EEC 210 | SOLUTIONS TO HW#4

(1) $L_{eff} = 1\,\mu m - 2(0.09\,\mu m) = 0.82\,\mu m$

$$r_o = \frac{1}{\lambda I_D} \quad \text{where } \lambda = \frac{dx_d/dV_{DS}}{L_{eff}} = \frac{0.02}{0.82} = 0.024\,V^{-1}$$

$$= \frac{1}{0.024(100\mu A)} = 417\,K\Omega$$

$$g_m = \sqrt{2K'\frac{W}{L_{eff}} I_D} = \sqrt{2(194)\left(\frac{100}{0.82}\right)100} = 2.18\,mA/V$$

$$R_o = r_{o2}\left[1 + (g_{m2} + g_{mb2})r_{o1}\right] + r_{o1} \qquad EQ\ (4.50)$$

Here $g_{mb2} = 0$ by assumption

$$R_o \simeq r_{o2}(g_{m2} r_{o1}) = 417K(2.18m \times 417K) = \boxed{379\,M\Omega}$$

From SPICE, $\boxed{R_o = 417\,M\Omega}$

(2) From condition (a), $V_{o\,min} = 0.2V$. This is split across M_1 & M_2

Therefore $V_{ov} = V_{GS} - V_t = 0.1V$

From this and condition (b), $50\mu A = \frac{194\mu}{2}\left(\frac{W}{L_{eff}}\right)(0.1)^2 \Rightarrow \frac{W}{L_{eff}} = 51.5$

$r_o = \frac{1}{\lambda I_D}$ where $\lambda = \frac{dx_d/dV_{DS}}{L_{eff}}$ So $r_o = \frac{L_{eff}}{50\mu(0.02\,\mu m/V)} = \frac{L_{eff}}{1\times10^{-12}}$

$$R_o = \frac{\Delta V_{OUT}}{\Delta I_{OUT}} = \frac{1V}{0.0002(I_D)} = \frac{1V}{0.0002(50\mu)} = 100\,M\Omega$$

From (4.50), $R_o \simeq r_{o2} g_{m2} r_{o1} = \left(\frac{L_{eff}}{10^{-12}}\right)^2 \sqrt{2(194)(51.5)(50)} = \frac{L_{eff}^2}{10^{-21}} = 100\,M\Omega$

Therefore, $L_{eff} = 0.32\,\mu m$

$L_{drawn} = L_{eff} + 2L_D = 0.32 + 2(0.09) = \boxed{0.5\,\mu m}$

$W = 51.5(0.32) = \boxed{16.5\,\mu m}$

From SPICE, $R_o = \boxed{109\,M\Omega}$

(3) $g_{o1} = \frac{I_{REF}}{L_{eff}}\frac{dx_d}{dV_{DS}} = \frac{I_{REF}(0.02)}{0.82} = 0.024\,I_{REF} \qquad g_{o2} = \frac{I_{REF}(0.04)}{0.82} = 0.049\,I_{REF}$

In strong inversion, $g_{m1} = \sqrt{2K'\left(\frac{W}{L_{eff}}\right)_1 I_{REF}} = \sqrt{2\times1.94\times10^{-4}\frac{(100)}{0.82} I_{REF}} = 0.22\sqrt{I_{REF}}$

$$A_v = -\frac{g_{m1}}{g_{o1}+g_{o2}} = -\frac{0.22\sqrt{I_{REF}}}{0.073\,I_{REF}} = -\frac{3.0}{\sqrt{I_{REF}}}$$

I_{REF} (μA)	A_v (hand calc)	A_v (SPICE)	$(V_{GS}-V_t)_1$ (mV) $= \sqrt{\frac{2I_{REF}}{K'\left(\frac{W}{L_{eff}}\right)_1}}$
1000	−95	−100	290
100	−300	−314	92
10	−950	−992	29
1	−3000	−3130	9

In the last 2 cases above, $(V_{GS}-V_t)_1$ calculated by $\sqrt{\frac{2I_{REF}}{K'\left(\frac{W}{L_{eff}}\right)_1}} < 78mV$

Therefore, M_1 operates in weak inversion for these cases.

From (1.253), $g_{m1} = \frac{I_{REF}}{nV_T} = \frac{I_{REF}}{(1.5)(26mV)}$

$$A_v = -\frac{g_{m1}}{g_{o1}+g_{o2}} = -\frac{\cancel{I_{REF}}}{(1.5)(26mV)}\frac{1}{0.073\cancel{I_{REF}}} = -350$$

I_{REF} (μA)	A_v (hand calc)	A_v (SPICE)
1000	−95	−85
100	−300	−263
10	−350	−316
1	−350	−316

(4) From (1.140), $V_{t2} = V_{t2,0} + \gamma\left[\sqrt{2\phi_f + V_{SB2}} - \sqrt{2\phi_f}\right]$ $V_{SB2} = 1V$

$$C_{ox} = \frac{3.9 \times 8.86 \times 10^{-14}}{80 \times 10^{-8}} = 4.3 \times 10^{-7} \frac{F}{cm^2}$$

$$\gamma = \frac{1}{4.3 \times 10^{-7}} \sqrt{2(1.6 \times 10^{-19})(11.6)(8.86 \times 10^{-14})(5 \times 10^{15})} = 0.094\ V^{1/2}$$

From (1.135), $2\phi_f = 2V_T \ln \dfrac{5 \times 10^{15}}{1.45 \times 10^{10}} = 663\,mV$

So $V_{t2} = -1 + 0.094\left[\sqrt{1.663} - \sqrt{0.663}\right] = -0.955$

$$I_{D2} = \frac{k'}{2}\left(\frac{W}{L_{eff2}}\right)(V_{GS2} - V_{t2})^2 = \frac{194}{2}\,\frac{10}{0.82}\,(0 + 0.955)^2 = 1.08\,mA$$

From (4.120) $\dfrac{v_o}{v_i} = -g_{m1}\left(r_{o1} \| r_{o2} \| \dfrac{1}{g_{mb2}}\right) = -\dfrac{g_{m1}}{g_{o1} + g_{o2} + g_{mb2}}$

$$g_{m1} = \sqrt{2(194)\frac{100}{0.82}(1080)} = 7.2\,mA/V$$

From (1.199), $g_{mb2} = \dfrac{\gamma k'\left(\frac{W}{L_{eff}}\right)_2 (V_{GS2} - V_{t2})}{2\sqrt{2\phi_f + V_{SB2}}} = \dfrac{0.094(194)\frac{10}{0.82}(0.955)}{2\sqrt{1.663}} = 82.3 \dfrac{\mu A}{V}$

$$g_{o1} = g_{o2} = r_{o1}^{-1} = r_{o2}^{-1} = \frac{I_D}{L_{eff}}\frac{dX_d}{dV_{DS}} = \frac{1.08m}{0.82}(0.02) = 26.3 \frac{\mu A}{V}$$

$$\frac{v_o}{v_i} = -\frac{7.2m}{2(26.3\mu) + 82.3} = -53$$