EEC 210 Fall 2005 Midterm

Rajeevan Amirtharajah
Dept. of Electrical and Computer Engineering
University of California, Davis

November 15, 2005

Name: Solutions

Instructions: This test consists of 4 problems and 13 pages, including the cover sheet. Please make sure that you have all of them. This is an open-book, open-notes test. State any assumptions you make and show complete work to receive credit. The time limit is 80 minutes. The problems are weighted as shown below

Grading:

Problem	Maximum	Score
1	12	
2	16	
3	10	
4	12	
Total	50	

Typo: Problem 4, $V_{DD} = 6V$

1 CMOS Inverter Amplifier

Figure 1 shows a CMOS inverter biased as a linear amplifier. For this problem, use the transistor parameters in Table 1.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	300 μA/V^2	100 μA/V^2
γ	0	0
W/L	2	6
λ	0.1 V^{-1}	0.1 V^{-1}

Table 1: Problem 1 Transistor Parameters.

Problem 1.1 (3 points) Assume $V_I = 1.5$ V. For what range of V_O will the circuit best act as an amplifier? What is the output voltage swing?

Want M1 + M2 in saturation:

$V_{GS1} = V_I = 1.5\text{ V}, \quad V_{OV1} = 0.5\text{ V} \Rightarrow V_{out} \geq 0.5\text{ V}$

$V_{GS2} = V_{DD} - V_I = 1.5\text{ V}, \quad V_{OV2} = 0.5\text{ V} \Rightarrow V_{out} \leq 2.5\text{ V}$

$\boxed{0.5\text{ V} \leq V_{out} \leq 2.5\text{ V}}$

Output Swing = 2V as amplifier
= 3V $(0 \leq V_O \leq V_{DD})$ for output levels

Problem 1.2 (2 points) Find the power dissipation assuming the bias conditions in Problem 1.1.

$$I_{D1} = I_{D2} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_n (V_I - V_{tn})^2 = \frac{300\,\mu A/V^2}{2}(2)(1.5\text{ V} - 1)^2$$

$= 75\,\mu A$ $(86.2\,\mu A$ if $\lambda = 0.1\text{ V}^{-1}$, $V_{DS} = 1.5\text{ V})$

$\boxed{P = I_{D1}V_{DD} = 225\,\mu W}$

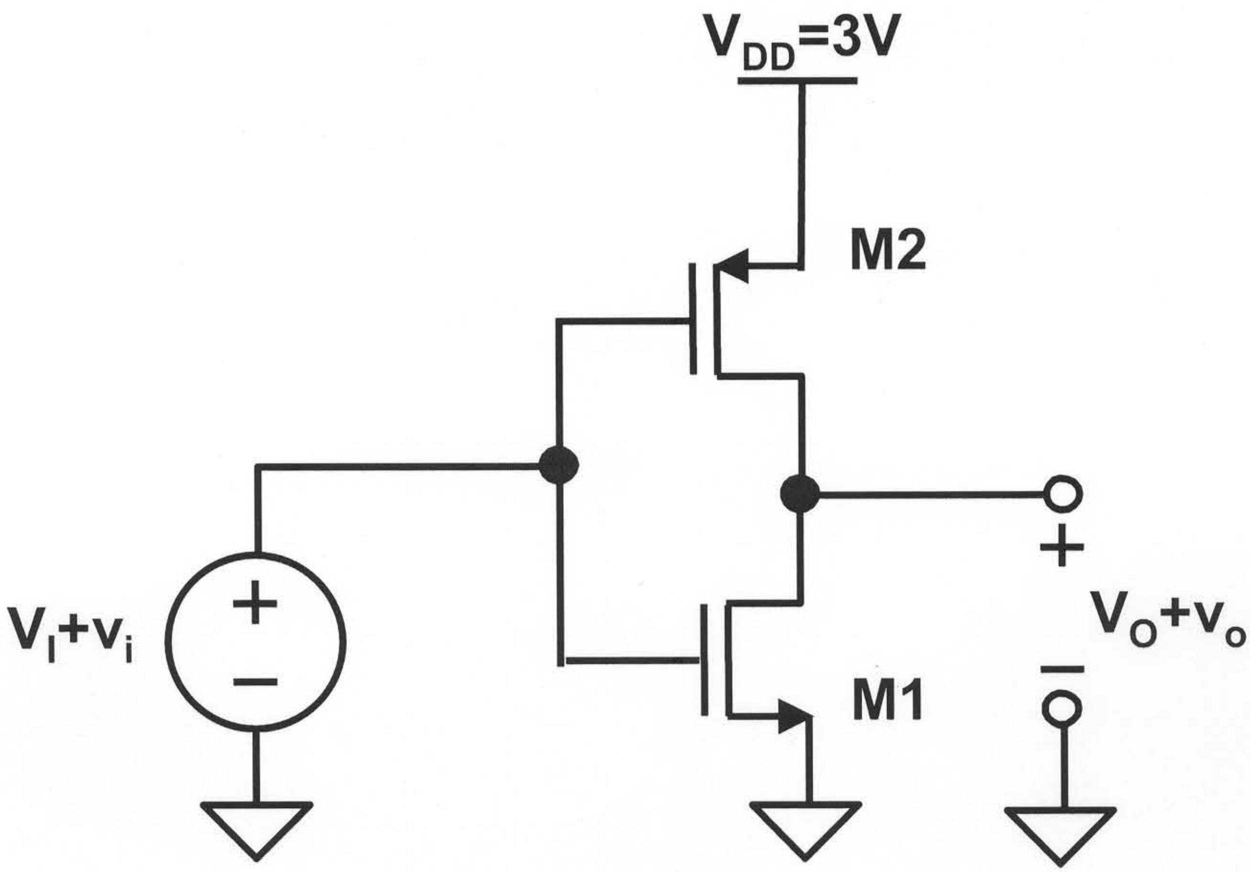
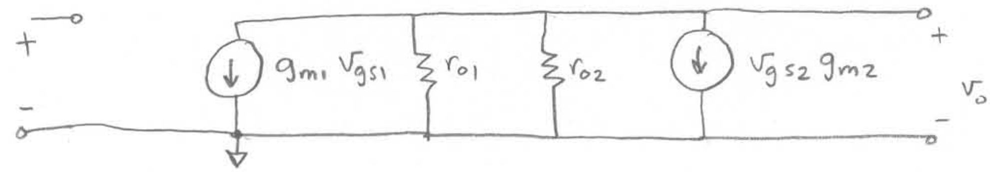


Figure 1: CMOS inverter as linear amplifier.

Problem 1.3 (**2 points**) Find the small-signal output resistance R_o assuming the bias conditions in Problem 1.1.

Small Signal model:

$V_i = V_{gs1} = V_{gs2}$



$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(75\,\mu A)} = 133\ k\Omega \quad (116\ k\Omega \text{ for } I_D = 78\,\mu A)$

$r_{o2} = r_{o1}$

$R_{out} = r_{o1} \| r_{o2} = \boxed{66.5\ k\Omega}$

$(58.0)\ k\Omega$

Problem 1.4 (**3 points**) Find the small-signal gain assuming the bias conditions in Problem 1.1.

$A_v = -G_m R_o = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})$

$g_{m1} = k_n' \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{tn}) = (300\,\mu A/V^2)(2)(1.5\,V - 1\,V) = 300\,\mu A/V$

$g_{m2} = k_p' \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{tp}) = (100\,\mu A/V^2)(6)(1.5\,V - 1V) = 300\,\mu A/V$

$\boxed{A_v = -39.9 \approx -40}$

Problem 1.5 (**2 points**) Suppose that γ is nonzero and that through source-well biasing, $V_{tn} = 1.25$ V and $V_{tp} = -1.25$ V. How does this affect the small-signal gain (assume the bias conditions in Problem 1.1)?

$A_v = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2}) = \frac{-(g_{m1} r_{o1})\, r_{o2}}{r_{o1} + r_{o2}} + \frac{-r_{o1}\,(g_{m2} r_{o2})}{r_{o1} + r_{o2}}$

$g_{m1} r_{o1} = \frac{1}{\lambda}\left(\frac{2}{V_{GS1} - V_{t1}}\right) = \left(\frac{1}{0.1}\right)\left(\frac{2}{1.5 - 1}\right) = 40$ for $V_{t1} = 1.0$, $g_{m1} r_{o1} = 80$ for $V_{t1} = 1.25$

$\boxed{A_v \text{ doubles},\ A_v \approx -80}$

2 Differential Amplifier

Figure 2 shows the circuit schematic for a differential amplifier. For this problem, use the transistor parameters in Table 2. Assume the following circuit parameters: $I_{TAIL} = 590\mu$A, $R_{TAIL} = 30$ kΩ, $R1 = 3$ kΩ, $R2 = 6$ kΩ, $(W/L)_1 = (W/L)_2 = 8$.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	300 μA/V^2	100 μA/V^2
γ	0	0
λ	0	0

Table 2: Problem 2 Transistor Parameters.

Problem 2.1 (**3 points**) Find V_I such that $V_s = 300$ mV.

$$I_1 = I_2 \Rightarrow I_1 = \frac{1}{2}\left(I_{TAIL} + \frac{V_s}{R_{TAIL}}\right) = \frac{1}{2}\left(590\mu A + \frac{300\,mV}{30\,k\Omega}\right) = 300\,\mu A$$

$$V_{ov1} = \sqrt{\frac{2I_1}{k_n'(W/L)_1}} = \sqrt{\frac{2\,(300\mu A)}{300\frac{\mu A}{V^2}(8)}} = 0.5\ V$$

$$V_I = V_s + V_{t1} + V_{ov1} = 0.3\ V + 1\ V + 0.5\ V = \boxed{1.8\ V}$$

For $I_1 = \frac{I_{TAIL}}{2}$, $V_I = 1.796$ V

Problem 2.2 (**3 points**) Find $(W/L)_3 = (W/L)_4$ such that $V_O = 1.5$ V, assuming the bias conditions you found in Problem 2.1.

$$I_3 = I_4\ ,\quad I_3 = I_1 + \frac{V_o}{R_1 + 2R_2} = 300\mu A + \frac{1.5\ V}{3\,k\Omega + 2(6\,k\Omega)} = 400\,\mu A$$

$\uparrow$ by symmetry

$$|V_{GS3}| = \left|\frac{2R_2 V_o}{R_1+2R_2} - V_{DD}\right| = \left|\frac{12\,k\Omega}{15\,k\Omega}1.5V - 3\ V\right| = 1.8\ V \Rightarrow 400\mu A = \frac{k_p'}{2}\left(\frac{W}{L}\right)_3 (|V_{GS3}| - |V_{tp}|)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \frac{(400\mu A)(2)}{(100\mu A/V^2)(1.8-1V)^2} = 12.5 \quad \boxed{\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 12.5}$$

For $I_3 = 295\mu A$, $(W/L)_3 = 23.6$

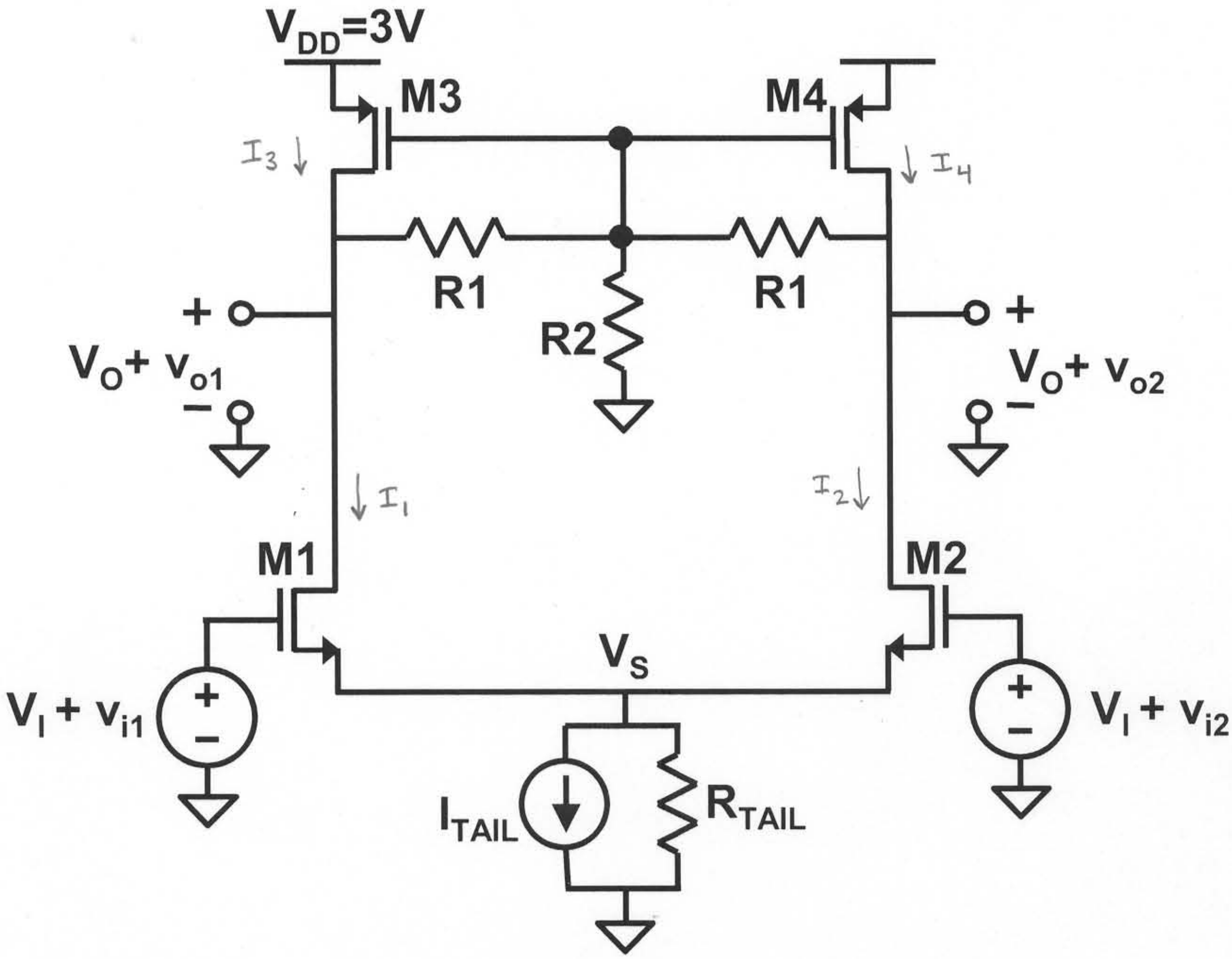
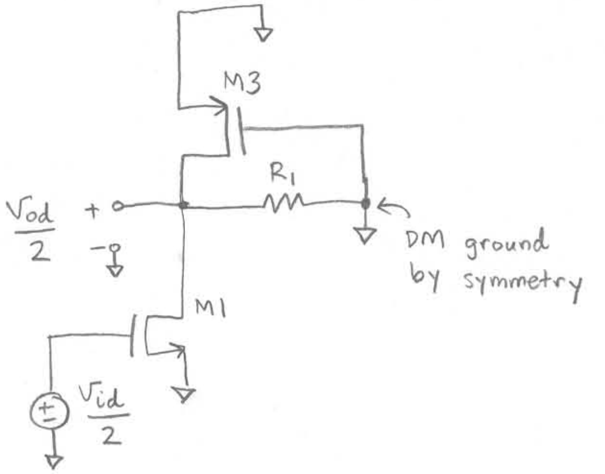


Figure 2: Differential amplifier.

Problem 2.3 (**4 points**) Draw and label the small-signal differential-mode half circuit, and find the differential-mode gain, $\frac{v_{od}}{v_{id}}$, where $v_{id} = v_{i1} - v_{i2}$ and $v_{od} = v_{o1} - v_{o2}$.



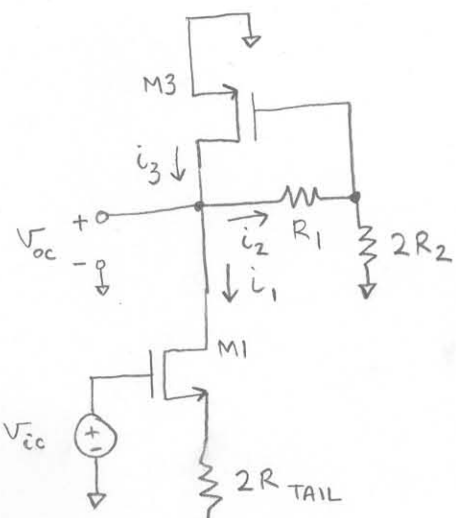
$$g_{m1} = k_n'\left(\frac{W}{L}\right)_1 V_{ov1} = \frac{300\,\mu A}{V^2}(8)(0.5\,V) = 1200\,\mu A/V$$

$$R_o = R_1 = 3\,k\Omega$$

$$\frac{v_{od}}{v_{id}} = -(1200\,\mu A/V)(3\,k\Omega)$$

$$\boxed{\frac{v_{od}}{v_{id}} = -3.6}$$

Problem 2.4 (**6 points**) Draw and label the small-signal common-mode half circuit, and find the common-mode gain, $\frac{v_{oc}}{v_{ic}}$, where $v_{ic} = 0.5(v_{i1} + v_{i2})$ and $v_{oc} = 0.5(v_{o1} + v_{o2})$.



$$i_1 = g_{m1}(v_{ic} - i_1 2R_{TAIL}) \Rightarrow i_1 = \frac{g_{m1} v_{ic}}{1 + 2R_{TAIL} g_{m1}}$$

$$i_2 = \frac{v_{oc}}{R_1 + 2R_2}$$

$$i_3 = -g_{m3}\left(\frac{2R_2}{2R_2 + R_1}\right) v_{oc}$$

$$i_1 = i_3 - i_2$$

$$i_1 = \frac{1200\,\mu A/V\,(v_{ic})}{1 + 2(30\,k\Omega)(1200\,\mu A/V)} = (1.64\times10^{-5})\,v_{ic}$$

$$i_2 = \frac{v_{oc}}{15\,k\Omega}$$

$$g_{m3} = k_p'\left(\frac{W}{L}\right)_3 V_{ov3} = 1\,mA/V$$

$$i_3 = -\frac{1\,mA/V}{5}\cdot 4\,v_{oc} = (8\times10^{-4})\,v_{oc}$$

$$\frac{v_{oc}}{v_{ic}} = -\frac{1.64\times10^{-5}}{(6.67\times10^{-5} + 8.0\times10^{-4})} \Rightarrow \boxed{\frac{v_{oc}}{v_{ic}} = 0.019}$$

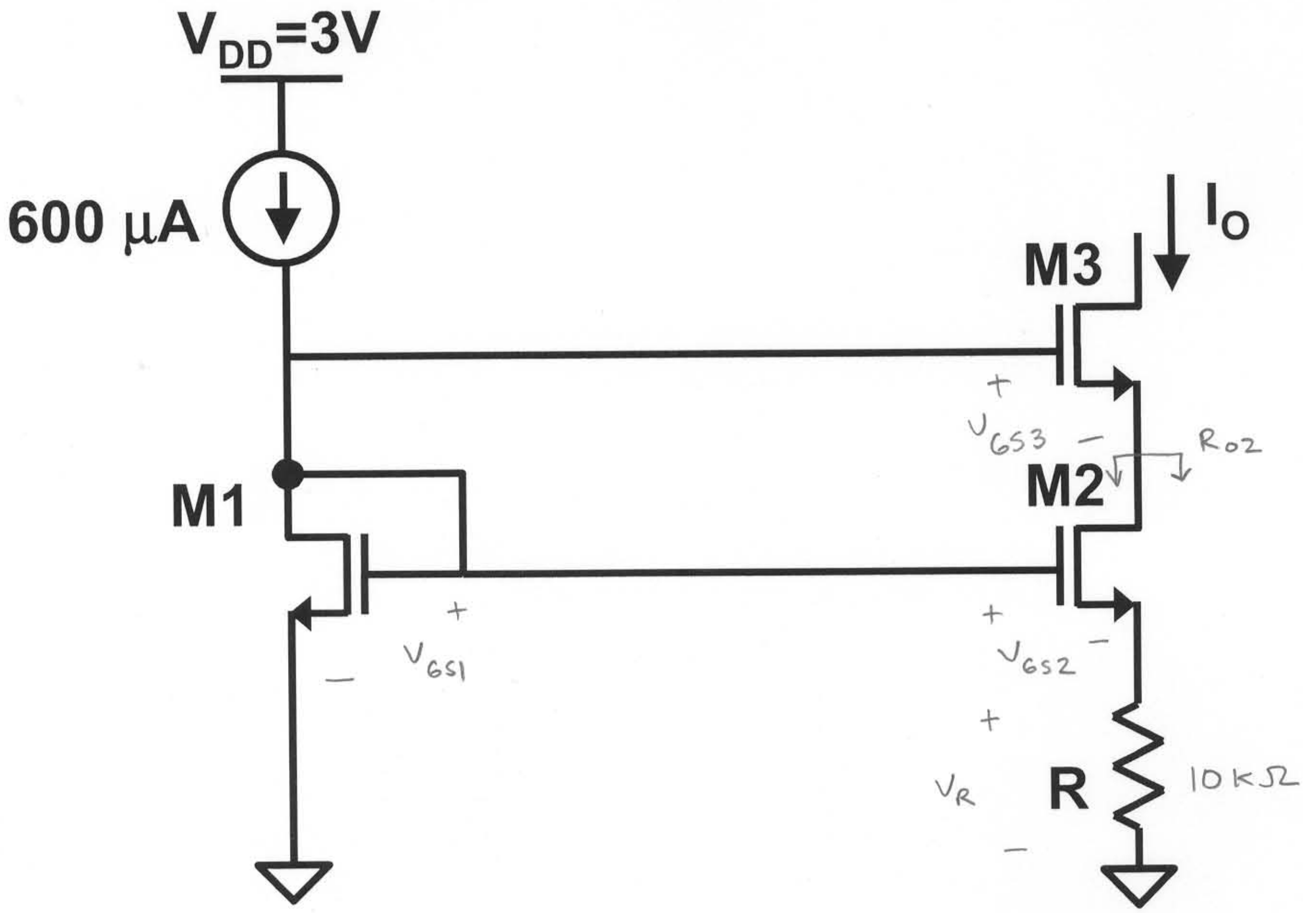


Figure 3: Current source.

3 Current Source

Figure 3 shows a current source circuit. For this problem, use the transistor parameters in Table 3. For the circuit in Figure 3, assume the following circuit parameters: $R = 10\ \mathrm{k}\Omega$, $(W/L)_1 = (W/L)_3 = 2$, $(W/L)_2 = 20$.

Problem 3.1 **(2 points)** Find the output current I_O.

$V_{GS1} = V_{GS2} + V_R = V_{GS2} + I_o R \Rightarrow V_{ov1} = V_{ov2} + I_o R \quad (V_{t1} = V_{t2} \text{ since } \gamma = 0)$

$$\sqrt{\frac{2(600\,\mu A)}{(300\,\mu A/V^2)(2)}} = 1.414\ V = \sqrt{\frac{2 I_o}{(300\,\mu A/V^2)(20)}} + I_o(10\ k\Omega)$$

Solve by iteration or quadratic formula (Widlar current source):

$\boxed{I_o = 121\,\mu A}$

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	300 μA/V^2	100 μA/V^2
γ	0	0
λ	0.1 V^{-1}	0.1 V^{-1}

Table 3: Problem 3 Transistor Parameters.

Problem 3.2 (**3 points**) Find the minimum output voltage $V_{OUT}(MIN)$.

$$V_{OUT}(\text{min}) = I_o R + V_{ov2} + V_{ov3}$$

$$V_{ov3} = \sqrt{\frac{2I_o}{k_n'(W/L)_3}} = \sqrt{\frac{2(121\mu A)}{(300\mu A/V^2)(2)}} = 0.635 \text{ V}$$

$$V_{ov2} = \sqrt{\frac{2(121\mu A)}{(300\mu A/V^2)(20)}} = 0.201 \text{ V}$$

$$I_o R = (121\mu A)(10k\Omega) = 1.21 \text{ V} \Rightarrow \boxed{V_{OUT}(\text{min}) = 2.046 \text{ V}}$$

Problem 3.3 (**3 points**) Find the output resistance R_o.

Output branch looks like cascode w/ common-source amp w/ source degeneration:

$$g_{m3} = \sqrt{k_n' 2I_D \left(\frac{W}{L}\right)_3} = 3.81 \times 10^{-4} \text{ A/V}$$

Cascode: $R_o = r_{o3}\left[1 + g_{m3} R_{o2}\right] + R_{o2}$

$$g_{m2} = \sqrt{k_n' 2I_D \left(\frac{W}{L}\right)_2} = 1.205 \times 10^{-3} \text{ A/V}$$

C.S. Amp w/ degeneration: $R_{o2} = R + r_{o2}(1 + g_{m2}R)$

$$r_{o2} = r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(121\mu A)} = 82.65 k\Omega$$

$R_{o2} = 1.09 \text{ M}\Omega$

$\boxed{R_o = 35.5 \text{ M}\Omega}$

or $R_o \cong g_{m3} r_{o3} (g_{m2} r_{o2} R) \cong 31.4 \text{ M}\Omega$

Problem 3.4 (**2 points**) Resistor R can be used to model the parasitic resistance associated with routing current sources for long distances on an IC. Find the largest value for R such that I_O is within 1% of the current reference value in Figure 3. Assume $(W/L)_1 = (W/L)_3 = (W/L)_2 = 2$.

$I_o(min) = (0.99)(600\,\mu A) = 594\,\mu A$

$I_o(min)\,R = V_{ov1} - V_{ov2}\big|_{I_o(min)} \Rightarrow R(594\,\mu A) = \sqrt{\dfrac{2(600\,\mu A)}{(300\,\mu A/V^2)(2)}} - \sqrt{\dfrac{2(594\,\mu A)}{(300\,\mu A/V^2)(2)}}$

$\boxed{R = 119\,\Omega}$

4 Current Source Reference

Figure 4 shows a current reference circuit. For this problem, use the transistor parameters in Table 4. For the circuit in Figure 4, assume the following circuit parameters: $R = 10\ \text{k}\Omega$, $(W/L)_1 = (W/L)_2$, $(W/L)_3 = 2$, and $(W/L)_4 = (W/L)_5 = (W/L)_6 = 6$.

Parameter	NMOS	PMOS
V_t	1 V	-1 V
L_d	0	0
X_d	0	0
k'	300 μA/V^2	100 μA/V^2
γ	0	0
λ	0	0

Table 4: Problem 4 Transistor Parameters.

Problem 4.1 **(6 points)** Find $(W/L)_1 = (W/L)_2$ and $(W/L)_7$ such that I_O is dependent on process parameters only (i.e., independent of bias point to first order).

KVL (bottom): $V_{GS3} + V_R = V_{GS1} + V_{GS2} + (-V_{GS7})$

$$V_R = \cancel{V_{tn}} + V_{ov1} + V_{tn} + V_{ov2} + |V_{tp}| + |V_{ov7}| - V_{ov3} - \cancel{V_{tn}}$$

$$= \underbrace{V_{ov1} + V_{ov2} + |V_{ov7}| - V_{ov3}}_{\text{bias dependent, want } = 0} + \underbrace{V_{tn} + |V_{tp}|}_{\text{process dependent}}$$

Current mirror (top): $(W/L)_4 = (W/L)_5 = (W/L)_6 \Rightarrow I_{REF} = I_2 = I_o$

$$V_R = \sqrt{\frac{2I_o}{k_n'(W/L)_1}} + \sqrt{\frac{2I_o}{k_n'(W/L)_2}} + \sqrt{\frac{2I_o}{k_p'(W/L)_7}} - \underbrace{\sqrt{\frac{2I_o}{k_n'(2)}}}_{-\sqrt{\frac{I_o}{k_n'}}} + 2\text{ V}$$

Choose $(W/L)_1 = (W/L)_2 = 9\,(W/L)_3$

Choose $(W/L)_7 = \frac{k_n'}{k_p'} \cdot 9\,(W/L)_3 = 27\,(W/L)_3$

$$\Rightarrow V_R = \frac{1}{3}\sqrt{\frac{I_o}{k_n'}} + \frac{1}{3}\sqrt{\frac{I_o}{k_n'}} + \frac{1}{3}\sqrt{\frac{I_o}{k_n'}} - \sqrt{\frac{I_o}{k_n'}} + 2\text{V} = 2\text{V} \Rightarrow I_o = \frac{V_R}{R} = \frac{2\text{V}}{10\text{k}\Omega}$$

$$\boxed{I_o = 200\mu A}$$

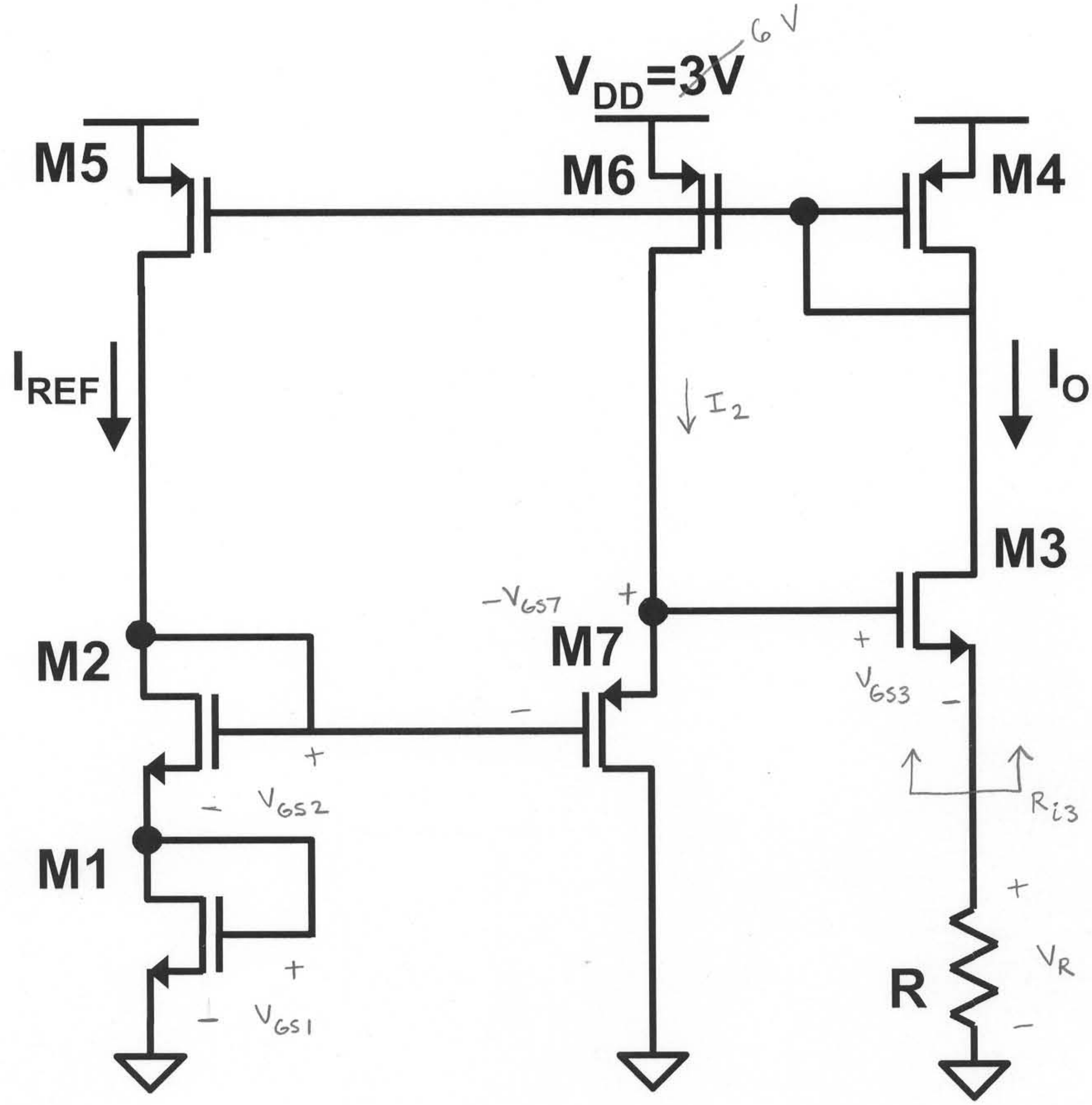


Figure 4: Current source reference.

Problem 4.2 (6 points) One can model the thermal noise of resistor R as a small-signal current source i_n in parallel with R. Quantify the impact of the noise current source on the total output current $I_O + i_o$, assuming the bias point you found in Problem 4.1 and $|i_n| = 1\mu A$.

Small-signal model:

Current gain for common-gate: $\frac{i_o}{i_i} = 1$

$R_{i3} \equiv$ common-gate input resistance $= \frac{1}{g_{m3}}$ $(\lambda = 0)$

$\frac{i_i}{i_n} = \frac{R}{R + R_{i3}}$ (current divider) $\Rightarrow \frac{i_o}{i_n} = (1)\frac{R}{R + 1/g_{m3}}$

$g_{m3} = \sqrt{2 I_o K_n' \left(\frac{W}{L}\right)_3}$

$= \sqrt{2(200\mu A)(300\frac{\mu A}{V^2})(2)}$

$= 4.90 \times 10^{-4}$ S

$\frac{i_o}{i_n} = \frac{10\,k\Omega}{10\,k\Omega + 2.04\,k\Omega} = 0.83 \Rightarrow |i_o| = 0.83\,\mu A$

$$\boxed{\frac{|i_o|}{|I_o|} = 0.415\,\%}$$