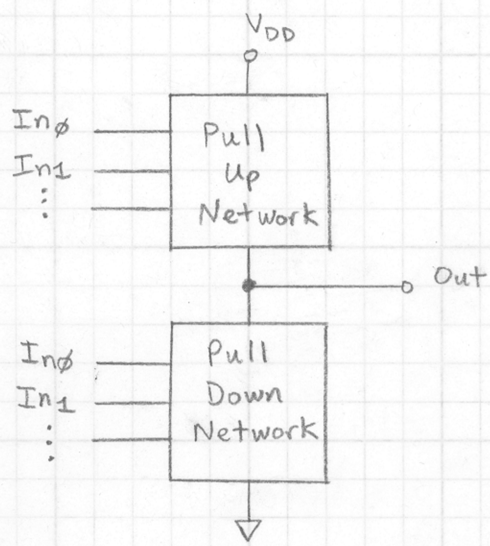
NMOS Inverter with Resistor Pull-Up



Truth Table:

V_{in}	V_{out}
0	1
1	0

General Logic Circuit Concept



In∅	In_1	...	In_N	Out	
0	0	...	1	0	← pull down network
1	0	...	1	0	
1	1	...	1	1	← pull up network

Each network can be implemented with transistors, resistors, sometimes diodes, etc.

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