(1) First, consider the balanced case in which a $10K\Omega$ resistor is present in the drain of M_1

Let V_S = voltage from source to ground

$V_S = 0 - V_t - \sqrt{\frac{I_{TAIL}}{K' W/L}}$ Since $\gamma = 0$, $V_t = V_{to} = 0.7V$

$K' = 650 \times \frac{3.9 \times 8.86\times10^{-14}}{250\times10^{-8}} = 90\,\mu A/V^2$

$I_{TAIL} = \frac{V_S+15}{10K}$

So $V_S = -0.7 - \sqrt{\frac{V_S+15}{(10K)(90\mu A/V^2)(100)}} \Rightarrow V_S = -0.7 - \sqrt{\frac{V_S+15}{90}}$

$(V_S+0.7)^2 = (V_S+15)/90 \Rightarrow V_S^2 + 1.3888 + 0.32333 = 0$

$V_S = \frac{-1.3888 \pm \sqrt{(1.3888)^2 - 4(0.32333)}}{2} = -0.30V$ or $-1.09V$

$V_S = -0.30V$ is not low enough then V_{GS} would be $0.3V$, which is $< V_t$

So V_S must equal $-1.09V$

Then $I_{TAIL} = \frac{-1.09+15}{10K} = 1.39mA$ $g_m = \sqrt{I_{TAIL} K' \frac{W}{L}} = \sqrt{1390(90)(100)} = 3.5mA/V$

$A_{dm} = -g_m(10K) = -3.5(10) = -35$

$A_{cm} = \frac{-g_m(10K)}{1+g_m(20K)} = \frac{-35}{1+70} = -0.49$

Second, the actual input is applied to the left side only (v_{i1}) and the actual output is taken from the right side only (v_{o2})

$v_{o2} = v_{oc} - v_{od}/2 = A_{cm} v_{ic} - (1/2) A_{dm} v_{id}$

$v_{ic} = (v_{i1} + v_{i2})/2 = v_i/2$ and $v_{id} = v_{i1} - v_{i2} = v_i$

So $v_{o2} = A_{cm}(v_i/2) - (1/2) A_{dm} v_i$

$v_{o2}/v_i = (1/2)(A_{cm} - A_{dm}) = (1/2)(-0.49+35) = \boxed{+17.2}$

Also $\boxed{R_i \to \infty}$ and $R_o = \boxed{10K\Omega}$ (because $\lambda = 0$)

(2) From SPICE, $v_{o2}/v_i = \boxed{+17.4}$ $R_i = \boxed{10^{20}\Omega}$ $R_o = \boxed{10K\Omega}$

(3) From (3.248), $V_{OS} = \Delta V_t + \frac{(V_{GS}-V_t)}{2}\left(-\frac{\Delta R_L}{R_L} - \frac{\Delta(W/L)}{W/L}\right)$

$V_{GS}-V_t = \sqrt{\frac{I_{TAIL}}{K' \frac{W}{L_{eff}}}}$ $K' = 450 \times \frac{3.9 \times 8.86\times10^{-14}}{80\times10^{-8}} = 194\,\mu A/V^2$

$L_{eff} = 1 - 2(0.09) = 0.82\,\mu m$

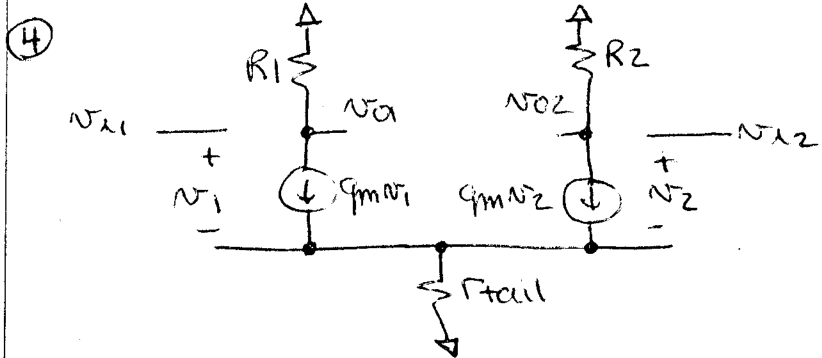
$V_{GS}-V_t = \sqrt{\frac{50\mu}{194\mu\left(\frac{10}{0.82}\right)}} = 0.145$

If $(W/L)_1 > (W/L)_2$, $V_{OS} < 0$

Since we don't know whether $(W/L)_1 > (W/L)_2$ or $(W/L)_2 > (W/L)_1$ we can only calculate the magnitude of V_{OS}

$|V_{OS}| = 0 + \frac{0.145}{2}(0 + 0.02) = \boxed{1.45\ mV}$

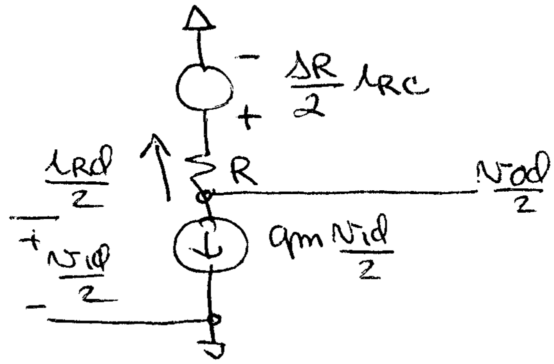
(4)



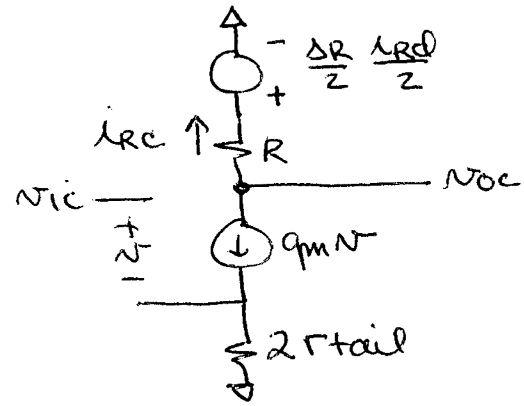
Let $R = \dfrac{R_1 + R_2}{2}$

$\Delta R = R_1 - R_2$

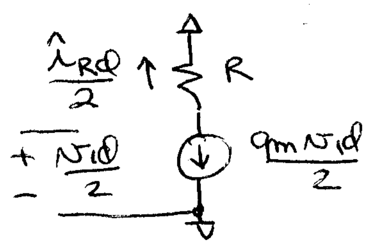
EXACT DM ½ CKT



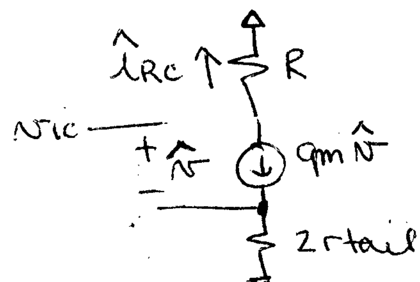
EXACT CM ½ CKT



DM ½ CKT WITHOUT MISMATCH



CM ½ CKT WITHOUT MISMATCH



$$\frac{\hat{i}_{Rd}}{2} = -g_m \frac{v_{id}}{2} \qquad \hat{i}_{Rc} = -\frac{g_m v_{ic}}{1 + 2 g_m r_{tail}}$$

From EXACT ½ CKts

$$\frac{v_{od}}{2} = \frac{\Delta R}{2} i_{Rc} + R \frac{i_{Rd}}{2} \simeq \frac{\Delta R}{2} \hat{i}_{Rc} + R \frac{\hat{i}_{Rd}}{2} = -\frac{\Delta R}{2} \frac{g_m v_{ic}}{1 + 2 g_m r_{tail}} - R g_m \frac{v_{id}}{2}$$

$$\therefore \; A_{dm} \simeq -g_m R = -1(10) = -10$$

$$A_{cm\text{-}dm} \simeq \frac{-g_m \Delta R}{1 + 2 g_m r_{tail}} = \frac{-1(0.2)}{1 + 2(1)(1000)} \simeq -1 \times 10^{-4}$$

$$v_{oc} = \frac{\Delta R}{2} \frac{i_{Rd}}{2} + R i_{Rc} \simeq \frac{\Delta R}{2} \frac{\hat{i}_{Rd}}{2} + R \hat{i}_{Rc} = -\frac{\Delta R}{2} g_m \frac{v_{id}}{2} - \frac{R g_m v_{ic}}{1 + 2 g_m r_{tail}}$$

$$\therefore \; A_{dm\text{-}cm} \simeq -\frac{g_m \Delta R}{4} = \frac{-1(0.2)}{4} = -0.05$$

$$A_{cm} \simeq \frac{-g_m R}{1 + 2 g_m r_{tail}} = \frac{-1(10)}{1 + 2(1)(1000)} \simeq -0.005$$

$$\frac{A_{dm}}{A_{cm}} \simeq \frac{10}{0.005} = 2000 \qquad \frac{A_{dm}}{A_{cm\text{-}dm}} \simeq \frac{10}{1 \times 10^{-4}} = 100{,}000 \qquad \frac{A_{dm}}{A_{dm\text{-}cm}} \simeq \frac{10}{0.05} = 200$$

(5) From Table 2.4, $k' = \mu_n C_{ox}$

$$= 450 \times \frac{3.9 \times 8.86 \times 10^{-14}}{0.08 \times 10^{-5}} = 194\ \mu A/V^2$$

$$I_{OUT} = 50\mu A\,(1 + \lambda \Delta V_{DS})$$

From condition (c), $\lambda \Delta V_{DS} \le 0.01$ and $\Delta V_{DS} = 1V \Rightarrow \lambda \le 0.01 V^{-1}$

From conditions (a) and (b)

$$50\mu A = \frac{k'}{2} \frac{W}{L_{eff}} (V_{GS} - V_t)^2 \quad \text{where } V_{GS} - V_t = 0.2V$$

$$50\mu = \frac{194}{2} \frac{W}{L_{eff}} (0.2)^2 \Rightarrow \frac{W}{L_{eff}} = 12.9$$

$\lambda \le \frac{1}{100V}$ Also from Table 2.4 $\frac{dx_d}{dV_{DS}} = 0.02$

$$\lambda = \frac{1}{V_A} = \frac{dx_d/dV_{DS}}{L_{eff}} \le \frac{1}{100V} \Rightarrow L_{eff} \ge 2\mu m$$

$L_{drawn} = L_{eff} + 2L_d + x_d$ where $x_d = 0$ (given)
$L_d = 0.09\mu m$ (Table 2.4)

$$L_{drawn} = 2\mu m + 2(0.09) = 2.18\mu m \simeq 2.2\mu m$$

The minimum length that satisfies the constraints minimizes the required gate area

Then $W = 12.9 \times 2.2\ \mu m = \boxed{28\mu m}$