EEC 216 Winter 2007 Midterm

Rajeevan Amirtharajah
Dept. of Electrical and Computer Engineering
University of California, Davis

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Name: Solutions

Instructions: This test consists of 3 problems and 10 pages, including the cover sheet. Please make sure that you have all of them. This is an open-book, open-notes test. State any assumptions you make and show complete work to receive credit. The time limit is 50 minutes. The problems are weighted as shown below:

Grading:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Maximum</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
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<td>3</td>
<td>18</td>
<td></td>
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<tr>
<td>Total</td>
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</table>
1 Logic Gate Design

For this problem, use the transistor parameters in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NMOS</th>
<th>PMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>1 V</td>
<td>-1 V</td>
</tr>
<tr>
<td>$k' = \mu C_{ox}$</td>
<td>300 $\mu$A/V$^2$</td>
<td>100 $\mu$A/V$^2$</td>
</tr>
<tr>
<td>$W_{min}$</td>
<td>2 $\mu$m</td>
<td>2 $\mu$m</td>
</tr>
<tr>
<td>$L_{min}$</td>
<td>1 $\mu$m</td>
<td>1 $\mu$m</td>
</tr>
</tbody>
</table>

Table 1: Problem 1 Transistor Parameters.

Problem 1.1 (6 points) Static CMOS. Design a static CMOS gate to implement the following logical expression using a minimum number of transistors:

$$F = A \cdot \overline{D} + B \cdot C \cdot \overline{D}$$

$$= \overline{(A + B \cdot C)} \cdot \overline{D}$$

$P_{UN}$: 3 pts,

$P_{DN}$: 3 pts.
Problem 1.2 (4 points) Static CMOS Sizing. Size the transistors in the circuit you designed for Problem 1.1 such that the worst case rise and fall times are equal while minimizing the input capacitance to the logic gate. Label the sizes in your schematic for Problem 1.1.

2 PMOS in series = 3 NMOS in series
Size PMOS 2x min. sized NMOS
2UN sizes: 2 pts.
2DN sizes: 2 pts.

Problem 1.3 (4 points) Pseudo NMOS Design. Design a pseudo-NMOS gate to implement Equation 1 such that the pullup network has the same resistance as the worst case pullup resistance for Problem 1.2 and \( V_{OL} \leq 0.2V_{DD} \) (assume \( V_{DD} = 2V \)). Assume that the PMOS and NMOS devices are in the saturation regime of operation at all times and neglect channel length modulation.

\[
\text{Need } \frac{W_n}{W_p} \geq \frac{4}{3} \quad \text{(voltage divider)}
\]
\[
\Rightarrow W_n \geq 12 \left( \frac{100 \mu A/V^2}{300 \mu A/V^2} \right) (2)
\]
\[
\geq 8
\]
Gate: 2 pts.
Sizes: 2 pts.
Problem 1.4 (3 points) Discussion. Which of the two circuits is better for low power design? Justify your answer.

Static CMOS (1 pt.)

No static power for any input combination. (2 pts.)

Less input capacitance.
Figure 1: Three signal shielded bus. Plan view (top) and cross-section (bottom).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Metal 1</th>
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<tbody>
<tr>
<td>Sheet Resistance ($R_{sq}$)</td>
<td>0.1 $\Omega$/sq.</td>
</tr>
<tr>
<td>Spacing ($S$)</td>
<td>500 nm</td>
</tr>
<tr>
<td>Thickness ($T$)</td>
<td>3 $\mu$m</td>
</tr>
<tr>
<td>Height ($H$)</td>
<td>500 nm</td>
</tr>
<tr>
<td>Dielectric Constant ($\epsilon_r$)</td>
<td>$4.3 \times 8.85\times10^{-12}$ F/m</td>
</tr>
<tr>
<td>Length ($L$, on figure)</td>
<td>1.0 cm</td>
</tr>
<tr>
<td>Width ($W$)</td>
<td>4.0 $\mu$m</td>
</tr>
</tbody>
</table>

Table 2: Problem 2 Interconnect Parameters.

2 Bus Power

Figure 1 shows a three signal bus driven by positive edge-triggered flip-flops with two ground shields on the metal 1 layer. Assume that the dimensions shown in Figure 1 and listed in Table 2 hold for every line in the bus.
Problem 2.1 (3 points) Coupling Capacitance. Calculate the line-to-line capacitance $C_c$. Ignore any contribution of fringing fields.

\[
C_c = \varepsilon_r \frac{T_L}{S} = \frac{4.3 \left( 8.85 \times 10^{-12} \text{ F/m} \right) \left( 3 \times 10^{-6} \text{ m} \right) \left( 1 \times 10^{-2} \text{ m} \right)}{500 \times 10^{-9} \text{ m}} \quad (3 \text{ pts.})
\]

\[= \frac{2.28 \text{ pF}}{}
\]

Problem 2.2 (3 points) Substrate Capacitance. Using the same data as Problem 2.1, calculate the line-to-substrate capacitance $C_L$. Ignore any contribution of fringing fields.

\[
C_L = \varepsilon_r \frac{W_L}{H} = \frac{4.3 \left( 8.85 \times 10^{-12} \text{ F/m} \right) \left( 4 \times 10^{-6} \text{ m} \right) \left( 1 \times 10^{-2} \text{ m} \right)}{500 \times 10^{-9} \text{ m}} \quad (3 \text{ pts.})
\]

\[= \frac{3.04 \text{ pF}}{}
\]
Problem 2.3 (6 points) Worst Case Power. Draw a timing diagram of the data pattern on signals A, B, and C, which results in the worst case power dissipation on line B. Calculate this worst case power based on the $C_C$ and $C_L$ you calculated earlier. Assume the flip-flops are clocked at 200 MHz with $V_{DD} = 1.5\, \text{V}$.

\[
C_{\text{tot}} = 2 \cdot 2 \cdot C_C + C_L = 12.16 \, \text{pF}
\]

\[
P = C_{\text{tot}} f V_{DD}^2 = (2 \cdot 2 \cdot C_C + C_L) (100 \, \text{MHz}) (1.5 \, \text{V})^2
\]

\[
P = 2.74 \, \text{mW}
\]

Problem 2.4 (3 points) Clock Skew. Suppose that there is significant clock skew at the flip-flops driving A, B, and C. How would this affect the worst case power you computed in Problem 2.3? Justify your answer.

Clock skew decreases the peak power of driving line B since A, B, and C don't switch simultaneously, but the total capacitance charged and discharged is the same, so total worst case power is unchanged. (3 pts.)
3 Manchester Carry Chain

Problem 3.1 (4 points) Static Activity Factor. Figure 2 shows a dynamic gate implementation of a carry-generation circuit for a full adder cell. Ignoring signal Φ and the circuit inside the dotted lines, calculate the activity factor (0→1 transition probability) for the following signals, assuming the full adder inputs $A$, $B$, and $C_i$ are independent, identically distributed binary random variables:

\[
G = A \cdot B \\
P = A \oplus B \\
C_o = G + P \cdot C_i \\
= A \cdot B + B \cdot C_i + A \cdot C_i
\]

\[
G: \quad P_{0\rightarrow1} = \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16} \\
\text{[2 pts.]} \\
\]

\[
P: \quad P_{0\rightarrow1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\
\]

\[
C_o: \quad P_{0\rightarrow1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \quad \text{(2 pts.)}
\]

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C_i$</th>
<th>$C_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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Figure 2: Manchester carry chain dynamic gate. Transistor sizes are shown as ratio $W/L$. 
Problem 3.2 (6 points) Dynamic Logic Timing. Sketch timing diagrams of signals $\Phi$, $\overline{G}$, $\overline{P}$, and $C_i$ for both input scenarios which result in $C_o$ evaluating to logic 1. Ignore the circuit inside the dotted lines.

Problem 3.3 (3 points) Dynamic Activity Factor. What is the activity factor for node $C_o$, taking into account the action of signal $\Phi$? Ignore the circuit inside the dotted lines.

Dynamic ck+ precharge low, therefore $P_{O \rightarrow 1} = \text{Prob}(1) = \frac{1}{2}$

(3 pts.)
Problem 3.4 (2 points) Mystery Circuit. What is the intended purpose of the circuit within the dotted lines?

Feedback ckt should keep $C_0$ node low when $C_0$ evaluates to $\phi$ and $\bar{\phi}$ low during evaluate phase.

Problem 3.5 (3 points) Suppose that the switching threshold of the inverter in Figure 2 is $V_{DD}/2$. Assume that the mobility of the PMOS devices is $\frac{2}{5}$ of the mobility of the NMOS devices, i.e. $5\mu_p = 2\mu_n$. Including the circuit within the dotted lines, will the transistor sizings ($W/L$) shown in Figure 2 result in a working circuit? Justify your answer.

Need $R_{\text{pullup}} \leq R_{\text{mo}}$ for $C_0$ to get above inverter threshold $(V_{DD}/2)$;

$$\frac{1}{2} \left(\frac{12}{L}\right) \mu_p \geq \frac{4}{1} \mu_n \Rightarrow 6 \left(\frac{2}{5}\right) \mu_n \geq \frac{4}{1} \mu_n \Rightarrow 2.4 \leq 4$$ (2 pts.)

Ckt will not work! (1 pt.)