When to Quit for A New Job: Quickest Detection of Spectrum Opportunities in Multiple Channels

Qing Zhao, Jia Ye
Department of Electrical and Computer Engineering
University of California, Davis, CA 95616
Email: {qzhao,jiaye}@ucdavis.edu

***Abstract*—We consider quickest detection of idle/off periods in multiple on-off processes. We show that this problem presents a fresh twist to the classic signal processing problem of quickest change detection that considers only one stochastic process. In particular, we demonstrate that the key to quickest change detection in multiple processes is to abandon the current process when its state is unlikely to change in the near future (as indicated by the measurements obtained so far) and seek opportunities in a new process. This problem arises in spectrum opportunity detection in cognitive radio networks where a secondary user searches for idle channels in the spectrum.**

A Bayesian formulation of quickest change detection in multiple on-off processes with geometrically distributed busy and idle times is obtained within a decision-theoretic framework. Based on the structure of the resulting sequentially decision problem, we propose a low-complexity threshold policy for channel switching and change detection and demonstrate its superior performance over the single-channel approach.

***Index Terms*—Quickest change detection, opportunistic spectrum access, cognitive radio, spectrum opportunity detection.**

I. Introduction

The classic framework of quickest change detection dates back to 1931 [1]. In the conventional setting, the problem is to detect abrupt changes in the distribution of a *single* stochastic process. Specifically, it is assumed that the observations $X_1, X_2, \cdots, X_{T_0-1}$ are i.i.d. according to a distribution f_0. After a random change point T_0, the observations $X_{T_0}, X_{T_0+1}, \cdots$, are i.i.d. according to a different distribution f_1. The objective is to detect the change point T_0 as quickly as possible subject to a reliability constraint, *i.e.,* a constraint on the probability of false alarm (declaring a change before it occurs).

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In this paper, we formulate a new form of quickest change detection by considering multiple independent on-off processes. The objective is to catch as quickly as possible an idle/off period in any of the stochastic processes. This problem arises in cognitive radio systems for opportunistic spectrum access, where secondary users need to quickly and reliably detect channels temporarily unused by primary users [2]. The objective there is to detect, as soon as possible, whether the sensed channel has become idle, in order to maximize the transmission time before primary users reclaim the channel. The design constraint is on the maximum probability of declaring a busy channel as idle in order to limit the interference to primary users.

A. Main Results

We obtain a Bayesian formulation of quickest change detection in multiple on-off processes within a decision-theoretic framework. We demonstrate that the key to quickest change detection in multiple processes is to abandon the current process when its state is unlikely to change in the near future (as indicated by the measurements obtained so far) and seek opportunities in a new process. The caveat here is the lost measurements obtained in the abandoned process. Given that change detection relies on the accumulation of "evidence" (measurements), the switching rule needs to be carefully chosen. *An analogy is climbing the corporate ladder: when a long-waited promotion has yet to come, should one quit, abandon the established seniority, and look for a greener pasture?*

We address the optimal joint design of the switching rule and the detection rule. We consider geometrically distributed busy and idle times and establish a decision-theoretic framework for the Bayesian formulation of this problem. Based on the structure of the resulting sequential decision process, we propose a low-complexity threshold policy for channel switching and change de-

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tection. The threshold structure is with respect to the *a posterior* probability λ_t (given the whole observation history) that the process currently being observed is idle at time t. Specifically, the user should switch to a new channel when $\lambda_t \in [0, \eta_s)$, should continue observing the current channel when $\lambda_t \in [\eta_s, \eta_d)$, and should declare that the current channel is idle and start transmitting when $\lambda_t \in [\eta_d, 1]$, where η_s and η_d are, respectively, the switching and detection thresholds. In the proposed threshold policy, the switching threshold η_s is set to the average fraction of time that a channel is idle when the channel switching time is negligible. The detection threshold η_d is determined by the maximum allowable probability ζ of declaring a busy channel as idle, and setting $\eta_d = 1 - \zeta$ is asymptotically optimal as the interference constraint becomes more strict: $\zeta \to 0$.

B. Related Work

There are two standard Mathematical formulations of the classic quickest change detection in a single stochastic process: Bayesian and minimax. The Bayesian formulation assumes a prior distribution of the change point. The first optimal Bayesian change detection algorithm was developed by Shiryaev in 1961 [3], [4], where the change point is assumed to have a geometric/exponential distribution. In the context of opportunity detection, this implies that the connection time (channel "on" time) of the primary system is geometrically/exponentially distributed. Generalizations of Shiryaev's algorithm to arbitrary prior distributions of the change point have been studied (see, for example, [5], [6]).

The minimax formulation does not assume any prior distribution of the change point and aims to minimize the worst-case detection delay. A classic change detection algorithm under this formulation is the cumulative sum algorithm (CUSUM) first proposed by Page in 1954 [7] and later proven to be asymptotically optimal (as the maximum allowable probability of incorrect detection goes to zero) by Lorden in 1971 [8].

Recently, the classic CUSUM algorithm for change detection is applied directly to detect the *return* of primary users in a given *single* channel [9], [10]. In this paper, we exploit the presence of multiple channels in quickest detection of opportunities, which requires a new formulation of this classic signal processing problem. To our best knowledge, this work is the first that considers quickest change detection in multiple stochastic processes.

For an overview of opportunistic spectrum access in cognitive radio systems, readers are referred to [2].

II. Quickest Opportunity Detection in a Single Channel

In this section, we illustrate the problem of quickest opportunity detection by first considering a single channel. Shiryaev's algorithm is then presented.

A. Problem Formulation

As shown in Figure 1, suppose that sensing starts at $t = 0$, and the channel becomes idle at a random time $t = T_0$ unknown to the secondary user. The sensing measurements obtained before and after T_0 thus have different distributions. Specifically, the sensing measurements $\{X_1, X_2, \cdots, X_{T_0-1}\}$ before the change point T_0 are i.i.d. random variables with distribution $f_0(x)$, and the sensing measurements $\{X_{T_0}, X_{T_0+1}, \cdots\}$ after the change point T_0 are i.i.d. random variables with distribution $f_1(x)$. The time unit here is the secondary user's sampling period (the time for taking one channel measurement).

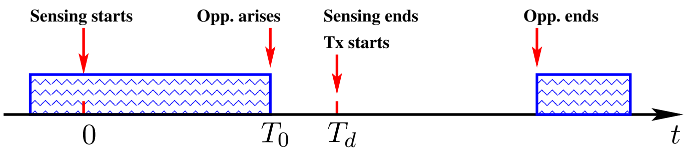

Fig. 1. Quickest detection of spectrum opportunities.

At each time instant t, the user aims to infer from measurements $\{X_1, X_2, \cdots, X_t\}$ whether a change in the channel state has occurred, *i.e.,* whether to start transmitting or to continue monitoring the channel and taking another measurement X_{t+1}.

Suppose that at time $t = T_d$, the user is convinced that an opportunity has arisen and proceeds to transmit. The problem of quickest opportunity detection can be formulated as choosing a *stopping rule* T_d under the following objective and constraint:

$$\min \mathbb{E}[(T_d - T_0)^+] \quad \text{subject to} \ \Pr[T_d < T_0] \le \zeta, \quad (1)$$

where $(T_d - T_0)^+ \triangleq \max\{0, T_d - T_0\}$, and $\mathbb{E}[(T_d - T_0)^+]$ represents the expected detection delay. The constraint in (1) is on the probability that the secondary user starts transmitting when the channel is still busy. It should be capped below an interference constraint ζ. Clearly, the optimal stopping rule T_d should strike a balance between detection delay and detection reliability.

B. Shiryaev's Algorithm for Quickest Change Detection

Shiryaev's algorithm for quickest change detection was developed within the Bayesian framework [3], [4], where the change point T_0 has a known geometric distribution. Specifically,

$$\Pr[T_0 = k] = p(1-p)^{k-1}(1-\lambda_0), \quad \forall k > 0,$$

where p is the parameter of the geometric distribution, and

$$\lambda_0 \stackrel{\Delta}{=} \Pr[T_0 = 0]$$

is the probability that change has already happened when we start to observe the process (*i.e.,* sensing starts during an idle period of the channel in the context of opportunistic spectrum access).

It has been shown by Shiryaev that a sufficient statistic for quickest change detection for geometrically/exponentially distributed change point is the *a posterior* probability λ_t that change has already occurred given the measurements obtained up to t ($t > 0$):

$$\lambda_t \stackrel{\Delta}{=} \Pr[T_0 \leq t | X_1, X_2, \cdots, X_t]. \tag{2}$$

Based on Bayes' rule, the sufficient statistic λ_t can be computed recursively at each time t using the new observation $X_t = x$ with the initial value λ_0.

Shiryaev's change detection algorithm is given by the following stopping rule on the *a posterior* probability λ_t.

$$T_d = \inf\{t : \lambda_t \geq \eta_d\}, \tag{3}$$

where the detection threshold η_d is determined by the reliability constraint ζ given in (1). Obtaining the detection threshold η_d in a closed-form is generally difficult. Setting $\eta_d = 1 - \zeta$ has been shown to be asymptotically optimal as the reliability constraint becomes more strict ($\zeta \to 0$).

In [5], [6], Shiryaev's algorithm has been shown to be asymptotically ($\zeta \to 0$) optimal when the change point has an arbitrary prior distribution.

III. Quickest Detection in Multiple Processes

In this section, we formulate the problem of quickest change detection in multiple on-off processes as a Partially Observable Markov Decision Process (POMDP). We then propose a low-complexity threshold policy for channel switching and change detection suggested by the structure of the resulting POMDP.

A. *Quickest Detection in Multiple On-Off Processes*

We consider a spectrum consisting of a large number of channels. In each channel, the channel usage of the primary users is an on-off process with alternating busy and idle periods. These on-off processes are stochastically independent and identical. Let $\{B_i\}_{i=-\infty}^{\infty}$ and $\{I_i\}_{i=-\infty}^{\infty}$ denote, respectively, the lengths of each busy and idle periods in a particular channel. We assume that the busy periods $\{B_i\}_{i=-\infty}^{\infty}$ have an identical geometric distribution with parameter p_B, and the idle periods $\{I_i\}_{i=-\infty}^{\infty}$ have an identical geometric distribution with parameter p_I. The average busy and idle times are denoted by $m_B = 1/p_B$ and $m_I = 1/p_I$ respectively. Let λ_0 denote the fraction of channel idle time. It is given by

$$\lambda_0 \stackrel{\Delta}{=} \frac{m_I}{m_B + m_I}. \tag{4}$$

A secondary user starts to sense a channel at $t = 0$. The objective is to catch an idle channel and start transmitting as quickly as possible subject to an interference constraint that caps the probability of transmitting over a busy channel below ζ. The user may switch to a different channel at any time. We assume that the number of channels is large enough so that the user can always switch to a channel that has not been visited. This is equivalent to the case that switching back to a channel is allowed but measurements obtained during previous visits to this channel are discarded. We also assume that channel switching time is negligible. The POMDP formulation and the threshold policy can be easily extended to general cases where channel switching costs τ_s units of time (see discussions in Sec. III-C).

Let L be the number of channels visited by the user before it declares, correctly or falsely, that an opportunity (an idle period) has arisen. It is a random variable depending on the switching and detection rules and the random observations in each channel. Let $T_s(l)$ ($l = 1, \cdots, L-1$) denote the time spent in the l-th channel before switching to the $(l+1)$-th channel. Let $T_d(L)$ denote the time spent in the last channel (the L-th channel) before declaring an opportunity. The problem of quickest change detection in multiple channels can be formulated as jointly choosing a sequence of switching rules $\{T_s(l)\}_{l=1}^{L-1}$ and a detection rule $T_d(L)$ under the following objective and constraint:

$$\begin{aligned} &\min \mathbb{E}[\textstyle\sum_{l=1}^{L-1} T_s(l) + T_d(L)] \\ &s.t. \quad \Pr[Z_L(\textstyle\sum_{l=1}^{L-1} T_s(l) + T_d(L)) = \text{busy}] \leq \zeta, \end{aligned} \tag{5}$$

where $\mathbb{E}[\sum_{l=1}^{L-1} T_s(l) + T_d(L)]$ represents the expected waiting time before catching an idle channel, and $Z_L(t)$ denotes the state of channel L at time t.

We can see from (5) that quickest change detection in multiple stochastic processes is fundamentally different from that in a single process, and is significantly more difficult in that a sequence of stopping rules $(T_s(1), T_s(2), \cdots, T_s(L-1), T_d(L))$ need to be designed.

B. A Decision-Theoretic Formulation

We now formulate the problem of quickest change detection in multiple on-off processes as a POMDP over a random horizon.

State Space: The underlying system can be in three states: 0, 1, and Δ, where 0 and 1 indicate, respectively, that the current process is busy and idle, Δ is an absorbing state, indicating the end of the decision horizon.

Action Space: There are three actions: S (switch and take a measurement in a new process), C (continue taking measurements in the current process), and D (declare that a change has already happened in the current process, *i.e.,* the current process is idle).

State Transition: The transition probabilities under all possible actions are given in Table I, where Z_t is the state of the process being considered at time t.

TABLE I
STATE TRANSITION PROBABILITIES

Z_t \ Z_{t+1}	0			1			Δ		
	S	C	D	S	C	D	S	C	D
0	$1-\lambda_0$	$1-p_B$	0	λ_0	p_B	0	0	0	1
1	$1-\lambda_0$	p_I	0	λ_0	$1-p_I$	0	0	0	1
Δ	0	0	0	0	0	0	1	1	1

Observation Model: The observation at time t is X_t under actions S and C. The distribution of X_t is given by either $f_0(x)$ or $f_1(x)$ depending on the current state Z_t. Under action D, no observations are available.

Cost: The cost structure is given in Table II, where the cost γ for declaring a busy channel as idle models the tradeoff between detection delay and detection reliability. It is set to satisfy the interference constraint ζ given in (5). All other costs models the delay in catching an idle channel. Note that it is not necessary to specify the value of γ based on ζ. As shown in Sec. III-C, the optimal detection rule is specified by a detection threshold chosen to satisfy the interference constraint ζ.

TABLE II
THE COST STRUCTURE

Z_t \ A_t	S	C	D
0	1	1	γ
1	1	1	0
Δ	0	0	0

The objective is to choose actions sequentially in time to minimize the expected total cost over an infinite horizon, or equivalently, over a random horizon defined by the hitting time of the absorbing state Δ. It is clear from the cost structure that the expected total cost is the expected delay in catching an idle channel.

A Sufficient Statistic: the Information State Since the underlying system state Z_t is not directly observable from the measurements $\{X_t\}$, what we have here is a POMDP. From the fundamental theory of stochastic control, we know that a sufficient statistic for choosing the optimal action at each time is the information state or the belief value: the *a posterior* probability λ_t that $Z_t = 1$ (the current process is idle) given the measurements obtained up to t. As discussed in the previous section, the same statement was obtained by Shiryaev for quickest change detection in a single process.

It is easy to see that the information state λ_t has the following recursive update depending on the action $a(t-1)$ and the observation $X(t)$.

$$\lambda_t = \begin{cases} \mathcal{T}(\lambda_0|x) & a(t-1) = \text{S}, X_t = x \\ \mathcal{T}(\lambda_{t-1}|x) & a(t-1) = \text{C}, X_t = x \end{cases}, \tag{6}$$

where $\mathcal{T}(\lambda|x)$ denotes the updated information state based on a new measurement x. Let $\bar{p} \triangleq 1 - p$ for $p \in [0,1]$. We obtain $\mathcal{T}(\lambda|x)$ as follows.

$$\mathcal{T}(\lambda|x) \triangleq \frac{(\lambda\bar{p}_I + \bar{\lambda}p_B)f_1(x)}{(\lambda\bar{p}_I + \bar{\lambda}p_B)f_1(x) + (\lambda p_I + \bar{\lambda}\bar{p}_B)f_0(x)}. \tag{7}$$

A channel switching and change detection policy π specifies a function that maps an information state $\lambda_t \in [0,1]$ to an action $a(t) \in \{\text{S, C, D}\}$ for each time t. Quickest change detection in multiple on-off processes can thus be formulated as the following stochastic control problem:

$$\pi^* = \arg\min_{\pi} \mathbb{E}_\pi[\sum_{t=0}^{\infty} R_{\pi(\lambda_t)} | \lambda_0 = \frac{m_I}{m_B + m_I}], \tag{8}$$

where $\pi(\lambda_t)$ is the action specified by policy π in information state λ_t, and $R_{\pi(\lambda_t)}$ is the cost incurred under this action and can be easily obtained from Table II by averaging over the two possible values of Z_t with λ_t.

C. Quickest Change Detection: A Threshold Policy

Referred to as the value function, $V(\lambda_t)$ denotes the minimum expected total remaining cost when the current information state is λ_t. It specifies the performance of the optimal policy π^* starting from the information state λ_t. Let $V_S(\lambda_t)$ denote the expected total remaining cost when we take action S at the current time and then follow the optimal policy π^*. Let $V_C(\lambda_t)$ and $V_D(\lambda_t)$ be similarly defined. We thus have

$$V(\lambda_t) = \min\{V_S(\lambda_t),\ V_C(\lambda_t),\ V_D(\lambda_t)\}. \tag{9}$$

From the cost structure, we obtain the following.

$$\begin{aligned} V_S(\lambda_t) &= 1 + \int_x P(x;\lambda_0)V(\mathcal{T}(\lambda_0|x))\mathrm{d}x, \\ V_C(\lambda_t) &= 1 + \int_x P(x;\lambda_t)V(\mathcal{T}(\lambda_t|x))\mathrm{d}x, \\ V_D(\lambda_t) &= (1-\lambda_t)\gamma. \end{aligned} \tag{10}$$

where

$$P(x;\lambda) = (\lambda\bar{p}_I + \bar{\lambda}p_B)f_1(x) + (\lambda p_I + \bar{\lambda}\bar{p}_B)f_0(x)$$

is the probability of observing x when the process has probability λ to be idle. It is easy to see that

$$V_S(\lambda_t) = V_C(\lambda_0)$$

and is independent of λ_t. Furthermore, $V_D(\lambda_t)$ is linearly decreasing with λ_t. The following lemma specifies the properties of $V_C(\lambda_t)$ that are key to the proposed threshold policy.

Lemma 1: $V_C(\lambda_t)$ is concave for $\lambda_t \in [0,1]$. It thus has at most two intersecting points with $V_S(\lambda_t)$, one of which is at $\lambda_t = \lambda_0$. It has only one intersecting point with $V_D(\lambda_t)$ denoted as η_d that satisfies $\eta_d \geq \lambda_0$.

proof: Omitted due to space limit.

One possibility of the relationship among $V_C(\lambda_t)$, $V_S(\lambda_t)$, and $V_D(\lambda_t)$ is illustrated in Fig. 2 where $V_C(\lambda_t)$ and $V_S(\lambda_t)$ intersects only at $\lambda_t = \lambda_0$. This scenario suggests the following simple threshold policy for channel switching and change detection. Specifically, the policy π^* is given by two thresholds η_s and $\eta_d \in (\eta_s, 1]$ as given below:

$$\pi^*(\lambda_t) = \begin{cases} \text{S}, & \lambda_t \in [0,\eta_s) \\ \text{C}, & \lambda_t \in [\eta_s,\eta_d) \\ \text{D}, & \lambda_t \in [\eta_d,1] \end{cases}. \tag{11}$$

The switching threshold $\eta_s = \lambda_0 = \frac{m_I}{m_B+m_I}$, the fraction of the idle time, and setting the detection threshold $\eta_d = 1-\zeta$ is asymptotically optimal as $\zeta \to 0$.

This simple threshold policy agrees with our intuition: switch to a new channel when the prospect of catching an opportunity in a new channel is better than staying in the current channel (*i.e.,* $\lambda_t \leq \lambda_0$). As demonstrated in Sec. IV, this simple threshold policy offers significant reduction in the detection delay over the single-channel strategy that employs the optimal change detection rule in a single channel. We are currently investigating the optimality of this threshold policy.

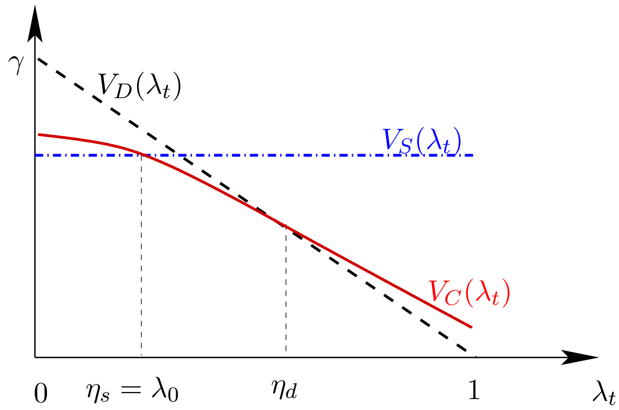

Fig. 2. A threshold structure of the resulting POMDP.

We point out that for the general case with an arbitrary channel switching time τ_s, we have $V_S(\lambda_t) = \tau_s + V_C(\lambda_0)$, still independent of λ_t. We can thus employ a similar threshold policy with a properly chosen switching threshold η_s. As can be seen from Fig. 2, the switching threshold η_s should be smaller than λ_0 when $\tau_s > 0$.

IV. Simulation Examples

In this section, we demonstrate the performance of the proposed threshold policy for the joint design of channel switching and change detection. The primary signals are modeled as Gaussian signals in Guassian noise, *i.e.,* $f_0(x)$ and $f_1(x)$ are both Gaussian distributions with zero mean and different variances. The signal to noise ratio is assumed to be $10dB$. In all these examples, we use the asymptotically optimal value of the detection threshold: $\eta_d = 1-\zeta$.

Shown in Fig. 3 is the expected time to catch an opportunity as a function of λ_0, the fraction of idle time in each channel. Compared to the strategy that stays in a single channel and uses Shiryaev's algorithm, the proposed threshold policy offers significant improvement for a large range of λ_0. When λ_0 is close to 1, these two strategies have comparable performance. This is because when λ_0 is close to 1, the first channel that the user observes is already in the idle state with high probability.

Fig. 4 illustrates the expected number of channels visited under the multi-channel detection strategy as a function of λ_0. It shows that the user only needs to visit a small number of channels, yet achieves significant reduction in detection time.

In Fig. 5, we compare the single-channel and multi-channel strategies for different on-off processes. Specif-

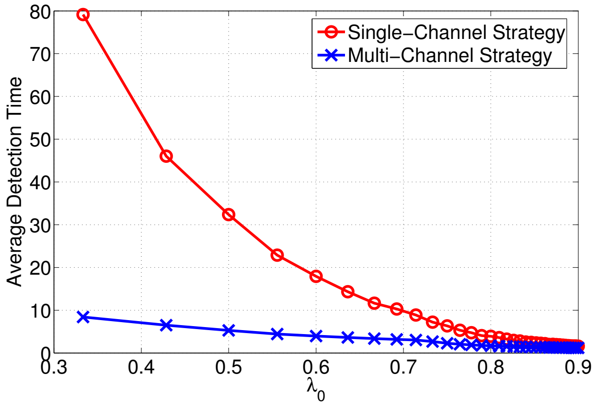

Fig. 3. Average detection time vs. the fraction λ_0 of channel idle time. ($p_I = 0.02,\ p_B = 0.01 : 0.005 : 0.2,\ \zeta = 0.1,$)

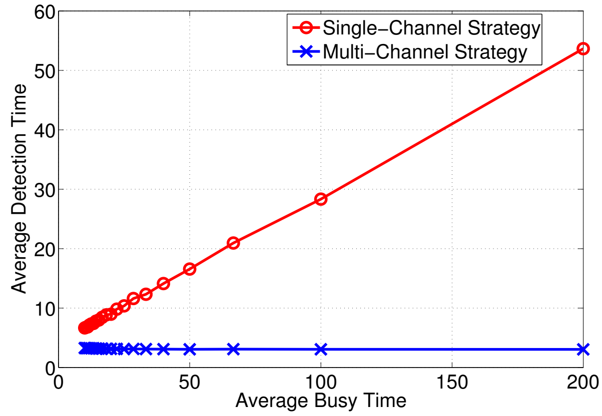

Fig. 5. Average detection time vs. the average busy time m_B. ($\lambda_0 = 0.7,\ p_B = 0.005 : 0.005 : 0.1,\ p_I = p_B \frac{1-\lambda_0}{\lambda_0}$)

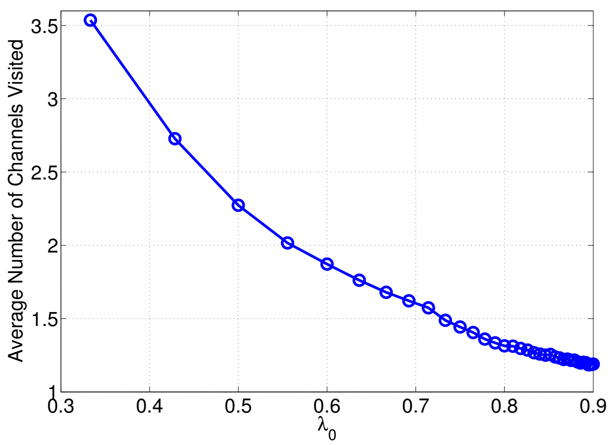

Fig. 4. Average number of channel visited vs. the fraction λ_0 of channel idle time. ($p_I = 0.02,\ p_B = 0.01 : 0.005 : 0.2,\ \zeta = 0.1$)

ically, we increase both the average busy time m_B and the average idle time m_I while keeping the fraction λ_0 of idle time unchanged. In this case, we observe that the average detection time of the single-channel strategy increases linearly with m_B, as suggested by our intuition. On the other hand, the multi-channel strategy can maintain the same small average detection time regardless of the increase in the length of busy periods in every channel. The performance improvement is thus dramatic when the average busy time is large. This is due to the channel switching strategy that avoids large realizations of busy time and fully exploits the presence of multiple channels.

V. Conclusion

In this paper, we have addressed quickest detection of spectrum opportunities in cognitive radio networks where a secondary user searches for idle channels in the spectrum. A Bayesian formulation of quickest change detection in multiple independent on-off processes has been obtained within a decision-theoretic framework. Based on the structure of the resulting POMDP, we proposed a low-complexity threshold policy for the joint design of the switching rule and the detection rule and demonstrated its superior performance over the single-channel approach.

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