EEC 210 Fall 2008 Midterm

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Name:

Instructions: This test consists of 4 problems and 14 pages, including the cover sheet. Please make sure that you have all of them. This is an open-book, open-notes test. State any assumptions you make and show complete work to receive credit. The time limit is 80 minutes. The problems are weighted as shown below:

Grading: Solutions

Problem	Maximum	Score
1	13	
2	13	
3	12	
4	12	
Total	50	

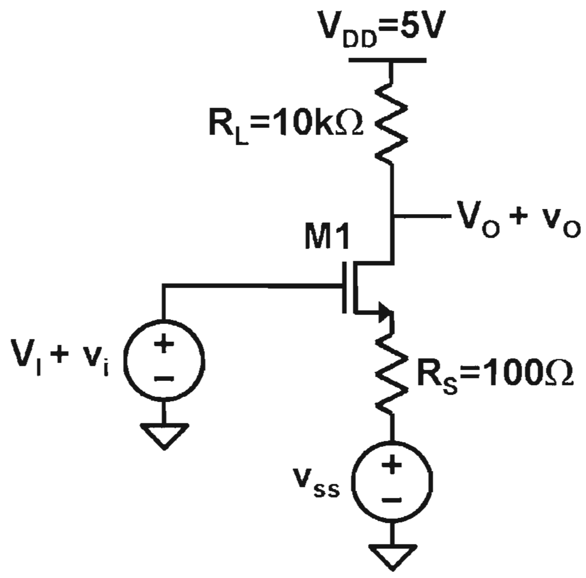


Figure 1: Resistive load amplifier.

1 Resistive Load Amplifier

Figure 1 shows a resistively loaded amplifier. Source v_{ss} is a small-signal source only. For this problem, use the transistor parameters in Table 1.

Parameter	NMOS
V_t	1 V
L_d	0
X_d	0
k'	300 μA/V^2
γ	0
W/L	16/3
λ	1/100 V^{-1}

Table 1: Problem 1 Transistor Parameters.

Problem 1.1 (**4 points**) Find the large-signal input voltage V_I such that the large-signal output voltage $V_O = 3$V. You may ignore channel length modulation for this part.

$V_O = 3V \Rightarrow I_D = \frac{V_{DD} - V_O}{R_L} = \frac{2V}{10k\Omega} = 0.2mA$, $V_{GS} = V_I + I_D R_S$ (1pt.)

$0.2mA = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2 = \frac{300\mu A/V^2}{2}\left(\frac{16}{3}\right)(V_I - (0.2mA)(100\Omega) - 1V)^2$ (2pts.)

$\boxed{V_I = 1.52\ V}$ (1pt.)

Problem 1.1 (cont.)

or $V_{ov} = \sqrt{\frac{2 I_D}{k'(W/L)}} = \sqrt{\frac{2(200\mu A)}{300\frac{\mu A}{V^2}\left(\frac{16}{3}\right)}} = 0.5V$ (1pt.)

$V_I = I_D R_S + V_t + V_{ov} = (200\mu A)(100\Omega) + 1V + 0.5V$ (1pt.)

$\boxed{V_I = 1.52\ V}$ (1pt.)

Problem 1.2 **(4 points)** Find the small-signal voltage gain $a_{v1} = \frac{v_o}{v_i}$.

CS Amp w/ Degeneration

$a_{v1} = G_m (R_o \| R_L)$ $g_m = \sqrt{2 I_D k'\left(\frac{W}{L}\right)} = \sqrt{2(200\mu A)\left(300\frac{\mu A}{V^2}\right)\left(\frac{16}{3}\right)}$ (1pt.)

$= 0.8\ mS$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01\ V^{-1})(200\mu A)} = 500\ k\Omega$

$G_m = \frac{g_m}{1 + (g_m + \underbrace{g_{mb}}_{0})R_S + \frac{R_S}{r_o}} \simeq \frac{g_m}{1 + (g_m)R_S} = 741\ \mu S$ (1pt.)

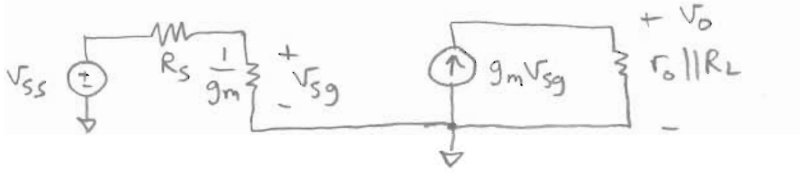
$R_o = R_S + r_o(1 + g_m R_S) \simeq r_o(1 + g_m R_S) = 540\ k\Omega$ (1pt.)

$R_o \| R_L = 9818\ \Omega$

$a_{v1} = (741\ \mu S)(9818\ \Omega) = \boxed{7.28}$ (1pt.)

Problem 1.3 (**4 points**) Find the small-signal voltage gain $a_{v2} = \frac{v_o}{v_{ss}}$.

CG Amp (2 pts.)



$$\frac{v_o}{v_{ss}} = \cancel{g_m}(r_o || R_L) \frac{1/\cancel{g_m}}{R_S + 1/g_m} = \frac{(r_o || R_L)}{R_S + 1/g_m} \quad \text{(1 pt.)}$$

$$a_{v2} = \frac{(500\,k\Omega \,||\, 10\,k\Omega)}{100\,\Omega + \frac{1}{800\,\mu S}} = \frac{9800}{1350} = \boxed{7.26} \quad \text{(1 pt.)}$$

Problem 1.4 (**1 point**) Find the power supply rejection ratio $\left|\frac{a_{v1}}{a_{v2}}\right|$.

$$PSRR^- = \frac{7.28}{7.26} = \boxed{1.00} \; !$$

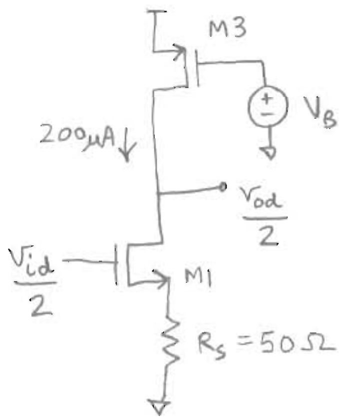
2 Differential Amplifier

Figure 2 shows the circuit schematic for a differential amplifier. For this problem, use the transistor parameters in Table 2. Assume that all transistors are biased in saturation and the following circuit parameters: $I_{TAIL} = 400\mu\text{A}$, $R_{TAIL} = 500\ \text{k}\Omega$, $R_S = 50\Omega$, $(W/L)_1 = (W/L)_2 = 10, (W/L)_3 = (W/L)_4 = 8$.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	400 μA/V^2	100 μA/V^2
χ	0.25	0.25
λ	0.01 V^{-1}	0.01 V^{-1}

Table 2: Problem 2 Transistor Parameters.

Problem 2.1 (**4 points**) Draw and label the small-signal differential-mode half circuit, and find the differential-mode gain, $\frac{v_{od}}{v_{id}}$, where $v_{id} = v_{i1} - v_{i2}$ and $v_{od} = v_{o1} - v_{o2}$.



(1pt.)

CS Amp w/ degen.

$I_D = 200\mu A$

$$\frac{v_{od}}{v_{id}} = G_m (R_o \| r_{o3})$$

$$g_m = \sqrt{k'\left(\frac{W}{L}\right) I_D \cdot 2} = \sqrt{2 \cdot 400 \frac{\mu A}{V^2} (10)(200\mu A)}$$

$$= 1.26\ mS$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{(0.01)V^{-1}(200\mu A)} = r_{o3} = 500\ k\Omega \qquad (1pt.)$$

$$G_m = \frac{g_m}{1 + (1+\chi) g_m R_s + \frac{R_s}{r_o}} \simeq \frac{g_m}{1 + (1+\chi) g_m R_s} = 1.17 mS \qquad (1pt.)$$

$$R_o = R_s + r_{o1}\left[1 + (1+\chi) g_m R_s\right] = 539\ k\Omega$$

$$\frac{v_{od}}{v_{id}} = (1.17mS)(539k\Omega \| 500k\Omega) = \boxed{303} \qquad (1pt.)$$

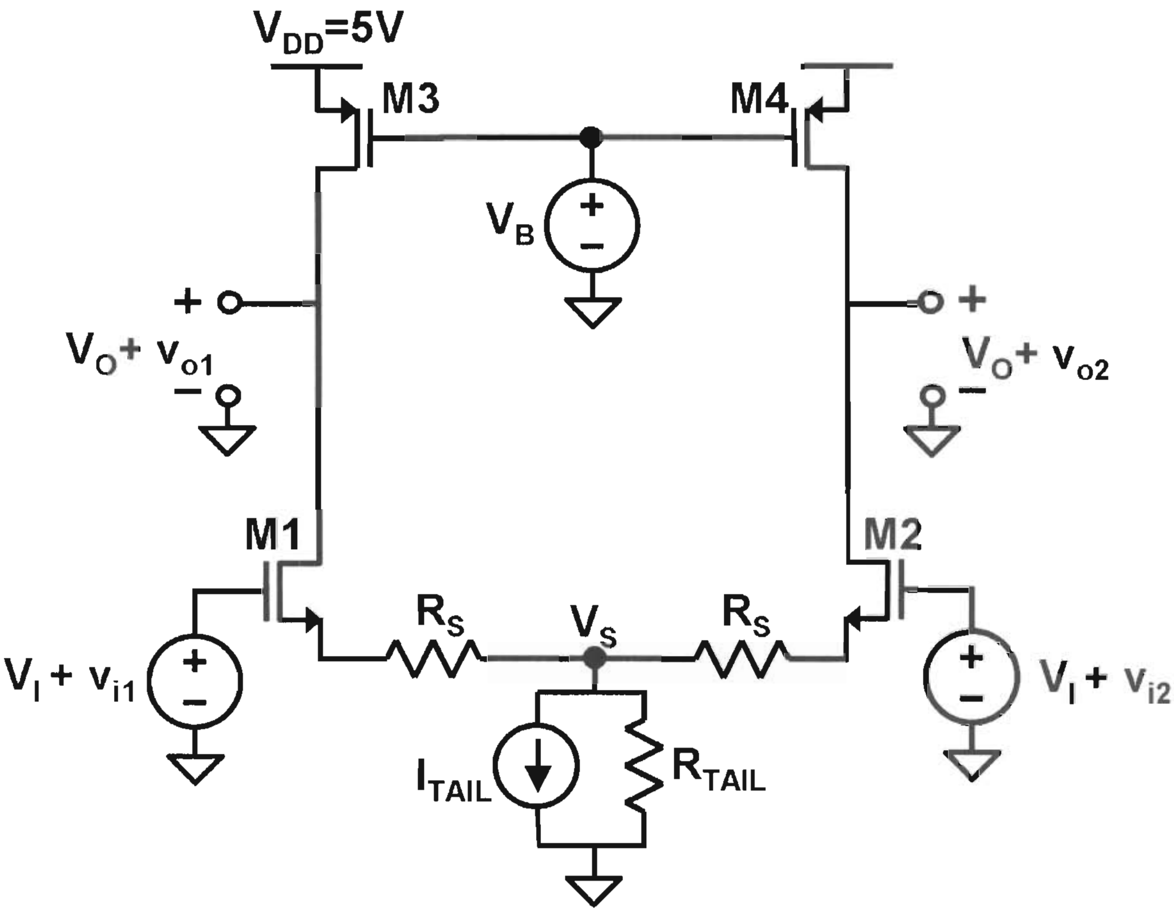
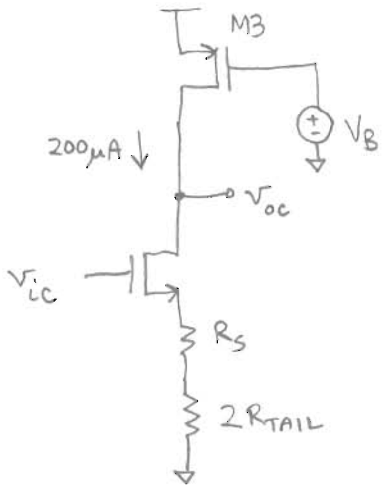


Figure 2: Differential amplifier.

Problem 2.2 (**4 points**) Draw and label the small-signal common-mode half circuit, and find the common-mode gain, $\frac{v_{oc}}{v_{ic}}$, where $v_{ic} = 0.5(v_{i1} + v_{i2})$ and $v_{oc} = 0.5(v_{o1} + v_{o2})$.



(1 pt.)

CS Amp w/ degeneration

$$\frac{v_{oc}}{v_{ic}} = G_m (R_o \| r_{o3})$$

$$G_m = \frac{g_m}{1 + (1+\chi) g_m (R_S + 2R_{TAIL}) + \frac{R_S + 2R_{TAIL}}{r_{o1}}} = 798\ nS \quad (1\ pt.)$$

$$R_o = R_S + 2R_{TAIL} + r_o(1 + (1+\chi) g_m (R_S + 2R_{TAIL})) = 789\ M\Omega \quad (1\ pt.)$$

$$\frac{v_{oc}}{v_{ic}} \cong G_m r_{o3} = (798\ nS)(500\ k\Omega) = \boxed{0.399} \quad (1\ pt.)$$

Problem 2.3 (**2 points**) Find the common-mode rejection ratio (CMRR).

$$\left| \frac{v_{od}/v_{id}}{v_{oc}/v_{ic}} \right| = \frac{303}{0.399} = \boxed{760}$$

(1 pt.) (1 pt.)

Problem 2.4 (3 points) Find the large-signal common-mode input voltage V_I assuming $V_{DS1} = V_{DS2} = 1.5\text{V}$ and the current through R_{TAIL} is $2\mu\text{A}$.

$R_{TAIL} = 500\,\text{k}\Omega$, $I_{R_{TAIL}} = 2\mu A \Rightarrow V_S = 1\,V$

$$I_{DS1} = (200\,\mu A + 1\,\mu A) = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{GS1} - V_t)^2(1 + \lambda V_{DS1})$$ (2 pts.)

$$= \frac{k'}{2}\left(\frac{W}{L}\right)(V_I - V_S - V_t)^2(1 + \lambda V_{DS1})$$

$$= \frac{400\,\mu A/V^2}{2}(10)(V_I - 1V - 1V)^2(1 + 0.01\,V^{-1}(1.5V))$$

Solve for $V_I = V_{ov1} + V_S + V_t = 0.315\,V + 2V$

$V_I = \boxed{2.32\,V}$ (ignore R_S since $V_{R_S} = 10\,mV$) (1 pt.)

or

$2.33\,V$ if IR_S included

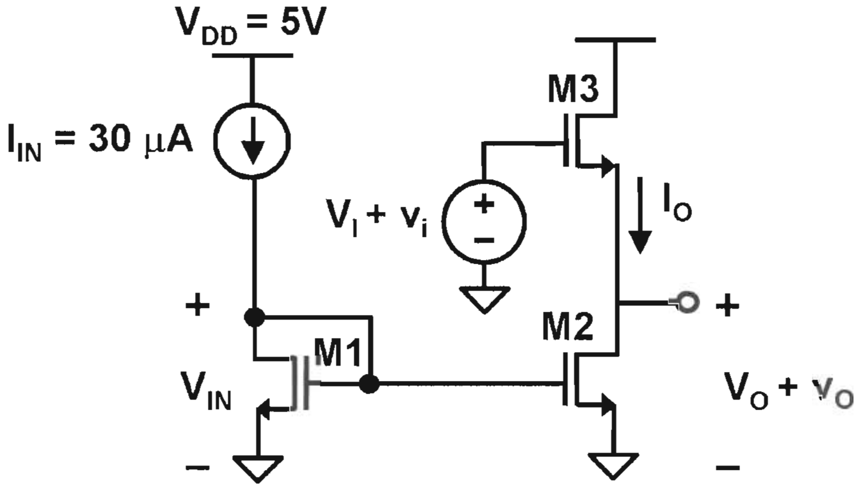


Figure 3: Active load amplifier.

3 Active Load Amplifier

Figure 3 shows an actively loaded amplifier circuit. For this problem, use the transistor parameters in Table 3. For the circuit in Figure 3, assume the following circuit parameters: $(W/L)_1 = 2$, $(W/L)_2 = 20$, and $(W/L)_3 = 30$.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	300 μA/V^2	100 μA/V^2
χ	0.2	0
λ	0.02 V^{-1}	0.02 V^{-1}

Table 3: Problem 3 Transistor Parameters.

Problem 3.1 (4 points) Find the large-signal input voltage V_I required to make the systematic current gain error 0. You can ignore λ when calculating the large-signal current.

$$V_{IN} = V_t + V_{ov_1} = V_t + \sqrt{\frac{2I_D}{k'\left(\frac{W}{L}\right)_1}} = 1V + \sqrt{\frac{2 \cdot 30\mu A}{\frac{300\mu A}{V^2}(2)}} = 1.32\,V \qquad (1\text{ pt.})$$

$$I_o = 10 \cdot I_{IN} = 300\mu A \qquad (1\text{ pt.})$$

Want $V_o = V_{IN}$ for 0 systematic offset $\Rightarrow V_o = V_I - V_{GS3} \Rightarrow V_I = V_{IN} + V_{GS3}$

$$V_{GS3} = V_t + \sqrt{\frac{2I_o}{k'\left(\frac{W}{L}\right)_3}} = 1V + \sqrt{\frac{2(300\mu A)}{\frac{300\mu A}{V^2}(30)}} = 1.26V \qquad (2\text{pts.})$$

Problem 3.1 (cont.)

$V_I = \boxed{2.57\,V}$ (1 pt.)

Problem 3.2 (**3 points**) Find the minimum large-signal input voltage $V_I(MIN)$.

$V_{I(MIN)} = V_{O(MIN)} + V_{GS3} = V_{OV2} + V_{GS3}$ (1 pt.)

$V_{OV2} = V_{OV1} = \sqrt{\dfrac{2\cdot(30\mu A)}{\frac{300\mu A}{V^2}(2)}} = 0.32V$ (1 pt.)

$V_{I(MIN)} = 1.26V + 0.32V = \boxed{1.58\,V}$ (1 pt.)

Problem 3.3 (**2 points**) Find the output resistance R_o.

Source Follower w/ active load

$$R_o = \frac{1}{g_m + g_{mb} + \frac{1}{r_o} + \frac{1}{R_L}} \quad \text{(1 pt.)}$$

$$g_{m3} = \sqrt{2k'\left(\frac{W}{L}\right)_3 I_o} = \sqrt{2\left(300\frac{\mu A}{V^2}\right)(30)(300\mu A)}$$

$$= 2.32 \text{ mS}$$

$$r_{o3} = r_{o2} = \frac{1}{\lambda I_o} = \frac{1}{(0.02V^{-1})(300\mu A)}$$

$$= 167 \text{ k}\Omega$$

$$R_o = \frac{1}{(1+\chi)g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o2}}} = \frac{1}{(1+0.2)(2.32\text{ mS}) + \frac{2}{167\text{ k}\Omega}} = \boxed{357\ \Omega} \quad \text{(1 pt.)}$$

Problem 3.4 (**3 points**) Find the small-signal voltage gain $a_v = \frac{v_o}{v_i}$.

$$a_v = \frac{v_o}{v_i} = g_m R_o = (2.32\text{ mS})(357\ \Omega) = \boxed{0.828} \approx \frac{1}{1+\chi}$$

(2 pts.)

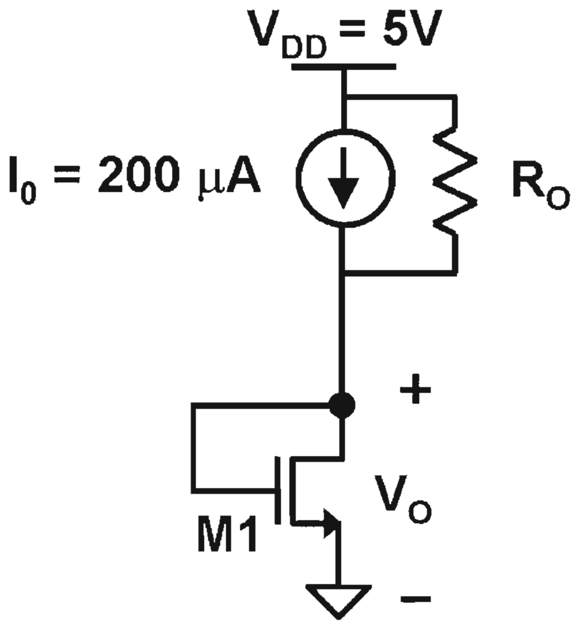


Figure 4: Voltage reference.

4 Voltage Reference

Figure 4 shows a proposed temperature and supply voltage-independent voltage reference circuit. For this problem, use the transistor parameters in Table 4. Note the temperature dependence of V_t and k'.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
$\frac{dV_t}{dT}$	-2.3 mV/K	-3 mV/K
L_d	0	0
X_d	0	0
k'	$100\ \mu A/V^2 \left(\frac{T}{300K}\right)^{-1.5}$	$50\ \mu A/V^2$
γ	0	0
λ	0	0

Table 4: Problem 4 Transistor Parameters.

Problem 4.1 (5 points) Find W/L such that $\frac{dV_O}{dT} = 0$ at $T = 300\text{K}$ in the limit as $R_O \to \infty$.

$$V_o = V_{GS1} = V_t + V_{ov1} = V_t + \sqrt{\frac{2I_o}{k'(W/L)_1}} = V_t + \sqrt{\frac{2I_o}{(W/L)_1}}\,(k')^{-1/2}$$

(2 pts.)

$$V_o = V_t + \sqrt{\frac{2I_o}{(W/L)_1}}\;\frac{(100\,\mu A/V^2)^{-1/2}}{(300\,K)^{3/4}}\;T^{3/4}$$

$$\frac{dV_o}{dT} = \frac{dV_t}{dT} + \sqrt{\frac{2I_o}{(W/L)_1}}\;\frac{(100\,\mu A/V^2)^{-1/2}}{(300\,K)^{3/4}}\;\frac{3T^{-1/4}}{4}$$

(2 pts.)

$$\left.\frac{dV_o}{dT}\right|_{T=300K} = 0 = -2.3\,mV/K + \frac{5\,mV/K}{\sqrt{(W/L)_1}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{5\,mV/K}{2.3\,mV/K}\right)^2$$

$$\left(\frac{W}{L}\right)_1 = \boxed{4.73}$$ (1 pt.)

Problem 4.2 (3 points) Find the target output voltage V_O at the operating point found in Problem 4.1.

$$V_o = V_t + \sqrt{\frac{2 I_o}{k'(W/L)}} = 1V + \sqrt{\frac{2 \cdot 200\mu A}{100\frac{\mu A}{V^2}(4.73)}} = \boxed{1.92\ V} \quad (1\ pt.)$$

(2 pts.)

Problem 4.3 (4 points) Suppose $R_O = 100\text{k}\Omega$. Find the approximate power supply gain $\frac{\Delta V_O}{\Delta V_{DD}}$ assuming $V_{DD} \gg V_O$. Hint: $(1+x)^{\frac{1}{2}} \approx 1 + \frac{x}{2}$ for small x.

$$V_o = V_t + \sqrt{\frac{2 I_{TOT}}{k'(W/L)}} = V_t + \sqrt{\frac{2}{k'(W/L)}}\left(I_o + \frac{V_{DD} - V_o}{R_o}\right)^{1/2} \quad (1\ pt.)$$

$$\approx V_t + \sqrt{\frac{2 I_o}{k'(W/L)}}\left(1 + \frac{V_{DD}}{2 I_o R_o}\right) \quad (1\ pt.)$$

$$\frac{\Delta V_o}{\Delta V_{DD}} \approx \sqrt{\frac{2 I_o}{k'(W/L)}}\frac{1}{2R_o} = \sqrt{\frac{2(200\mu A)}{100\frac{\mu A}{V^2}(4.73)}} \cdot \frac{1}{2 \cdot 100k\Omega(200\mu A)} \quad (1\ pt.)$$

$$= \boxed{2.3\%} \quad (1\ pt.)$$

Small-signal approach:

$$\frac{\Delta V_o}{\Delta V_{DD}} = \frac{1/g_m}{R_o + 1/g_m}$$

$$\approx \boxed{2.06\%}$$

$$g_m = \sqrt{2k'\left(\frac{W}{L}\right)I_{TOT}} = \sqrt{2\left(100\frac{\mu A}{V^2}\right)(4.73)\left(200\mu A + \frac{5V}{100k\Omega}\right)}$$

$$= 486\mu S$$