EEC 118 Lecture #5: CMOS Inverter AC Characteristics

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Outline

• Review: CMOS Inverter Transfer Characteristics

• CMOS Inverters: Rabaey 5.4-5.5 (Kang & Leblebici, 6.1-6.4, 6.7)
CMOS Inverter VTC: Device Operation

P linear
N cutoff

P linear
N sat

P cutoff
N linear

V_{DD}

V_{T0,p}

V_{T0,n}

V_{IL}

V_{IH}

V_{DD} + V_{T0,p}

V_{DD}

Output Voltage (V)

Input Voltage (V)

V_{out} = V_{in} - V_{T0,p}

V_{out} = V_{in} - V_{T0,n}

nMOS in saturation

pMOS in saturation
Logic Circuit Delay

- For CMOS (or almost all logic circuit families), only one fundamental equation necessary to determine delay:

\[ I = C \frac{dV}{dt} \]

- Consider the discretized version:

\[ I = C \frac{\Delta V}{\Delta t} \]

- Rewrite to solve for delay:

\[ \Delta t = C \frac{\Delta V}{I} \]

- Only three ways to make faster logic: \( \downarrow C, \downarrow \Delta V, \uparrow I \)
• Assume input transition is fixed, then delay determined by output

Capacitance on node f (output):

• Junction cap Cdb,p and Cdb,n
• Gate capacitance Cgd,p and Cgd,n
• Interconnect cap
• Receiver gate cap
CMOS Inverter Junction Capacitances

- Junction capacitances $C_{db,p}$ and $C_{db,n}$:

  - Equation for junction cap:

$$C_j(V) = \frac{AC_{j0}}{\left(1 - \frac{V}{\phi_0}\right)^m}, \quad C_{j0} = \left(\frac{\varepsilon q}{2} \frac{N_a N_d}{N_a + N_d} \frac{1}{\phi_0}\right)^m$$

  - Non-linear, depends on voltage across junction

  - Use $K_{eq}$ factor to get equivalent capacitance for a voltage transition

$$C_{db} = AK_{eq} C_j + PK_{eqsw} C_{jsw}$$
CMOS Inverter Gate Capacitances

- Gate capacitances $C_{gd,p}$ and $C_{gd,n}$:
  - Just after the input switches ($t = 0^+$), what regions are transistors in?
  - One is in cutoff: $C_{GD} = $ Overlap Cap
  - One is in Saturation: $C_{GD} = $ Overlap Cap
  - Therefore, gate-to-drain capacitance is due to overlap capacitance:

\[
C_{gd,p} = C_{gd,n} = C_{ox}WL_D
\]

However, also need to consider Miller effect ...
CMOS Inverter Capacitances: Miller Effect

- When input rises by $\Delta V$, output falls by $\Delta V$
  - Change in stored charge: $\Delta Q = C_{gd1}\Delta V - (-C_{gd1}\Delta V)$
  - Effective voltage change across $C_{gd1}$ is $2\Delta V$
  - Effective capacitance to ground is \textit{twice} $C_{gd1}$

- Including Miller effect:
  $$C_{gd,p} = C_{gd,n} = 2C_{ox}WL_D$$ (For transistor in Cutoff)
CMOS Inverter Capacitances: Receiver

- Receiver gate capacitance
  - Includes all capacitances of gate(s) connected to output node
  - Unknown region of operation for receiver transistor: total gate cap varies from $(2/3)WL_{Cox}$ to $WL_{Cox}$
  - Ignore Miller effect (taken into account on output)
  - Assume worst-case value, include overlap

\[
C_g = WL_{eff}C_{ox} + 2WL_{D}C_{ox}
\]

\[
C_g = WL \ C_{ox}
\]
Inverter Capacitances: Analysis

• Simplify the circuit: combine all capacitances at output into one lumped linear capacitance:

\[ C_{load} = 2*C_{gd,n} + 2*C_{gd,p} + C_{db,n} + C_{db,p} + C_{int} + C_g \]

- \( C_{gb,n} = C_{gb,p} = 0 \)

- \( C_{gs,n} \) and \( C_{gs,p} \) are not connected to the load. These are part of the gate capacitance \( C_g \)
First-Order Inverter Delay

- Suppose ideal voltage step at input
- Assume: Current charging or discharging capacitance $C_{load}$ is nearly constant $I_{avg}$
  
  
  - $t_{PHL} = C_{load} \frac{(Vdd - Vdd/2)}{I_{avg}}$
  
  - $t_{PLH} = C_{load} \frac{(Vdd/2 - Vss)}{I_{avg}}$
Inverter Delay: Falling

- Assume PMOS fully off (ideal step input, $I_{D,p} = 0$)

\[ I = C \frac{dV}{dt} \]

\[ I_{D,n} = C_{\text{load}} \frac{dV_{\text{out}}}{dt} \quad \Rightarrow \quad \text{Need to determine } I_{D,n} \]
Inverter Delay: Falling

- From $t_0$ to $t_1$: NMOS in saturation
- From $t_1$ to $t_2$: NMOS in linear region
- Find $I_D$ in each region
Inverter Delay: Falling $t_1 - t_0$

- Assumption: Input fast enough to go through transition before output voltage changes

- $V_{out}$ drops from $V_{OH}$ to $V_{DD} - V_{TN}$ (NMOS saturated)

\[
I_{DS} = k_n (V_{in} - V_{T0,n})^2 / 2 = k_n (V_{OH} - V_{T0,n})^2 / 2
\]

\[
\int_{t_0}^{t_1} dt = \frac{-2C_L}{k_n (V_{OH} - V_{T0,n})^2} \int_{V_{OH}}^{V_{OH} - V_{T0,n}} dV_{out}
\]

\[
t_1 - t_0 = \frac{2C_L V_{T0,n}}{k_n (V_{OH} - V_{T0,n})^2}
\]
Inverter Delay: Falling $t_2 - t_1$

- $V_{\text{out}}$ drops from $(V_{\text{OH}} - V_{T0,n})$ to $V_{\text{DD}}/2$
- NMOS in linear region

$$I_{DS} = k_n \left[ (V_{\text{OH}} - V_{T0,n})V_{\text{out}} - \frac{1}{2} V_{\text{out}}^2 \right]$$

$$t_2 - t_1 = -C_L \int_{V_{\text{OH}}-V_{T0,n}}^{(V_{\text{OH}}+V_{\text{OL}})/2} \frac{dV_{\text{out}}}{k_n \left[ (V_{\text{OH}} - V_{T0,n})V_{\text{out}} - \frac{1}{2} V_{\text{out}}^2 \right]}$$

$$t_2 - t_1 = \frac{C_L}{k_n (V_{\text{OH}} - V_{T0,n})} \ln \left[ \frac{2(V_{\text{OH}} - V_{T0,n}) - (V_{\text{OH}} + V_{\text{OL}})/2}{(V_{\text{OH}} + V_{\text{OL}})/2} \right]$$
Inverter Delay: Falling, Total

• Total fall delay = (t_1 - t_0) + (t_2 - t_1)

\[ t_{PHL} = \frac{C_L}{k_n(V_{OH} - V_{T0,n})} \left[ \frac{2V_{T0,n}}{V_{OH} - V_{T0,n}} + \ln \left( \frac{4(V_{OH} - V_{T0,n})}{V_{OH} + V_{OL}} - 1 \right) \right] \]
Inverter Delay: Rising

- Similar calculation as for falling delay
- Separate into regions where PMOS is in linear, saturation

\[
\begin{align*}
t_{PLH} &= \frac{C_L}{k_p (V_{OH} - V_{OL} - |V_{T0,p}|)} \left[ \frac{2|V_{T0,p}|}{V_{OH} - V_{OL} - |V_{T0,p}|} + \ln \left( \frac{4(V_{OH} - V_{OL} - |V_{T0,p}|)}{V_{OH} + V_{OL}} - 1 \right) \right] \\
&= \frac{2|V_{T0,p}|}{V_{OH} - V_{OL} - |V_{T0,p}|} + \ln \left( \frac{4(V_{OH} - V_{OL} - |V_{T0,p}|)}{V_{OH} + V_{OL}} - 1 \right)
\end{align*}
\]

- Note: to balance rise and fall delays (assuming \(V_{OH} = V_{DD}, V_{OL} = 0V, \) and \(V_{T0,n} = V_{T0,p}\)) requires

\[
\begin{align*}
\frac{k_p}{k_n} &= 1 \\
\frac{W}{L} &= \frac{\mu_n}{\mu_p} \approx 2.5
\end{align*}
\]
Inverter Rise, Fall Times

• Summary -- Exact method: separate into two regions

  – $t_1$
    - $V_{out}$ drops from $0.9V_{DD}$ to $V_{DD} - V_{T,n}$ (NMOS in saturation)
    - $V_{out}$ rises from $0.1V_{DD}$ to $|V_{T,p}|$ (PMOS in saturation)

  – $t_2$
    - $V_{out}$ drops from $V_{DD} - V_{T,n}$ to $0.1V_{DD}$ (NMOS in linear region)
    - $V_{out}$ rises from $|V_{T,p}|$ to $0.9V_{DD}$ (PMOS in linear region)

  – $t_{f,r} = t_1 + t_2$
CMOS Inverter Delay

- Review of approximate method
  - Assume a constant average current for the transition
  - $I_{avg} = \text{average of drain current at beginning and end of transition}$

\[
t_{PHL} = \frac{C_{load}}{I_{avg}} \left( V_{DD} - \frac{1}{2} V_{DD} \right)
\]

\[
t_{PLH} = \frac{C_{load}}{I_{avg}} \left( \frac{1}{2} V_{DD} - V_{SS} \right)
\]

$V_1 = V_{dd}$

$V_2 = \frac{1}{2} V_{dd}$

$I_{avg} = \frac{1}{2}(I_1 + I_2)$
CMOS Inverter Delay: 2nd Approximation

- Another approximate method:
  - Again assume constant $I_{\text{avg}}$
  - $I_{\text{avg}} = \text{current } I_1 \text{ at start of transition}$

$$t_{PHL} = \frac{C_{load} V_{DD}}{k_n (V_{DD} - V_{Tn})^2}$$

$$t_{PLH} = \frac{C_{load} V_{DD}}{k_p (V_{DD} - |V_{TP}|)^2}$$

- Why is this a good approximation (esp. for deep submicron)?
CMOS Inverter Delay: Finite Input Transitions

• What if input has finite rise/fall time?
  – Both transistors are on for some amount of time
  – Capacitor charge/discharge current is reduced

Empirical equations:

\[ t_{pHL}^{\text{actual}} = \sqrt{t_{pHL}^{\text{step}} + \left(\frac{t_r}{2}\right)^2} \]

\[ t_{plh}^{\text{actual}} = \sqrt{t_{plh}^{\text{step}} + \left(\frac{t_f}{2}\right)^2} \]
How to Improve Delay?

• Minimize load capacitances
  – Small interconnect capacitance
  – Small Cg of next stage

• Raise supply voltage
  – Increases current faster than increased swing $\Delta V$

• Increase transistor gain factor
  – Increase transistor drive current for charging/discharging output capacitance

• Use low threshold voltage devices
  – More subthreshold leakage power dissipation
Inverter Power Consumption

• **Static power consumption (ideal) = 0**
  
  – Actually DIBL (Drain-Induced Barrier Lowering), gate leakage, junction leakage are still present

• **Dynamic power consumption**

\[
P_{\text{avg}} = \frac{1}{T} \int_0^T v(t)i(t)\,dt
\]

\[
P_{\text{avg}} = \frac{1}{T} \left[ \int_0^{T/2} V_{\text{out}} \left( -C_{\text{load}} \frac{dV_{\text{out}}}{dt} \right) dt + \int_{T/2}^T (V_{DD} - V_{\text{out}}) \left( C_{\text{load}} \frac{dV_{\text{out}}}{dt} \right) dt \right]
\]

\[
P_{\text{avg}} = \frac{1}{T} \left[ \left( -C_{\text{load}} \frac{V_{\text{out}}^2}{2} \right) \bigg|_0^{T/2} + \left( V_{DD}V_{\text{out}}C_{\text{load}} - \frac{1}{2} C_{\text{load}}V_{\text{out}}^2 \right) \bigg|_{T/2}^T \right]
\]

\[
P_{\text{avg}} = \frac{1}{T} C_{\text{load}} V_{DD}^2 = C_{\text{load}} V_{DD}^2 f
\]
Next Time: Combinational Logic

• Combinational MOS Logic
  – DC Characteristics, Equivalent Inverter method
  – AC Characteristics, Switch Model