① (a) $V_o = 3 - V_t - (V_{GS} - V_t)$; $V_{GS} - V_t = \sqrt{\frac{2I}{K'\frac{W}{L}}}$; $K' = \mu_n C_{ox}'$

$C_{ox}' = \dfrac{3.9 \times 8.854 \times 10^{-14}}{250 \times 10^{-8}\ cm} = 1.38 \times 10^{-7} \dfrac{F}{cm^2} = 1.38 \dfrac{fF}{\mu^2}$

$K' = \mu_n C_{ox}' = 650 \dfrac{cm^2}{V s} \cdot 1.38 \times 10^{-7} \dfrac{F}{cm} = 89.7\ \mu A/V^2 \simeq 90\ \mu A/V^2$

$\therefore V_{GS} - V_t = \sqrt{\dfrac{2(200)}{90(10)}} \simeq 0.67V \rightarrow V_o = 3 - 0.7 - 0.67 = \boxed{1.63V}$

$\dfrac{v_o}{v_i} = \dfrac{g_m r_o}{1 + g_m r_o}$; $r_o = \infty$ because $\lambda = 0 \rightarrow \boxed{\dfrac{v_o}{v_i} = 1}$

(b) V_t affects V_o and vice versa. Solve iteratively

$V_t = V_{to} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

$\gamma = \dfrac{\sqrt{2 q \epsilon N_A}}{C_{ox}'} = \dfrac{\sqrt{2(1.6 \times 10^{-19})(11.7 \times 8.854 \times 10^{-14})(2 \times 10^{15})}}{1.38 \times 10^{-7}} = 0.19 \sqrt{V}$

$V_{SB} = V_o$ — use answer from (a) to start ; $\phi_F = V_T \ln \frac{N_A}{n_i} = 26m \ln \frac{2\times 10^{15}}{1.5 \times 10^{10}} \simeq 0.3$

$V_t = 0.7 + 0.19\left[\sqrt{0.6 + 1.63} - \sqrt{0.6}\right] = 0.84V$

so $V_o = 3 - 0.84 - 0.67 = 1.49$. Try Again

$V_t = 0.7 + 0.19\left[\sqrt{0.6 + 1.49} - \sqrt{0.6}\right] = 0.83$ — Not much change — OK

So $V_o = 3 - 0.83 - 0.67 \simeq \boxed{1.5\ V}$

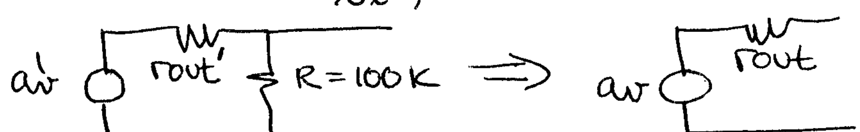
$\dfrac{v_o}{v_i} = \dfrac{g_m}{g_m + g_{mb}} = \dfrac{1}{1 + \chi}$; $\chi = \dfrac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = \dfrac{0.19}{2\sqrt{0.6 + 1.5}} \simeq 0.07$

$= \dfrac{1}{1 + 0.07} \simeq \boxed{0.93}$

(c) Use V_o from (b) at first $\rightarrow I = 200\mu A + \dfrac{1.5}{100K} \simeq 215\ \mu A$

$V_o = 3 - 0.83 - \sqrt{\dfrac{2(215)}{90 \cdot 10}} = \boxed{1.48V}$ (Not much change from (b) so don't bother to recalculate V_t.)

To find $a_v = \frac{v_o}{v_i}$, use $a_v' = OCVG$ from (b) $= 0.93$



$a_v = a_v' \dfrac{100K}{100K + r_{out}'}$

$r_{out}' = 1/(g_m + g_{mb}) = 1/g_m(1+\chi)$; $g_m = \sqrt{2 I K' \frac{W}{L}} = \sqrt{2(215)\, 90 \cdot 10} = 0.62\ mA/V$

$r_{out}' = \dfrac{1}{0.62(1.07)} = 1.5K\Omega \Rightarrow a_v = 0.93 \dfrac{100}{101.5} = \boxed{0.92}$

(d) Use V_o from (c) at first $\rightarrow I = 200 + \dfrac{1.48}{10K} = 348\ \mu A$

$V_o = 3 - 0.83 - \sqrt{\dfrac{2(348)}{90 \cdot 10}} = 1.29V$

$V_t = 0.7 + 0.19\left(\sqrt{0.6 + 1.29} - \sqrt{0.6}\right) = 0.81V$

$I = 200 + \dfrac{1.29}{10K} = 329\ \mu A$

$V_o = 3 - 0.81 - \sqrt{\dfrac{2(329)}{90 \cdot 10}} = \boxed{1.33\ V}$

$a_v' = 1/(1+\chi)$; $\chi = 0.19/(2\sqrt{0.6 + 1.33}) = 0.07 \rightarrow a_v' \simeq 0.93$

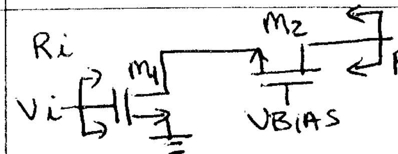
$r_{out}' = 1/[g_m(1+\chi)]$; $g_m = \sqrt{2(329)\,90(10)} = 0.77\ mA/V$

$r_{out}' = \dfrac{1}{0.77(1.07)} = 1.2k \rightarrow a_v = 0.93 \dfrac{10k}{11.2k} = \boxed{0.83}$

② I'll send my SPICE file to anyone who wants it.
Please send me e-mail if you want it but did not get it

ANSWER: $V_o = 1.32V$ $\quad \dfrac{v_o}{v_i} = 0.835$ (very close to hand calcs)

(3)



$\boxed{R_i \to \infty}$

$G_m = g_{m1} = \sqrt{2(90)(100)(250)} \simeq \boxed{2.1\ mA/V}$

$R_o \simeq (g_{m2}+g_{mb2})\, r_{o2} r_{o1} = g_{m2}(1+\chi)\, r_{o2} r_{o1}$

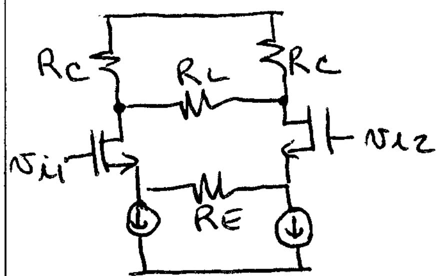
$r_{o1} = r_{o2} = \frac{1}{\lambda I} = \frac{10}{250} = 40\,k\Omega$

$R_o \simeq 2.1\,(1+0.1)(40k)^2 \simeq \boxed{3.7\ meg\Omega}$

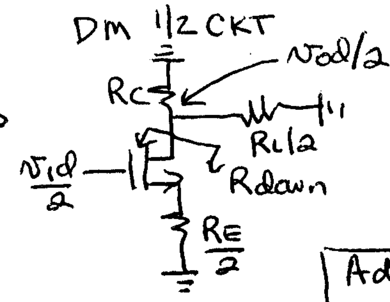
$OCVG = -G_m R_o = 2.1m(3.7meg)$

$\simeq \boxed{-7700}$

(4)



⇒



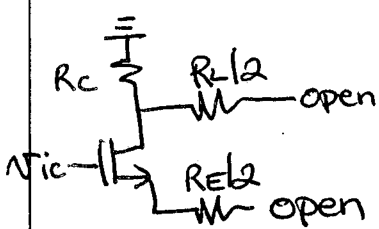
$A_{dm} = -G_m R_o$

$G_m = \dfrac{g_m}{1+g_m\frac{R_E}{2}}$

$R_o = R_c \| R_L/2 \| \underbrace{R_{down}}_{\infty}$

$$\boxed{A_{dm} = -\frac{g_m (R_c \| R_L/2)}{1 + g_m R_E/2}}$$

⇓ CM ½ CKT



Since the resistance connected to the source → ∞, Gm here = 0 and $\boxed{A_{cm} = 0}$

(5) From Table 2.4 $\mu_n C_{ox} = 450 \times \dfrac{3.9 \times 8.854 \times 10^{-14}}{0.08 \times 10^{-5}} = 194\ \mu A/V^2 = k'$

$\Delta I_d = 0.85\, I_{TAIL} = \dfrac{k'}{2}\dfrac{W}{L_{eff}}(0.2)\sqrt{\dfrac{2 I_{TAIL}}{\frac{k'}{2}\frac{W}{L_{eff}}} - (0.2)^2}$ EQ(1)

Also $G_m = 1.0\ \frac{mA}{V} = \sqrt{I_{TAIL}\, k' \frac{W}{L_{eff}}}$ EQ(2) $\rightarrow k' \dfrac{W}{L_{eff}} = \dfrac{1 \times 10^{-6}\ A/V^2}{I_{TAIL}}$

So we have 2 equations and 2 unknowns (I_{TAIL} and $\frac{W}{L_{eff}}$)

Substitute (2) into (1)

$$0.85\, I_{TAIL} = \frac{1\times10^{-6}}{2 I_{TAIL}}(0.2)\sqrt{\frac{2 I_{TAIL}}{1\times10^{-6}/2I_{TAIL}} - (0.2)^2}$$

Solve for I_{TAIL}^2: $7.225\times10^{13} I_{TAIL}^4 - 4\times10^6 I_{TAIL}^2 + 0.04 = 0$

$$I_{TAIL}^2 = \frac{4\times10^6 \pm \sqrt{(4\times10^6)^2 - 4(7.225\times10^{13})(0.04)}}{2(7.225\times10^{13})} = 2.768\times10^{-8} \pm 1.458\times10^{-8}$$

$I_{TAIL}^2 = 4.226\times10^{-8}$ or 1.310×10^{-8}

$I_{TAIL} = 205.6\ \mu A$ or $114.4\ \mu A$

If $I_{TAIL} = 205.6\mu A$, $\dfrac{W}{L_{eff}} = \dfrac{1\times10^{-6}}{194\times10^{-6}(205.6\times10^{-6})} = 25.1$

If $I_{TAIL} = 114.4\mu A$ $\dfrac{W}{L_{eff}} = \dfrac{1\times10^{-6}}{194\times10^{-6}(114.4\times10^{-6})} = 45.0$

If $I_{TAIL} = 205.6\ \mu A$ $V_{OV} = \sqrt{\dfrac{2(I_{TAIL}/2)}{k' W/L_{eff}}} = \sqrt{\dfrac{205.6}{194(25)}} = 0.206V$

If $I_{TAIL} = 114.4\mu A$ $V_{OV} = \sqrt{\dfrac{2(I_{TAIL}/2)}{k' W/L_{eff}}} = \sqrt{\dfrac{114.4}{194(45)}} = 0.114\ V$

This overdrive is too small because the range would be only $\pm\sqrt{2}(0.114)$ which is 162mV which is less than the 200mV input

$L_{eff} = L_{drawn} - 2L_D - X_d = 1\,um - 2(0.09) - 0 = 0.82\,um$

$\boxed{I_{TAIL} = 205.6\,\mu A}$ $\dfrac{W}{L_{eff}} = \dfrac{W}{0.82\,um} = 25.1 \Rightarrow \boxed{20.6\,um = W}$